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The String Tensions of (2+1)-dimensional $SU(2)$ Gauge Theory

Li Zhibing, Zheng Weihong and Guo Shuohong

(Department of Physics, Zhongshan University, Guangzhou)

We present a variational calculation of the string tensions in (2+1)-dimensional $SU(2)$ Lattice gauge theory using two kinds of Hamiltonians of which the exact ground states are known.

Due to the delocalization induced by the roughening transition, the calculations of string tension are rather difficult and thus results of string tension in the deep weak coupling region are still lacking (either for (3+1)-dimensional or for (2+1)-dimensional gauge models). For (2+1)-dimensional $SU(2)$ group, Monte Carlo simulation gave $\sigma a^2 = 0.2g^4$ [1] around $1/g^2 \sim 1.5$; the cluster expansion method showed that $\sigma a^2 = (0.14 \pm 0.01)g^4$ [2]. In this article, we adopt the Hamiltonians of Ref. [3], of which the ground states are exactly known, in the variational calculations of (2+1)-dimensional $SU(2)$ gauge group.

A state with two heavy quarks separated by a distance of L lattice units can be generated by the operator

$$Q(\Gamma) = \phi^+(0) \prod_{\Gamma} U\phi(L) \tag{1}$$

where U is the gauge link along the path Γ connecting the quarks. The only constrain on Q is the gauge invariant. The wave function is given by

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$$|q\bar{q}\rangle = \sum_{\Gamma} C_{\Gamma} Q(\Gamma)|\Omega\rangle \tag{2}$$

where $|\Omega\rangle$ is the ground state of the gauge field. The quark potential of this system is

$$\begin{aligned} V(L) &= \langle q\bar{q}|H|q\bar{q}\rangle / \langle q\bar{q}|q\bar{q}\rangle \\ &= \frac{\sum C_m^* C_n \langle \Omega | Q_m^+ H Q_n | \Omega\rangle}{\sum C_m^* C_n \langle \Omega | Q_m^+ Q_n | \Omega\rangle} \end{aligned} \tag{3}$$

If only the finite number of paths is included, the above equation gives an upper bound of the quark potential. The string tension is defined by

$$\sigma = \lim_{aL \to \infty} V(L)/aL \tag{4}$$

It can be verified that the Hamiltonian

$$\begin{aligned} H &= \frac{g^2}{2a}(e^R E_l^a e^{-R})^+ (e^R E_l^a e^{-R}) \\ R^+ &= R \end{aligned} \tag{5}$$

possesses an exact ground state.

$$|\Omega\rangle = e^R|0\rangle \tag{6}$$

where $|0\rangle$ is defined by $E_l^2|0\rangle = 0$. Using Eq.(5) in Eq.(3), we have

$$\begin{aligned} V(L) &= -\frac{g^2}{2a}\frac{\sum C_m^* C_n \langle \Omega | [E_l^a, Q_m^+][E_l^a, Q_n] | \Omega\rangle}{\sum C_m^* C_n \langle \Omega | Q_m^+ Q_n | \Omega\rangle} \\ &= \frac{g^2}{2a}\frac{\sum C_m^* C_n W_{mn}}{\sum C_m^* C_n D_{mn}} \end{aligned} \tag{7}$$

The variation with respect to $\{C_m\}$ gives an eigen equation

$$\det(W - \lambda D) = 0 \tag{8}$$

where $\lambda = 2a/g^2 V_{\min}(L)$, $\{C_m\}$ is the corresponding eigenvector.

In this article, we will calculate the string tensions of Hamiltonians H_1 and H_2. $H_i (i = 1,2)$ is given by Eq.(5), with R_i defined as

$$R_1 = \frac{1}{2g^4 C_N}\sum_p \operatorname{tr}(U_p + U_p^+) \tag{9a}$$

$$R_2 = -\frac{N}{(N^2+1)g^4}\sum_p \operatorname{tr}(U_p + U_p^+) + \frac{1}{4(N^2+1)g^4}\sum_p [\operatorname{tr}(U_p + U_p^+)]^2 \tag{9b}$$

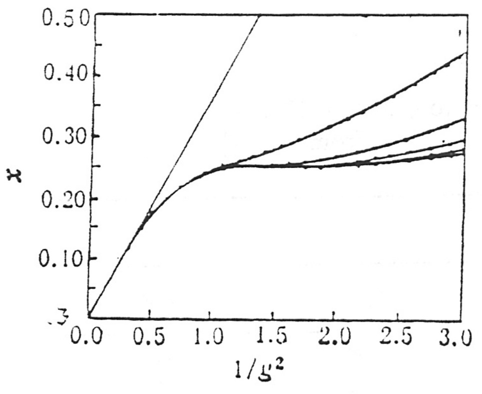


FIG.1 The relation of x vs. $1/g^2$ for Hamiltonian H_1, $L = 13$ the varia-tional state in each curve has the form:

$$|l\rangle = \left\{\sum_{j=0}^{l}\left[\sum_{m=0}^{L} C^a_{jm} Q^a_{jm} + \sum_{n=1}^{[L/2]} C^b_{jn} Q^b_{jn}\right]\right\}|\Omega\rangle .$$

The curves from top to bottom correspond to $l = 0,1,2,\ldots7$

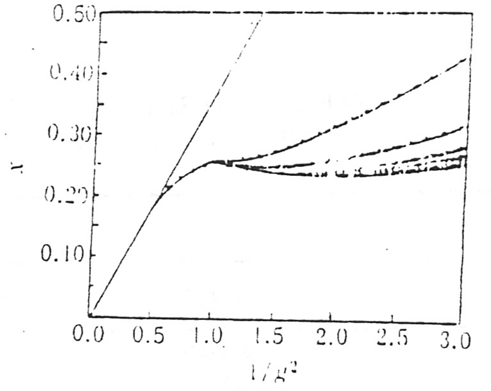
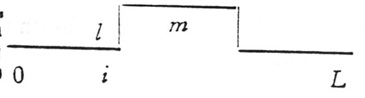


FIG.2 The relation of x vs. $1/g^2$ for Hamiltonian H_2, $L = 13$ see also the caption of Fig.1.

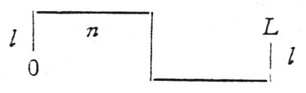
It has been proved[3] that H_2 is endowed with correct classical continuous limit both in (3+1)-dimension and in (2+1)-dimension, while H_1 only possesses correct classical continuous limit in (3+1)-dimension (in (2+1)-dimension, it can be regarded as a nontrivial interacting model).

In this article, the states as selected below are used

Q^a_{lm}: $\sum_{i=0}^{L-m}$


(10a)

$l = 0, 1, 2, \cdots;\ m = 0, 1, 2, \cdots, L$

Q^b_{ln}: $\frac{1}{2}\Big($


$+$ Left $\longleftrightarrow$ Right $\Big)$

$l = 1\ 2, \cdots;\ n = 1, 2, \cdots, \left[\frac{L}{2}\right]$ (10b)

TABLE 1. The Ratio of O^{++} Glueball Mass m to $\sqrt{\sigma}$ ($L = 13$, $l = 7$), where we have quoted the glueball mass $am = 2.28g^2$ from Ref.[4].

$1/g^2$	H_1		H_2	
	$a^2\sigma g^{-4}$	$m/\sqrt{\sigma}$	$a^2\sigma g^{-4}$	$m/\sqrt{\sigma}$
1.0	0.24276	4.627	0.25440	4.520
1.2	0.24837	4.575	0.24932	4.566
1.4	0.24958	4.564	0.24256	4.629
1.6	0.24956	4.564	0.23824	4.671
1.8	0.24978	4.562	0.23631	4.690
2.0	0.25085	4.552	0.23643	4.689
2.2	0.25297	4.533	0.23827	4.671
2.4	0.25615	4.505	0.24155	4.639
1.1~1.4[5]	Monte Carlo simulation to Wilson action gave $m/\sqrt{\sigma} = 5.8 \pm 0.7$			

The weak coupling expansion suggests that $a^2\sigma/g^4 \sim$ const. When L is large enough, $x \equiv \lambda/(2Lg^2) = a^2\sigma/g^4$. We depict the curves of x vs. $1/g^2$ for $L = 13$, $l \leq 7$ (see Fig.1 and Fig.2). From the figures one can see that the curves tend to scaling (horizontal lines) as predicted by the weak coupling expansion when the number of variational states increases. When $1/g^2$ becomes large, the needed number of states also increases enormously.

Let m be the glueball mass, then the ratio $m/\sqrt{\sigma}$ is a dimensionless quantity. In the scaling region, its value can be directly compared with the one evaluated in the continuum theory. Ref.[4] presented the mass of O^{++} to be $am = 2.28g^2$ (for both H_1 and H_2). Combining with our results, some values of $m/\sqrt{\sigma}$ are obtained as shown in the table.

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