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Study of Collective Spectrum for ^{157,159}Tb and ^{155,157}Gd

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A microscopic approach for studying collective states of odd-mass nuclei is proposed in terms of the interacting boson-fermion model. The model Hamiltonian is derived and used for calculating the spectrum of ^{157,159}Tb and ^{155,157}Gd isotopes. As a result, qualitative agreement between the numerical results and the experimental data is obtained.

1. INTRODUCTION

In the past years, several schemes were proposed to investigate the microscopic foundation of the interacting boson model (IBM)[1]. One of the schemes, which was based on the boson expansion theory and the so-called modified Jancovici-Schiff substitution, was described in detail in Refs.[2] and [3]. By using this scheme, preliminary progress on the investigations of Sm, Er and Os nuclei was obtained [3]. In order to test this scheme, further studies in various aspects should be made, for instance, extending this scheme to describe more collective properties of nuclei. In the interacting boson-fermion model (IBFM)[4], which is a natural extension of the interacting boson model, an odd-*A* nucleus is treated as a system where the even *s-d* core couples with odd fermions. In this area, a number of phenomenological investigations have been done. As an attempt at the investigation of the IBFM microscopic foundation, in this paper, this microscopic scheme is extended by introducing additional degrees of freedom of odd fermions to study, the collective states in odd-*A* nuclei intensively.

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In the microscopic IBM approach, one changes the original valence nucleon description to the ideal boson description by means of the boson expansion theory. After defining the s and d boson operators, one can truncate the collective state subspace [2,3]. Since each boson carries two units of valence nucleon number, for the odd nucleon system, it is natural to use the boson expansion theory [5,6]. Based on the assumption of the IBFM, our procedure is the following: In terms of the generalized Dyson boson expansion[6], we transform the valence nucleon description of the odd-A nucleus into the ideal boson-fermion description, then, we define the s and d bosons by using the same method employed for the double-even nucleus[2], take the equivalent subspace[6] of the single fermion as the s-d truncated space, namely the truncated collective state space, derive the effective operator of the ideal boson-fermion Hamiltonian in the s-d subspace, which is just the IBFM-2 Hamiltonian, and study the low-lying collective properties of the nucleus.

In the following section, we mainly discuss the outline of the scheme for the odd-A nucleus and no longer give the similar content for the even-even nucleus, as they can be found in Refs.[2] and [3].

In the third section, the deformed nuclei ^{157,159}Tb and ^{157,159}Gb are studied, the numerical method is discussed, and the results are given. Because of high dimensions of the state space, it is difficult to diagonalize the IBFM Hamiltonian numerically. Alternatively, an analytic approximation method developed in our earlier work [7] is adopted, i.e., the intrinsic wave functions of rotational bands are constructed, then, by means of self-consistent cranking calculation, the moments of inertia are determined and the excited spectra are computed. The calculated results are discussed and compared with the experimental data. Finally, concluding remarks are given in Section Four.

2. OUTLINE OF THE SCHEME

Let us define $a_\alpha^{(\sigma)+}$ and $a_\alpha^{(\sigma)}$ as the creation and annihilation operators of valence nucleons, where the index σ = n(p) specifies the neutron (proton) and the subscript $\alpha \equiv$ (im) denotes the quantum numbers (nljm) of single particle states where the single particle is bound onto the major shell. The symbol $|0\rangle$ represents the vacuum state of valence nucleons, describing the closed major shell. Then, we can express an arbitrary state vector of valence nucleons

$$|\psi\rangle = \Sigma c\, a_{\alpha_1}^{(n)+} \cdots a_{\alpha_x}^{(n)+} a_{\beta_1}^{(p)+} \cdots a_{\beta_{x'}}^{(p)+} |0\rangle. \tag{1}$$

For definiteness, we treat the odd-n nucleus in which the neutron number x is odd and the proton number x' is even. The valence nucleon Hamiltonian with two-body effective interaction has the following general form

$$H_f = \sum_\sigma \left\{ \sum_\alpha E_\alpha^{(\sigma)} a_\alpha^{(\sigma)+} a_\alpha^{(\sigma)} + \sum_{\alpha\beta\gamma\delta} P_{\alpha\beta\gamma\delta}^{(\sigma)} a_\alpha^{(\sigma)+} a_\beta^{(\sigma)+} a_\gamma^{(\sigma)} a_\delta^{(\sigma)} \right\} + \sum_{\alpha\beta\gamma\delta} P_{\alpha\beta\gamma\delta}^{(np)} a_\alpha^{(p)+} a_\beta^{(n)+} a_\gamma^{(p)} a_\delta^{(n)}, \tag{2}$$

where $P_{\alpha\beta\gamma\delta}$ are matrix elements of the interaction.

According to the IBFM theory, we first transform the valence nucleon description into the boson-fermion description in terms of the ideal boson operators $A_{\alpha\beta}^+$ and $A_{\alpha\beta}$ and fermion operators η_a^+ and η_α[6]. These operators satisfy the following relations:

$$A_{\alpha\beta} = -A_{\beta\alpha}, \quad [A_{\alpha\beta}^+, A_{\alpha'\beta'}^+] = 0,$$
$$[A_{\alpha\beta}, A_{\alpha'\beta'}^+] = \delta_{\alpha\alpha'}\delta_{\beta\beta'} - \delta_{\alpha\beta'}\delta_{\beta\alpha'},$$

$$\{\eta_\alpha^+, \eta_{\alpha'}^+\} = 0, \quad \{\eta_\alpha, \eta_{\alpha'}^+\} = \delta_{\alpha\alpha'}, \tag{3}$$
$$[A_{\alpha\beta}, \eta_\gamma] = [A_{\alpha\beta}, \eta_\gamma^+] = 0.$$

Then, by performing the generalized Dyson expansion and using the operator U [6], we can map the nuclear state vector $|\psi\rangle$ in Eq.(1) onto the generalized state vector $|\psi)$

$$|\psi) = U|\psi\rangle = U_n U_p |\psi\rangle, \tag{4}$$

$$U_\sigma = \langle 0| e^{\frac{1}{2}\sum_{\alpha\beta} A_{\alpha\beta}^{(\sigma)+} a_\beta^{(\sigma)} a_\alpha^{(\sigma)} + \sum_\alpha \eta_\alpha^{(\sigma)+} a_\alpha^{(\sigma)}} |0). \tag{5}$$

Consequently, we obtain the condition

$$UH_f = H_{bf}U. \tag{6}$$

which determines the boson-fermion image H_{bf} of Hamiltonian H_f and quarantees that the Hamiltonian matrix elements between the corresponding states are equal. Thus, in case no approximation is introduced, two different descriptions are equivalent.

The expression of H_{bf}

$$H_{bf} = \sum_\sigma H_{bf}^{(\sigma)} + H_{bf}^{(np)}. \tag{7}$$

can be directly deduced from Eq.(6)

According to the types of operators, the terms in the r.h.s. of Eq.(7) can be written in three parts:

$$H_{bf}^{(\sigma)} = H_{gb}^{(\sigma)} + H_{gf}^{(\sigma)} + V_{bf}^{(\sigma)}, \tag{8}$$

and

$$H_{bf}^{(np)} = H_{gb}^{(np)} + H_{gf}^{(np)} + V_{bf}^{(np)}, \tag{9}$$

where the sum of the pure-boson operator terms $H_{gb}^{(\sigma)}$ and $H_{gb}^{(np)}$ is the same as the effective ideal boson Hamiltonian for even-even nucleus[3]. The sum of the pure-fermion operator terms $H_{gf}^{(\sigma)}$ and $H_{gf}^{(np)}$ describes the single particle energies and the interaction of ideal fermions and has the same form but different meaning and properties from H_f in Eq.(2). The last terms in Eqs.(8) and (9) represent the boson-fermion interaction and have the forms

$$V_{bf}^{(\sigma)} = -2\sum_{\alpha\beta\gamma\delta} P_{\alpha\beta\gamma\delta}^{(\sigma)} \eta_\alpha^{(\sigma)+} \eta_\gamma^{(\sigma)} \sum_\lambda A_{\beta\lambda}^{(\sigma)+} A_{\delta\lambda}^{(\sigma)}, \tag{10}$$

and

$$V_{bf}^{(np)} = \sum_{\alpha\beta\gamma\delta} P_{\alpha\beta\gamma\delta}^{(np)} \eta_\beta^{(n)+} \eta_\delta^{(n)} \sum_\lambda A_{\alpha\lambda}^{(p)+} A_{\gamma\lambda}^{(p)}$$
$$+ \sum_{\alpha\beta\gamma\delta} P_{\alpha\beta\gamma\delta}^{(np)} \eta_\alpha^{(p)+} \eta_\gamma^{(p)} \sum_\lambda A_{\beta\lambda}^{(n)+} A_{\delta\lambda}^{(n)}. \tag{11}$$

It should be mentioned that these interaction terms are phenomenologically introduced in the IBFM, but naturally arisen, in the effective valence nucleon Hamiltonian in the generalized state space. Because the boson operators in Eqs.(10) and (11) are not s and d operators, it is necessary to choose a complete set of basis vectors with definite boson number (equivalently, with definite fermion number) to describe the nucleus, and furthermore, to define the s and d operators to truncate the state space into the collective subspace.

According to the generalized expansion theory, there exist several physical subspaces which are equivalent to each other[6]. The difference between them is that the complete sets of basis vectors have different boson number n_B and fermion number n_F, which satisfy the particle number conservation law. Since each boson carries two units of nucleon number, $2n_B + n_F$ should be equal to the number of valence nucleons. This condition holds for neutrons and protons respectively. However, if we make the state space truncation, these subspaces may no longer be equivalent, depending on the truncation form. We notice that, in the IBFM, an odd-A nucleus is treated as a system in which the single nucleon couples with an s-d even core. This implies that we may work in the subspace with one ideal fermion only. Consequently, for odd-n nucleus, σ in Eq.(10) equals p, the second term in Eq.(11) does not contribute and the pure-fermion terms contribute one neutron energy only.

As a direct extension of the IBM, one always takes the boson terms in the IBFM Hamiltonian to be the IBM Hamiltonian for the adjacent even-even nuclei. This implies that, for both odd-A and adjacent even-even nuclei, the s and d bosons have similar microscopic structures respectively. Therefore, defining the s and d bosons in the same way as in the even-even nucleus case is reasonable. The procedure presented in detail in Ref.[2] is the following: By using the specific unitary transformation, construct a set of collective bosons from ideal bosons. Then, define the s and d bosons in terms of this set of collective bosons and the properties of the s and d boson in the phenomenological model. The identical definitions of the s and d operators indicate that the method for the odd-A nucleus is just a natural extension of the method for the even-even nucleus.

After defining the s and d bosons, the s-d truncation, i.e. the collective subspace truncation, can be made. The effective Hamiltonian in the s-d subspace, which is just the microscopic IBFM Hamiltonian in our scheme can be easily derived

$$H_{\mathrm{IBFM}} = h_{\mathrm{IBM}} + h_f + h_{bf}. \tag{12}$$

where h_{IBM} has the form of the general IBM-2 Hamiltonian[3], h_f gives the energy of the odd nucleon, and h_{bf} is the boson-fermion interaction term with the form

$$\begin{aligned} h_{bf} = \sum_{\sigma} \Big\{ & \sum_i k_{0i}^{(\sigma)} [(s^{(\sigma)+} s^{(\sigma)})(\eta_i^{(n)+} \tilde{\eta}_i^{(n)})_0] \\ & + \sum_{ii'} k_{2ii'}^{(\sigma)} [(d^{(\sigma)+} s^{(\sigma)} + s^{(\sigma)+} \tilde{d}^{(\sigma)})(\eta_i^{(n)+} \tilde{\eta}_{i'}^{(n)})_2]_0 \\ & + \sum_{Jii'} x_{Jii'}^{(\sigma)} [(d^{(\sigma)+} \tilde{d}^{(\sigma)})_J (\eta_i^{(n)+} \tilde{\eta}_{i'}^{(n)})_J]_0 \Big\}. \end{aligned} \tag{13}$$

All the coefficients in Eq.(12) can be obtained from H_{bf} matrix elements between corresponding states. Thus, once the shell model single particle energies, wave functions, and the parameters of the nucleon-nucleon interaction are given, the coefficients in the valence nucleon Hamiltonian in Eq.(2) can be calculated, and consequently, the low-lying collective states of the odd-A nucleus can be studied.

3. NUMERICAL METHOD AND RESULTS

Systematic regularities exist among the excitation spectra of deformed nuclei, which is very useful to test a scheme. However, the dimension of the state space for the deformed nuclei is always so large that numerical calculation for diagonalizing the Hamiltonian H_{IBFM} is impractical. Therefore, we have to make some sort of approximation to solve the eigenvalue equation. In our earlier work[9], we studied several nuclei by the method of determining the projection wave functions of intrinsic states. Although the calculated results are quite good, but the method is only suitable for studying the rotational bands of the ground state but not for the entire spectrum. To this end we develop another analytic approximation method which was first proposed by *H.* Schasser et al. for the IBM[7]. The basic idea of this method is the following:

For the ground state of an N s-d boson system, we can write the intrinsic state as[10]:

$$|N;\,a\rangle = \mathcal{N}(B^{+})^{N}|0\rangle, \tag{14}$$

with

$$B^{+} = s^{+} + \sum_{\mu} a_{\mu} d_{\mu}^{+}. \tag{15}$$

and the intrinsic state of the IBFM system as a product of the state in Eq.(14) and the intrinsic wave function of the odd nucleon as

$$|N;\,a;\,\chi_K\rangle = |N;\,a\rangle|\chi_K\rangle, \tag{16}$$

where K is the z-component of the angular momentum in the intrinsic coordinate system, and a_μ are the intrinsic parameters which can be determined by variational calculation. Let us choose the principal axes of the quadropole tensor as the axes of the intrinsic coordinate system and assume that the nucleus is rotating around the x-axis which frequency ω. In order to determine the moment of inertia of the system, we should solve the following variational problem according to the cranking model,

$$\delta\langle N;\,a;\,\chi_K|H - \omega l_x|N;\,a;\,\chi_K\rangle = 0. \tag{17}$$

When ω is small enough we can expand the expectation values of $<H>$ and $<l_x>$ around $a_\mu^{(0)}$, which is the solution at $\omega = 0$. As a consequence, we can rewrite Eq.(17) approximately into the following set of linear equations:

$$\begin{aligned}
&\Delta_0\left[\frac{\partial^2\langle H\rangle}{\partial a_0^2}\right]_{a^{(0)}} + \Delta_2\left[\frac{\partial^2\langle H\rangle}{\partial a_2\,\partial a_0}\right]_{a^{(0)}} = 0,\\
&\Delta_0\left[\frac{\partial^2\langle H\rangle}{\partial a_0\,\partial a_2}\right]_{a^{(0)}} + \Delta_2\left[\frac{\partial^2\langle H\rangle}{\partial a_2^2}\right]_{a^{(0)}} = 0,\\
&\Delta_1\left[\frac{\partial^2\langle H\rangle}{\partial a_1^2}\right]_{a^{(0)}} = \omega\left[\frac{\partial\langle l_x\rangle}{\partial a_1}\right]_{a^{(0)}},
\end{aligned} \tag{18}$$

where $a_\mu = a_\mu^{(0)}\,\Delta +_\mu$. In this set of equations the first two equations are independent of the third one and only have a trivial solution $\Delta_0 = \Delta_2 = 0$, because the coefficient determinant does not vanish.

TABLE 1.
The Levels of Valence Nucleons

levels of neutrons		levels of protons	
nlj	E(MeV)	nlj	E(MeV)
3 p 1/2	8.88	3 s 1/2	7.21
2 f 5/2	7.64	2 d 3/2	6.76
1 i 13/2	6.85	1 h 11/2	5.32
1 h 9/2	5.80	2 d 5/2	5.00
3 p 3/2	4.30	1 g 7/2	4.00
2 f 7/2	4.00		

Solving $\Delta_1$from the third equation and comparing it with the definition of the moment of inertia, we can express the moment of inertia of the ground-band as:

$$I = \left[\frac{\partial \langle I_x \rangle}{\partial a_1}\right]^2_{a^{(0)}} \Big/ \left[\frac{\partial^2 \langle H \rangle}{\partial a_1^2}\right]_{a^{(0)}} . \tag{19}$$

consequently, we can obtain the energies of excited states in the ground band in terms of the rotational energy formula.

The discussion for excited bands is similar. If $|\chi_K>$in Eq.(16) is the wave function describing the intrinsic excited state of the odd nucleon, Eq.(19) gives the moment of inertia of the related excited band, and the expectation value of $<H>$ represents the intrinsic excited energy. If one replaces the state $|N;a>$in Eq.(16) with the state describing the intrinsic ß- or Δ- excitation of the even-core[10], the same procedure is applicable for the ß- or γ-band[7].

It should be mentioned that in the above discussion we did not distinguish a neutron from a proton. In order to obtain the IBFM-1 Hamiltonian in which the neutron and proton are treated as identical particles and refer to the knowledge obtained from the calculations of adjacent even-even nuclei[3], in this investigation, we make the maximum F-spin truncation[11]. However, this truncations is not necessary. We will publish the result without truncation in the near future.

TABLE 2.
The Parameters of Nucleon-nucleon Effective Interaction (MeV)

nucleus	g_n	G'_n	K'_n	g_p	G'_p	K'_p	K'_{np}
^{157}Tb, ^{159}Tb	0.048	0.052	0.021	0.028	0.046	0.031	0.005
^{155}Gd, ^{157}Gd	0.049	0.054	0.02	0.029	0.048	0.03	0.004

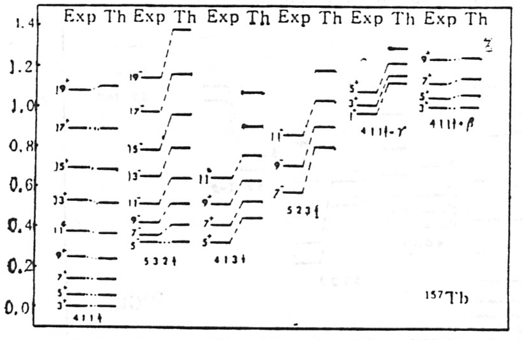

FIG.1

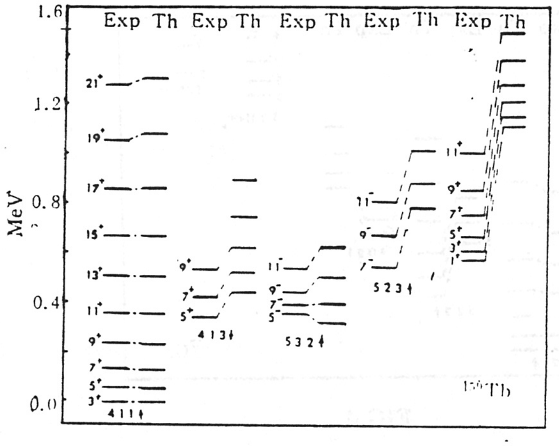

FIG.2

The neutron-neutron or proton-proton interaction of the valence nucleon Hamiltonian in Eq.(2) in our calculation are taken to be the pairing force, quadrupole pairing force and quadruple-quadrupole force, while the neutron-proton interaction is taken to be the quadrupole-quadrupole force. The single particle energy levels for neutrons and protons are shown in Table 1. The choice of the interaction parameters are based on those used in the calculations for the even-even nuclei[3] . Then, they are slightly modified to fit the data, and tabulated in Table 2. Moreover, the Nilsson wave functions[12] are employed to describe the intrinsic wave functions of the odd nucleon.

Figs.1-4 show the calculated results for ^{157}Tb, ^{159}Tb, ^{155}Gd and ^{157}Gd nuclei, respectively. Two of them are odd-n nucleus systems and the rest are odd-Z nucleus systems. In these figures, "Exp" and "Th" represent the experimented and theoretical bands, respectively, and "C" represents the theoretical prediction. The numbers beside the levels are two times the angular momentum quantum numbers.

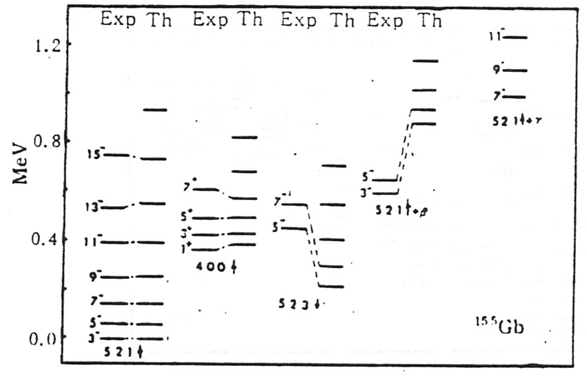

FIG.3

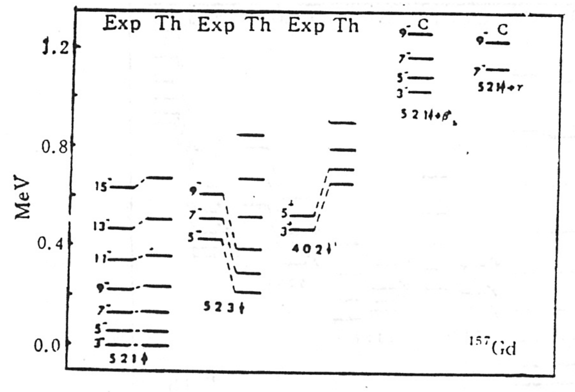

FIG.4

4. CONCLUSION

From Figs.1-4, one can see that the theoretical results qualitatively agree with the experimental data. For the ground bands, the calculated energy levels even quantitatively agree with the experimental data. Since there exist a quite large amount of experimental information for the nucleus ^{157}Tb, such as the rotational bands with positive or negative parity as well as the ß and γ-excited bands, the study of this nucleus is especially meaningful. In this investigation we calculate the energies of the excited states bands in terms of the obtained moment of inertia and the formula of the rotational energy. The important quantities which should be compared with the experimental data are the calculated values of the moment of inertia and the intrinsic excitation energy for each band, because the former determines the intervals between the energy levels within the band and the latter gives the location of the band head. From Fig.1, one can see that the locations of most band heads generally agree with the experimental data, while the calculated values of the moments of inertia agree very well with the experimental results. Therefore, we believe that, at least in the deform region, this method can be used to describe the odd-A nucleus. Moreover, we think the above positive conclusion also confirms the approach for the even-even nucleus because our theoretical approach is a direct extension of the latter. This work, of course, is preliminary. In order to further test our approach, we should do more work in various aspects.

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