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Rotational Effect on Electromagnetic Transitions in Nuclei

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The signature-splitting of the energies, $B(M1)$-values and dynamical quadrupole moment in ^{159}Tm are investigated by means of the extended particle-rotor model. In particular, the effect of second-order Coriolis interaction on the signature-dependence of $B(M1)$-values is discussed. It was noted that the calculated values are in qualitative agreement with the measured data.

In order to investigate the triaxial deformation of nuclei, which probably exists in high spin states, the rare-earth odd-Z nuclei ($N \sim 90$, $Z \sim 70$) have of late been extensively studied. Progress has been achieved in the understanding of the nuclear shape at high spin states, both experimentally[1-6] and theoretically[7-9]. In general, the γ-values are estimated from the measurements of the energy spectra. However, as Hamamoto has pointed out[10], various kinds independent experimental evidence must be obtained before a correct conclusion that the nucleus has a triaxial shape can be drawn, because a single kind of data can be interpreted in different ways. Consequently, we must study not only the energy spectra but also the electromagnetic transition probabilities in order to determine the triaxial shape of a nucleus. However, as it has been pointed out in Ref.[11], that $M1$ and $E2$ transitions are not only effected by the γ-deformation (including γ-fluctuation) but also by the hexadecapole deformation of the core and the second-order Coriolis

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interaction in the rotational Hamiltonian. Since the data of energy levels, $B(M1; I \rightarrow I\text{-}1)$-values and $B(E2; I \rightarrow I\text{-}2)$-values as well as $B(E2; I \rightarrow I\text{-}1)$-values for $\alpha = +1/2$ in ^{159}Tm have been reported, the purpose of this paper is to calculate the energies, $B(M1)$-values and $B(E2)$-values of the yrast states with negative-parity before the first bandcrossing in ^{159}Tm in terms of the extended particle-rotor model, to compare with the experimental data, and to estimate the possible γ-value of the triaxial deformation. It seems that the second-order Coriolis interaction is partly equivalent to triaxiality[12].

The extended particle-rotor Hamiltonian is written as[11]

$$H_{EPR} = H_R + H_{intr} + H'_C, \tag{1}$$

where H_R, H_{intr} and H'_C refer to the rotor Hamiltonian, the intrinsic Hamiltonian and the second-order Coriolis interaction, respectively. H_R has the form

$$H_R = \sum_{K=1}^{3} \frac{R_K^2}{2J_K} = \sum_{K=1}^{3} \frac{1}{2J_K} (\mathbf{I} - \mathbf{j})_K^2, \tag{2}$$

where **I** is the total angular momentum, **j** the single-particle angular momentum and J_K the moment of inertia. In numerical calculations the moments of inertia of hydrodynamical type

$$J_K = \frac{4}{3} J_0 \sin^2\left(\gamma + \frac{2}{3}\pi K\right), \quad K = 1, 2, 3 \tag{3}$$

are used.

The intrinsic Hamiltonian is written as

$$H_{intr} = \sum_{\nu} (\varepsilon_\nu - \lambda) a_\nu^+ a_\nu + \frac{\Delta}{2} \sum_{\mu\nu} \delta_{(\bar{\mu}\nu)} (a_\mu^+ a_\nu^+ + a_\nu a_\mu) \tag{4}$$

where ε_ν denotes the single-particle energies in the triaxially-deformed quadrupole potential V, (the hexadecapole deformation is not important for ^{159}Tm), λ is the Fermi energy, Δ refers to the pairing gap parameter, and $\bar{\mu}$ denotes the time-reversal of state μ. The triaxially-deformed quadrupole potential is written as

$$V = -\kappa\left[\cos\gamma Y_{20}(\theta, \varphi) - \frac{\sin\gamma}{\sqrt{2}}(Y_{22}(\theta, \varphi) + Y_{2-2}(\theta, \varphi))\right] \tag{5}$$

where κ is used as an energy unit in a single high-j shell model and it depends on the value of the deformation parameter ß. The typical value of κ is about 2--2.5 MeV in the studied region.

Since the rotor is not ideal, i.e., the moment of inertia varies with the rotational frequency, the rotational energy does not satisfy the $I(I+1)$ dependence. Therefore, higher-order powers of $I(I+1)$ should be contained in the rotational energy so that higher-order couplings between the particle and rotor should be included in the Hamiltonian. The second-order term H'_C, which is called second-order Coriolis interaction, is taken as[11]

$$H'_C = B_0(I_+^2 + I_-^2)(j_+^2 + j_-^2) \tag{6}$$

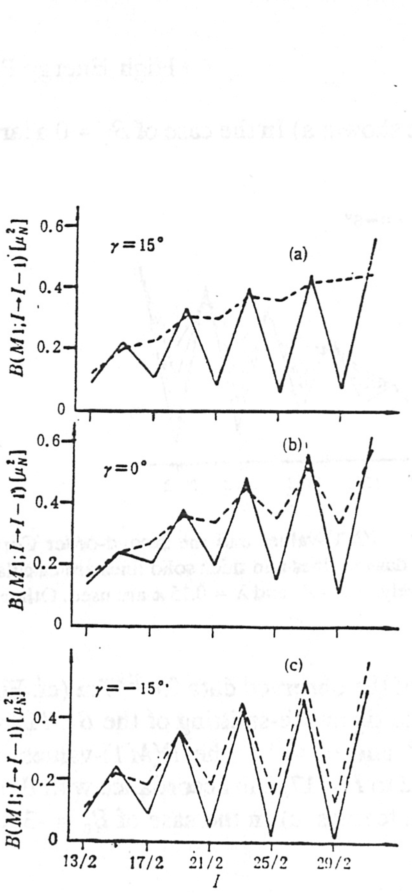

FIG.1 The effect of the second-order Coriolis interaction on $B(M1)$-values for different γ-values. Solid lines are calculated with the second-order Coriolis interaction strength $B_0 = -3 \times 10^{-5}\ \kappa$, while dashed lines are for $B_0 = 0$. Other parameters used are $\lambda = 0.1\ \kappa$, $\Delta = 0.45\ \kappa$, $J_0 = 72/\kappa$, $g_l = 1.0$, $g_s = 3.91$, $g_R = 0.42$.

where B_0 is the second-order inertial parameter which could be estimated by the Harris parameters. The order of magnitude of B_0 is about $10^{-5}\ \kappa$ for odd-Z nuclei with $A \sim 160$.

It is noticed that Hamamoto and Sagawa[13] have analyzed the experimental data for ^{159}Tm by using the particle-rotor model. In their analysis parameter J_0 is taken as a smooth function of total spin I in order to reproduce the observed energy spectra. With $\gamma = -16°$ in their calculation a considerable signature-splitting is obtained in the energy spectra and $B(M1;\ I \rightarrow I\text{-}1)$-values. However, the calculated $B(M1;\ I \rightarrow I\text{-}1)$-value has a phase opposite to the experimental one at $I = 17/2$. It is doubtful whether such a large γ-deformation really has to be used in order to fit the observed data of ^{159}Tm. What role does the higher-order coupling between the particle and rotor play? Our calculated results obtained by using the extended particle-rotor model are as follows:

(1) The effect of the second-order Coriolis interaction on $B(M1)$.

In Fig.1, we show the calculated $B(M1)$-values as a function of the total angular momentum I for different γ-values by using the particle-rotor model (namely, $B_0 = 0$, as shown by dashed lines) and the extended particle-rotor model ($B_0 = -3 \times 10^{-5}\ \kappa$ is used, and as shown by solid lines). From this figure the following features are shown: a) In the case of $B_0 = 0$ a large signature-splitting of the

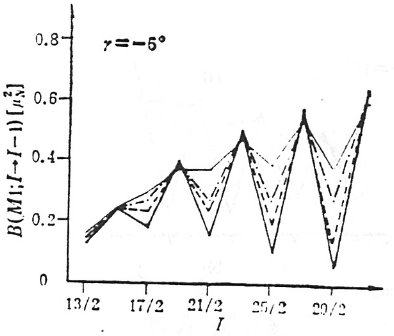

FIG.2 Relation between the calculated $B(M1)$-values and the second-order Coriolis interaction strength $|B_0|$. Thin solid lines, dot-dashed lines, dashed lines and thick solid lines are calculated with $B_0 = 0, -1\times10^{-5}\,\kappa, -3\times10^{-5}\,\kappa$ and $-5\times10^{-5}\,\kappa$, respectively. $\gamma = -5°$ and $\lambda = 0.15\,\kappa$ are used. Other parameters are the same as those used in Fig.1.

$B(M1)$-values, which is the feature of the observed data for ^{159}Tm (cf. Fig.3), appears only for $\gamma = -15°$, while for $\gamma = 0°$ and $\gamma = 15°$ the signature-splitting of the $B(M1)$-values becomes smaller. b) In the case of $B_0 = 0$, for $\gamma = 0°$ and $\gamma = 15°$ the $B(M1)$-values increase when the angular momentum I changes from $I = 15/2$ to $I = 17/2$ in accordance with the observed feature of ^{159}Tm, while for $\gamma = -15°$ there is no such a feature. c) In the case of $B_0 = -3\times10^{-5}\,\kappa$ a strong signature-

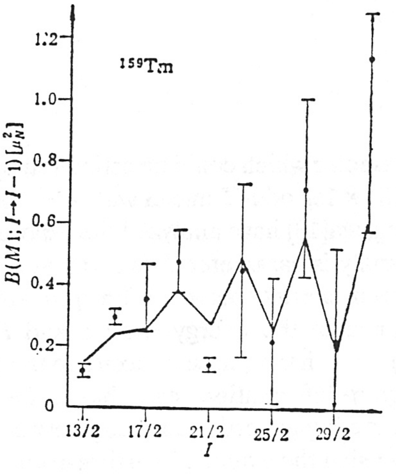

FIG.3 Calculated and measured $B(M1)$-values of the negative-parity yrast states for nucleus ^{159}Tm as function of total angular momentum I. Parameters used in calculation are $\gamma = -5°$, $B_0 = -2\times10^{-5}\,\kappa$, $\lambda = 0.15\,\kappa$, $\Delta = 0.45\,\kappa$, $J_0 = 72/\kappa$, $g_l = 1.0$, $g_s = 3.91$ and $g_R = 0.42$. The experimental data[3] are shown by full circles, while the calculated values are connected by solid lines.

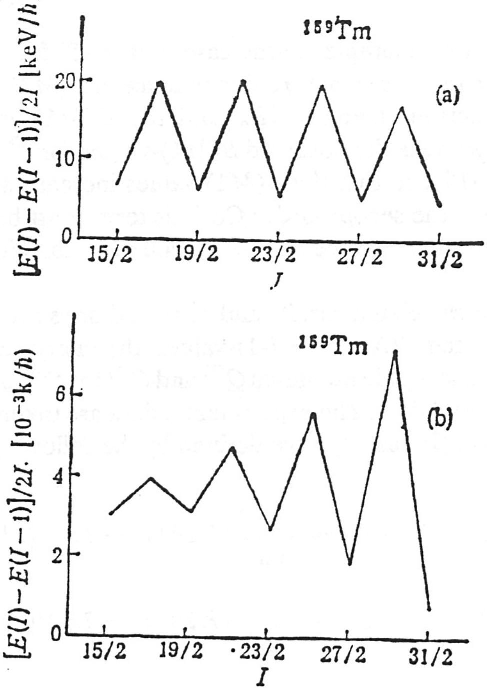

FIG.4 Experimental and calculated energy difference divided by twice the total angular momentum I as a function of I for nucleus ^{159}Tm. Parameters used in the calculation are the same as those used in Fig.3. (a) Experimental data[3]. (b) Calculated values.

dependence of $B(M1)$-values is shown for all given γ values, but the signature-averaged values decrease as compared with the result of $B_0 = 0$. By examining numerical calculations, it is found that the decrease of $B(M1)$-values comes mainly from the transition from the unfavored-signature states (u-states) to favored-signature states (f-states). The signature of u-state and f-state is defined as

$$\alpha_u = \frac{1}{2}(-)^{j+\frac{1}{2}}, \quad \alpha_f = \frac{1}{2}(-)^{j-\frac{1}{2}}, \tag{7}$$

and the relation between the total spin I and signature α is

$$I = \alpha \mathrm{mod} 2. \tag{8}$$

d) With the increase of I, the effect of B_0 on $B(M1)$-values increases and the signature-dependence is strengthened. e) An important result is that the signature-dependence of $B(M1)$ caused by including the second-order Coriolis term for $\gamma = 0°$ (as shown by the solid line in Fig.1b) is quite similar to the one obtained for $\gamma = -15°$ without the second-order Coriolis term (as shown by the dashed line in Fig.1c). We have pointed out[11] that the $B(E2)$-values exhibit the same behavior. Therefore, it implies that the higher-order coupling between the particle and rotor is partly equivalent to the triaxiality of the nuclear shape.

(2) Relation between interaction strength $|B_0|$ and $B(M1)$-values.

Fig.2 shows the calculated examples in the case of γ = -5° for B_0 = 0, -1 × 10^{-5} κ, -3 × 10^{-5} κ and -5 × 10^{-5} κ. It can be seen that the signature-dependence of $B(M1)$-values becomes stronger with the increase of the interaction strength $|B_0|$ and the $B(M1)$-values increases I increases. Accordingly, in order to reproduce the observed $B(M1)$-values for ^{159}Tm one should not use a large negative γ-value (say, γ = -15°) so that the $B(M1)$-values increase as I increases in the region of $I \leq 19/2$. On the other hand, the second-order Coriolis term must be taken into account so that a larger and stronger signature-dependence of $B(M1)$-values exhibits in the region of higher angular momentum I.

(3) Comparison of the calculated results and observed ones for ^{159}Tm.

In Figs.3--5, the calculated $B(M1; I \rightarrow I\text{-}1)$-values, the energy differences $[E(I) - E(I\text{-}1)]/2I$ and the dynamical electric quadrupole moments $Q^{(1)}$ and $Q^{(2)}$ for ^{159}Tm are given and compared with the corresponding experimental data. The experimental data are taken from Ref.[3]. The dynamical electric quadrupole moments $Q^{(1)}$ and $Q^{(2)}$are defined by the following formula:

$$B(E2; I \rightarrow I-1) = \frac{5}{16\pi} \langle IK20|I-1K\rangle^2 Q^{(1)^2},$$

$$B(E2; I \rightarrow I-2) = \frac{5}{16\pi} \langle IK20|I-2K\rangle^2 Q^{(2)^2}. \qquad (9)$$

From these figures, we notice that the calculated energy spectra, $B(M1)$-values, $Q^{(1)}$ and $Q^{(2)}$-values for ^{159}Tm are qualitatively in agreement with the observed data. However, the calculated signature-splitting of energies is smaller than the experimental ones. Considering that we adopt the same parameter J_0 (compared with Ref.[13], where J_0 is a function of I) and the obtained signature-splitting is in phase with that of the observed data, the quantitative differences should not be considered as a serious problem.

(4) Since the second-order Coriolis interaction plays an important role in the signature-dependence of the electromagnetic transition probabilities, it should be taken into account when one analyzes the triaxiality of the nuclear shape. From our numerical examples it is doubtful that the γ-deformation of ^{159}Tm is -20°[2] or -16°[13]. The information extracted from the experimental data may imply that the nucleus of ^{159}Tm has a nearly axially-symmetric shape. However, the higher-order coupling between the particle and rotor resulting from the rotation is partly equivalent to γ-deformation.

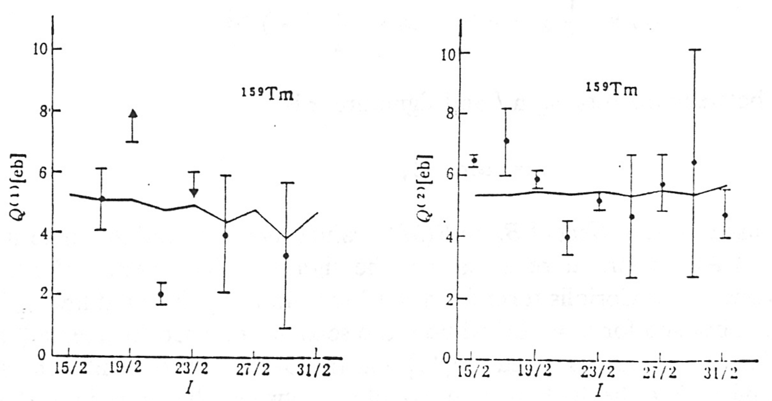

FIG.5 Experimental and calculated dynamical quadrupole moments $Q^{(1)}$ and $Q^{(2)}$ of the negative-parity yrast states for ^{159}Tm as a function of total angular momentum I. The effective charge of the $h11/2$-proton state is taken to be $e_{eff}\langle r^2\rangle/Q_0$ = 0.28e. Value of Q^a used in the calculation is 30.3 e^2b^2. Other parameters used are the same as those used in Fig.3. The experimental data[3] are shown as filled circles.

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