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A Model of Four Dimensional Heterotic String

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We constructed a new model of D = 4 heterotic string with the gauge group $[SU(2)]^6$, which consists of the right-movers of the D = 4 fermionic string and the left-movers of the D = 10 N-S bosonic string. The massless ground state of the string contains the 4 dimensional supergravity multiplet and the super-Yang-Mills gauge field multiplet. The new model is tachyon-free, Lorentz invariant and supersymmetric.

INTRODUCTION

In recent years, the superstring theory as the most hopeful candidate for the grand unified theory for all interactions has been widely investigated. Firstly, combining the supermmetric theory with the string theory, Green and Schwarz constructed a superstring theory in the ten dimensional (D = 10) space-time[1-2]. A new model of D = 10 superstring-heterotic string has been proposed by Gross et al.[3], which is a hybrid of the Green and Schwarz superstring and the D = 26 bosonic string[4]. The ground states of this model comprise the D = 10 supergravity multiplet and the super-Yang-Mills gauge field multiplet. This implies the possibility of studying quantum gravity and unified theory for all interactions in the theoretical frame of superstring theory.

As the physical space-time is 4-dimensional, it is perhaps more important to construct the string model in 4-dimensional space-time. This is the motivation of this paper.

From a modified Green-Schwarz covariant superstring action[5], Chen Wei and Zhao Weidong constructed a model of free four-dimensional fermionic string[6]. As a try to look for the unified theory, according to the mechanism of the D = 10 heterotic string, we construct a new D = 4 heterotic string model consisting of the right-movers of the D = 4 fermionic string and the left-

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movers of the D = 10 Neveu-Schwarz bosonic string[7]. The new heterotic string is tachyon-free, Lorentz invariant and supersymmetric.

In Section 1, we construct a free four-dimensional heterotic string in the light-cone gauge. In Section 2, the internal gauge symmetry in the model is discussed, followed by some discussions in the last section.

1. THE FOUR-DIMENSIONAL HETEROTIC STRING

Analogous to the case of the D = 10 heterotic string, the physical degrees of freedom of the right-moving sector of the closed fermionic string consist of two transverse coordinates $X^i(\tau\text{-}\sigma)(i = 1,2)$ and two Majorana spinous $S^a(\tau\text{-}\sigma)(a = 1,2)$[6], and those of the left-moving sector of the bosonic string consist of eight transverse coordinates $X^i(\tau+\sigma)(i = 1,2)$, $X^I(\tau+\sigma)(I = 1,2,\cdots6)$ and eight world-sheet spinors $^l(l = 1,\cdots8)$ which are space-time vectors[7]. They comprise the physical degrees of freedom of the heterotic string. The normal-modes expansions are

$$X^i(\tau-\sigma) = \frac{1}{2}x^i + \frac{1}{2}p^i(\tau-\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha_n^i \exp[-2in(\tau-\sigma)],$$

$$X^i(\tau+\sigma) = \frac{1}{2}x^i + \frac{1}{2}p^i(\tau+\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^i \exp[-2in(\tau+\sigma)],$$

$$X^I(\tau+\sigma) = x^I + \tilde{p}^I(\tau+\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^I \exp[-2in(\tau+\sigma)],$$

$$S^a(\tau-\sigma) = \sum_n S_n^a \exp[-2in(\tau-\sigma)],$$

$$\lambda^l(\tau+\sigma) = \sum_r \tilde{b}_r^l \exp[-2in(\tau+\sigma)]. \tag{1.1}$$

where the n and r sum over all integers and all half-integers respectively.
In Eq.(1.1), we have already chosen

$$\alpha_0^i = \tilde{\alpha}_0^i = \frac{1}{2}p^i,$$

$$\tilde{\alpha}_0^I = \tilde{p}^I. \tag{1.2}$$

Quantizing the model in the light cone gauge, we get the following commutation relations among the normal-mode operators

$$[\alpha_m^i, \alpha_n^i] = [\tilde{\alpha}_m^i, \tilde{\alpha}_n^i] = m\delta_{m+n,0}\delta^{ii},$$

$$\{S_m^a, \bar{S}_n^b\} = (\gamma^+)^{ab}\delta_{m+n,0}, \quad [x^i, p^i] = i\delta^{ii},$$

$$[\tilde{\alpha}_m^I, \tilde{\alpha}_n^J] = m\delta_{m+n,0}\delta^{IJ},$$

$$\{\tilde{b}_r^l, \tilde{b}_s^g\} = \delta_{r+s,0}\delta^{lg}, \quad [x^I, \tilde{p}^J] = \frac{1}{2}i\delta^{IJ}, \tag{1.3}$$

and other commutators and anticommutators vanish.

The factor 1/2 in the commutator of x^I and $\tilde{p}^I$, the center-of-mass coordinate and the momentum in the internal space, arise from the fact that X^I's depend only on $(\tau+\sigma)$. This means that $2\tilde{p}^I(I = 1,\cdots 6)$ are the generators of translation in the internal space.

For D = 4, the fermionic states are described by the Majorana spinous $S^{Aa}(A = 1,2,\ a = 1,2)$ in the light cone gauge, i.e.

$$\gamma^+ S = 0 \tag{1.4}$$

where γ_μ are the 4×4 Dirac matrices in the Majorana representation. In the light-cone gauge, the string coordinates may be written as

$$\begin{aligned} X^+(\sigma,\tau) &= x^+ + p^+\tau, \\ X^-(\sigma,\tau) &= x^- + p^-\tau + \frac{i}{2}\sum_{n\neq 0}\left[\alpha_n^- e^{-2in(\tau-\sigma)} + \tilde{\alpha}_n^- e^{-2in(\tau+\sigma)}\right] \end{aligned} \tag{1.5}$$

$$\alpha_n^- = \frac{1}{p^+}\sum_m \alpha_{n-m}^i\alpha_m^i + \frac{1}{2p^+}\sum_m (m - n/2)S_{n-m}\gamma^- S_m + \frac{1}{2}:nCE_{ij}R_n^{ij}.$$

$$R_n^{ij} = \frac{i}{8}\sum_{m=-\infty}^{\infty} :S_{n-m}\gamma^{ij} - S_m: \tag{1.6}$$

where $c^2 = 6$, which is determined by the Lorentz invariance of the 4-dimensional fermionic string, and E_{ij} is the two-dimensional Levi-Civita symbol and $\tilde{\alpha}_n^-$ is constructed in the N-S bosonic string:

$$\tilde{\alpha}_n^- = \frac{1}{p^+}\sum_m (\tilde{\alpha}_{n-m}^i\tilde{\alpha}_m^i + \tilde{\alpha}_{n-m}^I\tilde{\alpha}_m^I) + \frac{1}{2p^+}\sum_{r\in z+1/2}(r - n/2)\tilde{b}_{n-r}^I\tilde{b}_r^I \tag{1.7}$$

From Eq.(1.5), we find that:

$$p^- = \alpha_0^- + \tilde{\alpha}_0^- \tag{1.8}$$

then the mass shell condition for the operator $M^2 = 2p^+p^- - (p^i)^2$ of string is

$$\frac{1}{4}M^2 = N + \tilde{N} - \frac{1}{2} + \frac{1}{2}\tilde{p}^I\tilde{p}^I. \tag{1.9}$$

Moreover, the invariance of the closed string under an arbitrary shift in σ implies the constraint:

$$N = \tilde{N} - \frac{1}{2} + \frac{1}{2}\tilde{p}^I\tilde{p}^I. \tag{1.10}$$

where N and $\tilde{N}$ are the normally ordered number operators for the right- and left-movers, respectively

$$N = \sum_{n=1}^{\infty}\left(\alpha^i_{-n}\alpha^i_n + \frac{n}{2}S_{-n}\gamma^- S_n\right). \tag{1.11}$$

$$\tilde{N} = \sum_{n=1}^{\infty}(\tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n + \tilde{\alpha}^I_{-n}\tilde{\alpha}^I_n) + \sum_{r=1/2}^{\infty} r\tilde{b}^I_{-r}\tilde{b}^I_r. \tag{1.12}$$

The term (1/2) on the right hand side of Eq.(1.9) comes from the normal ordering of $\tilde{N}$ and (1/2 $\tilde{p}^I\tilde{p}^I$) from the compactifiaction of the internal space and must be an integer by virtue of Eq.(1.10).

Because of Eqs. (1.9) and (1.10), the ground states of the 4-D heterotic string are massless

$$M^2 = 0,$$

$$N = \tilde{N} - \frac{1}{2} + \frac{1}{2}\tilde{p}^I\tilde{p}^I = 0. \tag{1.13}$$

This implies that the heterotic string is tachyon-free.

The right-moving and the left-moving components of the string coordinates are independent of each other and the theory has no symmetry under transformation $\sigma \rightarrow \sigma - \pi$. Thus the 4-D heterotic string is an orientated closed one. The physical states of the heterotic string are formed by the direct products of the quantum states of the right- and left-movers and satisfy the constraint(1.10).

$$|\rangle = |\rangle_R \otimes |\rangle_L. \tag{1.14}$$

The right-movers' ground states consist of two bosonic states $|i\rangle_R$ and two fermionic states (annihilated by S^a_n, α^i_n $(n>0)$ and N); The left-movers' ground state is a tachyon, but it is removed since the constraint (1.10) is not satisfied. Hence, the spectra of ground states of the heterotic string are:

$$|i\rangle_R \otimes \tilde{b}^i_{-1/2}|0\rangle_L, \qquad (2\times 2)$$
$$|a\rangle_R \otimes b^i_{-1/2}|0\rangle_L. \qquad (2\times 2)$$

which form a four-dimensional supergravity multiplet, and

$$|i\rangle_R \otimes \tilde{b}^I_{-1/2}|0\rangle_L, \qquad (2\times 6)$$
$$|i\rangle_R \otimes |\tilde{p}^I\tilde{p}^I = 1\rangle_L, \qquad (2\times n)$$
$$|a\rangle_R \otimes \tilde{b}^I_{-1/2}|0\rangle_L, \qquad (2\times 6)$$
$$|a\rangle_R \otimes |\tilde{p}^I\tilde{p}^I = 1\rangle_L. \qquad (2\times n)$$

which form a super Yang-Mills gauge multiplet. Here 6 corresponds to the number of zero roots and n the number of nonzero roots of the gauge group and the gauge multiplet $|\tilde{p}^I\tilde{p}^I = 1\rangle$ stems from the compactification.

Taking the spectra of the ground states into account, it can be directly verified that the heterotic string is supersymmetric in 4-dimensions and the generators are

$$Q^a = i\sqrt{p^+}(\nu^+ S_0)^a + i\frac{1}{\sqrt{p^+}}\sum_n (\nu_i S_{-n})^a \alpha_n^i ,$$
$$\{Q^a, \bar{Q}^b\} = -2(\nu \cdot p)^{ab}. \tag{1.15}$$

which acts on the right-moving sector.

Similarly, the generators of the 4-dimensional Lorentz transformation, $J^{\mu\nu}$, can be written as $J^{\mu\nu} = l^{\mu\nu} + J_R^{\mu\nu} + J_L^{\mu\nu}$ (μ,ν = 0,1,2,3), where $l^{\mu\nu}$ and $J_R^{\mu\nu}$ ($J_L^{\mu\nu}$) are the Lorentz generators of the right- and left-moving sector of the fermionic and bosonic strings respectively. Since the Lorentz invariances for 4-D fermionic string and the 10-D N-S bosonic string hold separately, it is easy to prove that the generators $J^{\mu\nu}$ satisfy the 4-D Lorentz algebra and the model of the 4-D heterotic string, hence, possesses the 4-D Lorentz symmetry.

2. INTERNAL GAUGE INVARIANCE OF THE 4-D HETEROTIC STRING

The internal gauge group deals with the six dimensional internal space resulting from the compactification of the left movers of the N-S bosonic string from 10-D into 4-D space-time. We will choose this compact internal space as a six dimensional flat torus with radius R and let e_I be a unit basic vector in the I-th direction (I = 1,...6) on the torus. After compactification, the boundary conditions for the internal compact space coordinates X^I become[8]

$$X^I(\pi + \sigma, \tau) = X^I(\sigma, \tau) + 2R\pi\sum n_i e_i^I \ (e_i^I = \delta_i^I). \tag{2.1}$$

where n_i are integers, denoting the winding number--the number of times of the string winding around the I-th direction when the variable σ changes from 0 to π. The mode expansion for X^I are

$$X^I(\sigma, \tau) = x^I + p^I\tau + 2L^I\sigma + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha_n^I \exp[-2in(\tau - \sigma)]$$
$$+ \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^I \exp[-2in(\tau + \sigma)]. \tag{2.2}$$

where

$$L^I = R\sum n_i e_i^I. \tag{2.3}$$

The generators of the translations of X^I are p^I(I = 1,...6)

$$p^I = \frac{1}{R}\sum_{i=1}^{6} n_i e_i^I. \tag{2.4}$$

The coordinates $X^I(\sigma,\tau)$ may be devided into the right- and left- movers, i.e.,

$$X^I(\tau - \sigma) = \frac{1}{2}x^I + \left(\frac{1}{2}P^I - L^I\right)(\tau - \sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha_n^I \exp[-2in(\tau - \sigma)], \tag{2.5}$$

$$X^I(\tau + \sigma) = \frac{1}{2}x^I + \left(\frac{1}{2}P^I + L^I\right)(\tau + \sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^I \exp[-2in(\tau + \sigma)]. \tag{2.6}$$

Note that the left- and right-moving modes are independent when the winding number terms exist ($L^I \neq 0$)[3]. The 4-D heterotic string has only the left-moving modes so only $X^I(\sigma+\tau)$ is retained

$$X^I(\tau+\sigma) = x^I + \tilde{p}^I(\tau+\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}^I_n \exp[-2in(\tau+\sigma)]. \tag{2.7}$$

where $2\tilde{p}^I$ generates the translation of x^I, and the corresponding momentum is

$$\tilde{p}^I = \frac{1}{2R}\sum_i n_i e^I_i. \tag{2.8}$$

Comparing Eq.(2.6) with Eq.(2.7), we have

$$\tilde{p}^I = 2L^I. \tag{2.9}$$

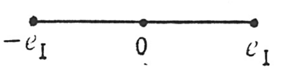
So we determine the radius R = 1/2 and get

$$\tilde{p}^I = \sum_i n_i e^I_i. \tag{2.10}$$

For the massless gauge states $|\tilde{p}^I\tilde{p}^I = 1>$, one finds

$$\tilde{p}^I\tilde{p}^I = 1 = \sum n_i^2 \quad (i = 1, \cdots, 6). \tag{2.11}$$

which has the solutions n_i = ±1 (i = 1,...6) so that the internal gauge group has twelve vector weights($\pm e_I$) and six zero weights. For each fixed I there are two vector following figure:



This is just the root graph of the Lie algebra of $SU(2)$. Thus, considering the compactification from 10 to 4 dimensions, the internal gauge group of the 4-D heterotic string is $[SU(2)]^6$. The eighteen gauge bosons belong to the adjoint representation of $[SU(2)]^6$.

Now, we verify this $[SU(2)]^6$ internal gauge symmetry in our model with the Kac-Moody Lie algebra[9].

In order to acquire the physical states satisfying the gauge conditions, the generators $x(r) = X^I r^I$ ($r^I = \pm e_I$) of the internal gauge group should commute with the Virasoro operators L_n and the super-gauge operators G_s[10]

$$[L_n, x(r)] = 0, \quad n = 0, \pm 1, \pm 2, \cdots \tag{2.12}$$

$$[G_S, x(r)] = 0, \quad S = \pm 1/2, \pm 3/2, \cdots \tag{2.13}$$

(In (2.12), L_0-1/2 is the Hamiltonian operator of the N-S bosonic string and commutes with $x(r)$. It explains that the $x(r)$ are the generators of the internal gauge group. Therefore $x(r)$ must be constructed by the vertex operators $V(r,z)$ of the N-S bosonic string tachyonic state[7]:

$$V(r, Z) = r^I \sum_{S=-\infty}^{\infty} \tilde{b}^I_S Z^{-S} : \exp[2ir^I \cdot x^I(Z)]:. \tag{2.14}$$

where

$$Z = \exp[2i(\tau + \sigma)]$$

$$:\exp[2ir^I x^I(Z)]: = \exp\left[r^I \sum_{n=1}^{\infty} \frac{1}{n} \tilde{\alpha}^I_n Z^n\right] \exp[2ir^I \cdot x^I(Z)]$$

$$\cdot\, Z^{r^I \cdot \tilde{p}^I} \exp\left[-r^I \sum_{n=1}^{\infty} \frac{1}{n} \tilde{\alpha}^I_n Z^{-n}\right] Z^{(r^I)^2/2}. \tag{2.15}$$

The operator: $\exp[2ir^I x^I(z)]$: translates the internal momenta p^I to $p^I + r^I$ with $(r^I)^2 = e^2 = 1$. According to (2.15), we define the operators[11].

$$x(r) = \frac{C_r}{2\pi i} \oint \frac{dz}{z} V(r, Z). \tag{2.16}$$

which denote the generators of the gauge group acting on the left-moving modes of the heterotic string, where the contour integration is around the unit circle and C_r is a Kac-Moody factor.

With (1.3), it can directly verify that

$$\begin{aligned} &[x(r), x(r')] = 0, \quad (r, r' = 0, 1) \\ &[x(r), x(-r)] = r^I \cdot \tilde{p}^I, \\ &[\tilde{p}^I, x(r)] = r^2 x(r). \end{aligned} \tag{2.17}$$

by making use of

$$\begin{aligned} &C_r C_{r'} = (-1)^{r \cdot r' + 1} C_{r'} C_r, \\ &C_r C_{-r} = 1. \end{aligned} \tag{2.18}$$

Hence, Eq.(2.17) is just the commutation relations of the generators of the Lie algebra of $[SU(2)]^6$ as long as $r^I = \pm e_I = \pm 1$. For each fixed I, $x(r)$, $x(-r)$ and $\tilde{p}^I$ form an $SU(2)$ Lie algebra. It follows that the 4-D heterotic string has $[SU(2)]^6$ gauge invariance.

3. DISCUSSION

We have constructed a model of the 4-D heterotic string and given some satisfactory results. However, a number of questions still remain. For instance, whether the heterotic string has $SU(3)$ $SU(2)$ $SU(1)$ gauge symmetries or not, how the interaction is introduced into the model, and so on. These problems remain to be investigated in the future.

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