High Energy Physics and Nuclear Physics
Volume 13, No. 1

Multipole Giant Resonances in Highly Excited Nuclei

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The isoscalar giant surface resonance and giant dipole resonance of highly excited nuclei are discussed. Excitation energies of the giant modes in ^{208}Pb are calculated in a simplified model, using the concept of energy weighted sum rule (EWSR), and the extended Thomas-Fermi approximation at finite temperature is employed to describe the finite temperature equilibrium state. It shows that EWSR and the energy of the resonance depend weakly on the temperature of the system. The weak dependence is analyzed.

1. INTRODUCTION

Preliminary evidences of giant resonances based on states with high excitation energy or high spin have been established in the last decade in heavy-ion fusions and deep inelastic reactions, whose energy locations and widths are comparable to those of the giant resonance based on the ground state (nuclear temperature T = 0 MeV) [1]. Many theoretical approaches have been developed to study the nuclear dynamics far from the ground state and to describe the properties, e.g. the energy, width as well as the strength distribution of these highly excited collective modes and their dependence on nuclear temperature T, total spin I and deformation. These developments associated with the experimental efforts have opened up a new field for the investigation of nuclear spectroscopy.

Since the nuclear compound system formed in heavy-ion collisions has high intrinsic excitation energy, and the time required to establish thermal equilibrium within a nucleus appears to be small compared to the de-excitation time, it is reasonable to introduce the statistical concept. Thus, the properties of these highly excited states are to be described by means of a statistical ensemble consisting of all nuclei with the same excitation energy. The use of the statistic approach will simplify calculations. First, one may obtain the equilibrium distribution of the quasiparticle system at finite

Received on April 15, 1987.

temperature in the mean-field approximation and then study the response of this system to the perturbation of a time dependent external field [2]. Some RPA calculations have shown that until $T \sim 6$ MeV, the dependence of isovector dipole strength function on temperature is very weak, but the isoscalar strength function, due to the smooth Fermi surface, is more affected by temperature in the lower energy region [3]. Barranco et al. [4] discussed the giant quadrupole and octupole resonances of the spherical nuclei from ^{40}Ca to ^{208}Pb by using an HF + RPA sum rule to calculate the first order moment m_1 and the third order moment m_3 of energy. They found that the energy location of resonances drops down gradually with the increase of temperature, which is more remarkable for the cases of heavy nuclei. Di. Toro et al. [5] used the quantum phase space method to study the properties of giant resonances, which greatly simplified the calculation. We have already indicated in Ref. [6] that the extended Thomas-Fermi model at finite temperature (TETF) could be well employed to treat the phase transition of nuclear matter. The goal of this paper is to further apply the TETF approximation to evaluate the sum rule as well as the energies of nuclear multipole resonances at different temperatures.

2. PARTICLE-HOLE RESPONSE FUNCTION

The response of the nuclear many-body system to an external field can be realized through the excitation of quasiparticles or collective modes [7]. Due to the residual interaction, the nuclear system will respond to the perturbation of the external field $F_{\lambda\mu}$ which couples weakly to it, resulting in a collective mode excitation, i.e., a correlated superposition of particle-hole excitations. The collective phonon energy $\hbar\omega_\lambda$ satisfies the following dispersion relation:

$$\frac{2\lambda+1}{2K_{\lambda,\tau}} = \sum_{ki} \frac{|\langle k\|F_{\lambda,\tau}(\boldsymbol{r})\|i\rangle|^2 \cdot (\varepsilon_k - \varepsilon_i)}{(\varepsilon_k-\varepsilon_i)^2 - (\hbar\omega_\lambda)^2} \tag{1}$$

where ε_k and ε_i are the Hartree-Fock single particle energies for particles and holes, respectively.

Using the self-consistency condition between the variation of the nuclear density $\delta\rho$ and the variation of the average potential δV, one can evaluate the coupling constants K_λ for the isoscalar as well as the isovector density oscillation from

$$K_{\lambda,\tau=0} = \frac{4\pi M\omega_0^2}{A(2\lambda+1)\langle r^{2\lambda-2}\rangle}, \tag{2}$$

$$K_{\lambda,\tau=1} = -\frac{\pi V_1}{A\langle r^{2\lambda}\rangle}, \tag{3}$$

where $\hbar\omega_0$ is the energy spacing of the harmonic oscillator major shells, $V_1 \approx -130$ MeV is the symmetry potential, and A the nucleon number.

The energy weighted sum rule(EWSR) associated with the transition operator F is defined as the expectation value of its double commutator with the Hamiltonian H in the ground state, i.e.

$$S(F) \equiv \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle = \sum_i (E_i - E_0)B(F;0\rightarrow i), \tag{4}$$

where $B(F;0\rightarrow i)$ is the reduced transition probability of the field operator F. Summing over the contribution of all states i that can be reached when F operates on the ground state 0, one obtains from Eq.(4) the following electric multipole sum rule

$$S(E\lambda) = \frac{(2\lambda + 1)^2 \cdot \lambda}{4\pi} \cdot \frac{\hbar^2 Z e^2}{2M} \langle r^{2\lambda-2} \rangle_{prot.} \quad (5)$$

Similarly, for the isoscalar and the isovector multipole sum rule, we get

$$S(\lambda, \tau = 0) = S(\lambda, \tau = 1) = \frac{(2\lambda + 1)^2 \cdot \lambda}{4\pi} \cdot \frac{A\hbar^2}{2M} \langle r^{2\lambda-2} \rangle. \quad (6)$$

which can be used to simplify further the dispersion relation (1). Assuming that the particles are moving in a harmonic oscillator potential without spin-orbit coupling, all the particle-hole excitation energies determined by ΔN degenerate at $\varepsilon_k - \varepsilon_i = \varepsilon_{ph}$. The expression of the dispersion relation (1) can now be written as

$$\frac{2\lambda + 1}{2K_{\lambda,\tau}} = \frac{S(\lambda, \tau)}{\varepsilon_{ph}^2 - (\hbar\omega_\lambda)^2}, \quad (7)$$

The expectation value in Eqs.(2), (3), (5) and (6) is defined for any radial function $g(r)$ as

$$\langle g(r) \rangle = \int_0^\infty \rho_0(r) g(r) r^2 dr \Big/ \int_0^\infty \rho_0(r) r^2 dr \quad (8)$$

where $\rho_0(r)$ is the nuclear density distribution in the ground state.

3. THOMAS-FERMI APPROXIMATION AT FINITE TEMPERATURE

At finite temperature, instead of a unique state with well-defined energy a statistical ensemble will be used, as mentioned in Sect.1. Assuming that the system is in equilibrium at temperature T, the probability of finding the system in the eigenstate $|m\rangle$ of the Hamiltonian, with energy E_m, is

$$W(m, N) = \frac{e^{-\beta(E_m - \varepsilon_F N)}}{Z} \quad (9)$$

where

$$Z = \sum_m e^{-\beta(E_m - \varepsilon_F N)}, \quad \beta = \frac{1}{T} \quad (10)$$

In this way, the electric multipole transition sum rule at temperature T can be written as

$$S(E\lambda, \lambda) = \frac{(2\lambda + 1)^2 \cdot \lambda}{4\pi} \cdot \frac{\hbar^2 Z e^2}{2M} \sum_m W(m, N) \langle m | r^{2\lambda-2} | m \rangle \quad (11)$$

which obviously depends only on the shape and the size of the nucleus. Thus, to the extent that the temperature T is low compared with the energy spacing between major shells ($T < 5$ MeV), in analogy to Eg.(5) the multipole sum rule $S(E\lambda, T)$ at the given temperature T reads

$$S(E\lambda, T) = \frac{(2\lambda + 1)^2 \cdot \lambda}{4\pi} \cdot \frac{\hbar^2 Z e^2}{2M} \langle r^{2\lambda-2} \rangle_{prot, T \neq 0} \quad (12)$$

Making use of the definition of average values at temperature $T \neq 0$, we get the coupling constants and the multipole sum rule directly from Eqs. (2), (3) and (6), both for the isoscalar and the isovector field,

$$K_{\lambda,\tau=0}(T) = \frac{4\pi M\,\omega_0(T)^2}{A(2\lambda+1)\langle r^{2\lambda-2}\rangle_T} \tag{13}$$

$$K_{\lambda,\tau=1}(T) = -\frac{\pi V_1}{A\langle r^{2\lambda}\rangle_T} \tag{14}$$

$$S(\lambda,\tau,T) = \frac{(2\lambda+1)^2-\lambda}{4\pi}\cdot\frac{A\hbar^2}{2M}\langle r^{2\lambda-2}\rangle_T \tag{15}$$

where the oscillator energy $\hbar\omega_0(T)$ depends also on the temperature. The average values of the T-dependent radial function have a similar form to Eq.(8):

$$\langle g(r)\rangle_T = \int_0^\infty \rho_T(r)g(r)r^2dr \Big/ \int_0^\infty \rho_T(r)r^2dr \tag{16}$$

where $\rho_T(r)$ is the density distribution of the system at the assigned temperature T.

We follow the extended Thomas-Fermi approximation at finite temperature (TETF) [8] to evaluate the nuclear density distribution $\rho_T(r)$ at each assigned T. The functional of Hamiltonian (see Ref. (9)), which has already been used to fit bulk properties of a large number of stable nuclei, reads

$$\begin{aligned}\mathscr{H}[\rho] = E_{\text{nucl}}[\rho] + \frac{\hbar^2}{8m}\xi\{(\nabla\rho)^2 + \theta[\nabla(\rho_n-\rho_p)]^2\} \\ + \frac{e}{2}\rho_p\cdot V_c - \frac{3}{4}\left(\frac{3}{\pi}\right)^{1/3} e^2\rho_p^{4/3}\end{aligned} \tag{17}$$

where $E_{\text{nucl}}[\rho]$ is the energy density of nuclear matter which is treated as a Fermi gas system with an interaction potential $V[\rho]$. Eq.(17) also contains the Coulomb potential terms and terms related to the fact that nucleus has a finite size. Assuming a Fermi-type nuclear density distribution

$$\rho_T(r) = \frac{\rho_{0T}}{1+e^{(r-R_T)/a_T}} \tag{18}$$

a variational calculation on the free energy $\mathscr{F} = \mathscr{H} - TS$ is carried out, resulting in an equilibrium density distribution with $\mathscr{F}$ at a minimal value. The details of the calculation can be found in Ref. 6.

4. RESULTS AND DISCUSSIONS

We evaluated the electric multipole sum rules of ^{208}Pb by using Eq.(12). Fig.1 displays the results of the multipole excitation for $\lambda = 2$ and $\lambda = 3$ as a function of temperature T. It can be seen from the figure that the sum rule is essentially independent of T as long as the temperature is small ($T = 2 \sim 3$ MeV) compared with the energy differences between major shells ($\sim 41A^{1/3}$ MeV). The result is strikingly similar to that obtained from the HF approach for the equilibrium density distribution at finite temperature [3]. It is noticed that the deviation of $S(E\lambda, T)$ for $\lambda = 3$ and T = 4 MeV from the zero-temperature value $S(E\lambda, T = 0)$ reaches about 14%, while the deviation is much smaller in the case of $\lambda = 2$. This is in good agreement with the result from the RPA sum rule calculations with SKM interaction [4]. It is well known that the smooth T-dependence of the sum rule is related to the Fermi type distribution of the occupation probability of the orbits at finite temperature, since the number of the excited particle-hole pairs, which might participate in the transition and contribute to the sum rule, increases. In the Thomas-Fermi approach, the temperature dependence of the nuclear density reflects the variations of both the occupation probability and the radial wave function with temperature, although the occupation probability is not calculated directly. This is clearly shown by the fact that the diffused edge of the calculated density distribution extends as the temperature increases.

The straightforward extension of the simplified dispersion relation (7) to the finite temperature gives

$$\frac{2\lambda + 1}{2\kappa_{\lambda,\tau}(T)} = \frac{S(\lambda,\tau,T)}{\varepsilon_{ph}^2 - [\hbar\omega_\lambda(T)]^2} \tag{19}$$

Making use of Eqs.(13) and (15) for $\lambda = 2$, $\Delta N = 2$ with a degenerate energy $\varepsilon_{ph} = 2\hbar\omega_0(T)$, we have

$$\hbar\omega_2(T) = \sqrt{2}\,\hbar\omega_0(T) \tag{20}$$

In the octupole case of $\lambda = 3$, there are two particle-hole excitation degenerate energies $\varepsilon_{ph} = \hbar\omega_0(T)$ and $\varepsilon_{ph} = 3\hbar\omega_0(T)$ associated with $\Delta N = 1$ and $\Delta N = 3$, respectively. Two solutions of the resonance energy are obtained from Eq. (19). One corresponds to a low-lying octupole oscillation with zero energy due to the crude approximation in our approach, and the other corresponds to a high octupole resonance with energy at

$$\hbar\omega_3(T) = \sqrt{7}\,\hbar\omega_0(T) \tag{21}$$

From the expression [10] of the distance between the harmonic oscillator major shells, i.e.

$$\hbar\omega_0(T) = \frac{\hbar^2}{2M}\frac{1}{\langle r^2\rangle_T}\left(\frac{3}{2}\right)^{4/3} A^{1/3} \tag{22}$$

also considering the temperature dependence, along with Eqs.(20) and (21), we get

$$\hbar\omega_2(T) = 50.6A^{1/3}/\langle r^2\rangle_T \tag{23}$$

$$\hbar\omega_3(T) = 94.2A^{1/3}/\langle r^2\rangle_T \tag{24}$$

In the case of $\lambda = 1, \tau = 1$ dipole resonance with the particle-hole excitation degenerate energy of $\varepsilon_{ph} = \hbar\omega_0(T)$, we can also get, from Eqs.(14), (15) and (19), the following T-dependent expression of the resonance energy

$$\hbar\omega_1(T) = \frac{35.6A^{1/3}}{\langle r^2\rangle_T}\left[1 + \frac{3.19\langle r^2\rangle_T}{A^{2/3}}\right]^{1/2} \tag{25}$$

TABLE 1
Giant Resonance Energy for the Isovector Dipole, Isoscalar Quadrupole and Octupole Resonances in ^{208}Pb as a Function of Temperature T (unit: MeV)

T(MeV)	0	1	2	3	4	5
$\hbar\omega_1(T)$	13.54	13.51	13.42	13.25	13.01	12.59
$\hbar\omega_2(T)$	9.97	9.93	9.81	9.62	9.35	8.88
$\hbar\omega_3(T)$	18.55	18.49	18.27	17.92	17.41	16.52

The calculations of resonance energies have been performed in the case of ^{208}Pb. The results are collected in Table 1. Obviously, the energies are less sensitive to T.

Here we would like to stress some argument for our results. We have started from the particle-hole response function to evaluate the resonance energies. This is valid as far as the temperature is small compared with the distance between major shells, which is $\hbar\omega_0 \sim 7$ MeV for ^{208}Pb. The main feature of our calculation is that we consider not only the variation of the occupation probability with T, which leads to an increase in the number of the particle-hole excitation configurations included in the transition (resulting in the increase of the sum rule with T), but also the dependence of coupling constants $K_{\lambda\tau}$ on T, i.e., the decrease of the residual interaction with the temperature T. Cancellation will take place between these effects. The reason for a weak T-dependence of the giant resonance energies can therefore be understood in terms of the compensation of a weakening of the effective residual interaction and an increasing in the number of the particle-hole configurations. In particular, the surface resonance energies are dominated by the variation of the oscillator energy $\hbar\omega_0$ with T.

To conclude this section we would like to give some discussions on the limitation of the semiclassical description of the T-dependent nuclear features. We realize that the functionals in the semiclassical Thomas-Fermi approach (such as the kinetic energy density functional $\tau(\rho)$ and the entropy density functional $S(\rho)$) appear to be the average part of the functionals calculated from the Skyrme-HF approach. The Skyrme-HF type functional of density can be expanded up to the order of $\hbar^2$ and expressed as

$$\begin{aligned} \tau(\rho) &= \tau_{TF}(\rho) + \tau_2(\rho) \\ S(\rho) &= S_{TF}(\rho) + S_2(\rho) \end{aligned} \tag{26}$$

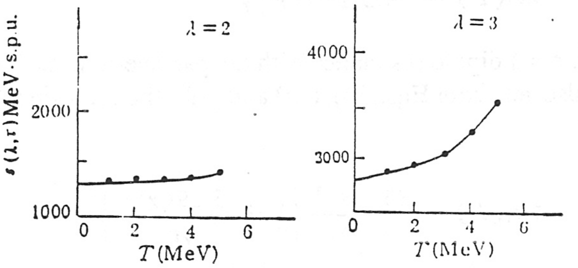

FIGURE 1 Total electric sum rule for the multipolarities $\lambda = 2$ and $\lambda = 3$ of ^{208}Pb as a function of temperature T.

with the well-known Thomas-Fermi terms $\tau_{TF}(\rho)$ and $S_{TF}(\rho)$ and the gradient correction terms $\tau_2(\rho)$ and $S_2(\rho)$ which are related to the finite nuclear size. In the TETF approximation used in the present work, we choose the TF functional $\tau_{TF}^{T\neq 0}(\rho)$ and $S_{TF}^{T\neq 0}(\rho)$, together with the gradient corrections $\tau_2(\rho)$ and $S_2(\rho)$ known from the $T = 0$ case (for $T \to 0$, $S_2(\rho) \to 0$). This is the so-called "cold" gradient approximation. Although it fails to yield reasonable average deformation energies at $T > 0$, this approximation leads to fairly good results for spherical systems such as ^{208}Pb [11].

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