

# Calculation of the Effective Moment of Inertia of Fissioning Nuclei at the Saddle Point

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The effective moment of inertia, charge density distributions of the fissioning nucleus  $^{170}\text{Yb}$  and  $^{254}\text{Fm}$  at the saddle point are calculated by using the continuous medium model with variable densities.

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## 1. INTRODUCTION

To describe nuclear deformations at the saddle point, the effective moment of inertia  $J_{\text{eff}}$  given by  $J_0/J_{\text{eff}} = J_0(J_{\perp} - J_{\parallel})/(J_{\perp} J_{\parallel})$ , is one of the important quantities which can be obtained by analyzing the experimental data. In the above formula,  $J_{\parallel}$  and  $J_{\perp}$  stand for the moments of inertia along and perpendicular to the symmetrical axis and  $J_0$  is the moment of inertia of the corresponding spherical nucleus. Theoretically, one usually applies the rotating liquid drop model (RLDM)[1] to calculate the effective moment of inertia. But the density in the nucleus is not uniform. For the fissioning nucleus with large saddle point deformation, change in the density distributions has a considerable effect on the value of  $J_{\text{eff}}$ . Hence, the assumption of a uniformly charged liquid drop may effect the reliability of the calculation. In order to consider the change in nuclear density, we have calculated the effective moment of inertia and the charge distributions at the saddle point of the fissioning nuclei  $^{170}\text{Yb}$  and  $^{254}\text{Fm}$  by using the continuous medium model.

## 2. THE MODEL

According to the continuous medium model[2], for a rotating nucleus with angular momentum  $I\hbar$ , proton density  $\rho_p$  and neutron density  $\rho_n$ , the part of the energy/functional related to the nuclear shape can be written as

$$E[\rho_p, \rho_n; I] = \alpha \int |\nabla \rho_0| dV + \beta \int [(\rho_p - \rho_n)^2 + s(\rho_p + \rho_n - \rho_0)^2] / (\Phi \rho_0) dV \\ + \frac{e^2}{2a} \iint \frac{\rho_p(\mathbf{r}_1) \rho_p(\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|} dV_1 dV_2 + \frac{1}{2} \frac{I^2}{Ja^2}, \quad (1)$$

In the above equation, the four terms on the right hand side represent, respectively, the surface energy, compressibility energy, Coulomb energy and the rotating energy of the nucleus with the moment of inertia  $J$  given by

$$J = M \int (x^2 + y^2)(\rho_p + \rho_n) dV, \quad (2)$$

here  $\Phi = (1 - r|\nabla \rho_0|/\rho_0)^{-1}$  and  $\alpha, \beta, \gamma$  and  $s$  are adjustable parameters. We have assumed the reference density  $\rho_0$  as a modified Fermi distribution

$$\rho_0 = \frac{t}{4\pi a^3} \frac{1 + \exp(-R/a)}{1 + \exp[(r/\mu - R)/a]}, \quad (3)$$

where

$$R = R_0 f / \mu, \\ f = 1 + \alpha_{20} Y_{20}(\theta) + \alpha_{22} [Y_{2-2}(\theta, \varphi) + Y_{22}(\theta, \varphi)] / \sqrt{2} + \dots, \\ \mu = \left[ 1 + \left( \frac{\partial f}{\partial \theta} / f \right)^2 + \left( \frac{\partial f}{\partial \varphi} / f \right)^2 / \sin^2 \theta \right]^\sigma. \quad (4)$$

Taking  $\sigma \sim 0.5$  in the factor  $\mu$  assures that the diffusion layer at the nuclear surface is approximately independent of nuclear deformations.

Varying the functional (1) under the restriction  $\int \rho_p dV = Z$  and  $\int \rho_n dV = N$ , an integral equation for  $\rho_p$  and  $\rho_n$  can be derived. We can obtain the potential energy surface of the fissioning nucleus, the density distribution at the saddle point and  $J_{eff}$  by solving the integral equation.

The parameters appearing in (1) and (3) are given in Table 1. They are determined by fitting the nuclear masses of 1526 nuclei, with the addition of suitable volume and exchange energy terms and microscopic energies given by Möller and Nix[3]. The rms deviation of the fitting is 0.9 MeV and reasonable fission barriers can be predicted.

TABLE 1

$a(\text{fm})$	$t$	$\alpha(\text{MeV})$	$\beta(\text{MeV})$	$\gamma$	$s$	$\sigma$
0.5361	0.325	14.5843	26.1003	0.45	0.5347	0.545

### 3. RESULTS OF CALCULATIONS

The relations between  $J_0/J_{\text{eff}}$  and  $\langle I^2 \rangle$  are given in Fig.1. The solid curves are the results of this paper and the broken curves are calculated with RLDM[4,5]. It is clear from the figure that the change of density lowers the value of  $J_0/J_{\text{eff}}$  by approximately 0.3 for  $^{170}\text{Yb}$ , while in the case of  $^{254}\text{Fm}$ , the effect is rather small. The saddle point deformation of  $^{170}\text{Yb}$  is much larger than the one of  $^{254}\text{Fm}$  (see Fig.2), and the effect of the change of density is more prominent in  $^{170}\text{Yb}$ . In Fig.2, the saddle point deformations and the non-uniformity in charge densities are illustrated by equal density contour plot in the  $Y$ - $Z$  (rotating axis) plane. In Fig.3, the calculated fission barrier  $B_f$  is given as a function of  $I$ .

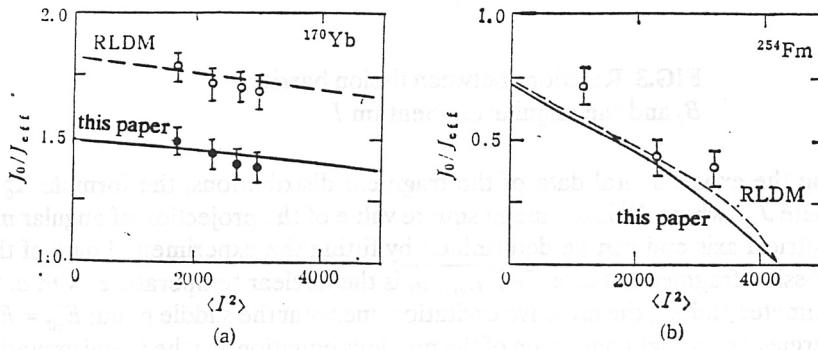


FIG.1 Relations between  $J_0/J_{\text{eff}}$  and  $\langle I^2 \rangle$ .

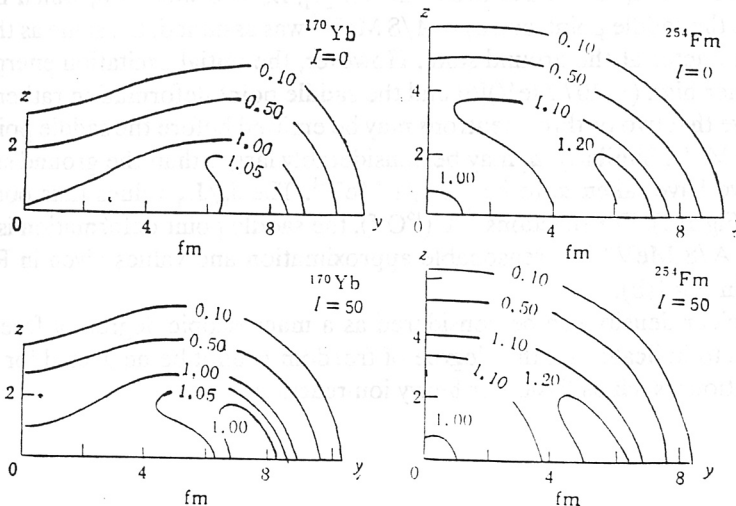


FIG.2 Charge distributions of fissioning nuclei at the saddle point.