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Non-Relativistic BUU Equation

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The Boltzmann-Uehling-Uhlenbeck (BUU) equation, which is the time evolution of the Wigner function of the single particle Green's function, is derived by using the closed-time Green's function approach. The quantum mechanical approximation in deriving the BUU equation is discussed.

1. INTRODUCTION

Intensive studies have been done in recent years of the collisions between heavy ions at medium energy following the discoveries of various new phenomena. The theories of non-relativistic heavy ion collisions are mainly devoted to the investigation of reaction mechanism and the exploration of new phenomena. At low energies ($E < 10$ MeV/A), the two-body interaction can be neglected in comparison with the mean field generated by the nucleons. However, at medium energies both the N-N collision and the mean field play important roles in heavy-ion collisions (HIC) and the Pauli principle cannot be excluded. Several theories have been proposed for the description of the HIC in the region of medium energies. Especially, the use of the Boltzmann-Uehling-Uhlenbeck (BUU) equation has achieved some satisfactory results for this purpose[1--3].

The theory of the BUU is essentially the Boltzmann equation for the single-particle distribution function, which includes both a mean field and a collision integral of the Uehling-Uhlenbeck form. This theory has the satisfactory attributes of reducing to the Vlasov equation governed by the mean field and to the Intranuclear Cascade (INC) model governed by the two-body collisions. The Vlasov equation is the time dependent Hartree-Fock (TDHF) equation in the classical approximation, while the BUU equation, we may say, is the extended time-dependent Hartree-Fock (ETDHF) equation in the classical approximation. Although the BUU equation is one of the classical form, its solution contains the quantum effects under the initial condition of quantum mechanics. It consists of three basic factors of reaction mechanism at medium energy and is one of the non-

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equilibrium dynamic equations, which is different from the master and the Forker-Plank equations both of which are not self-consistent.

After the Boltzmann equation was proposed for studying the non-equilibrium physical problems in the 19th century, Uehling and Uhlenbeck introduced the Pauli principle effects into the Boltzmann equation and obtained the BUU equation[4]. Since then, there have been several derivations for the BUU equation either from the statistical principle or from the equation of quantum mechanics[5,6]. Recently new ways of deriving this equation have been published[7--12]. However, the connection between the mean field and the collision term is an assumption, which is neither natural nor net.

The application of the non-equilibrium Green function technique in many-body theory has proved to be highly successful in studying the time evolution of many-particle quantum systems. The Vlasov and Boltzmann type equations have been derived and discussed under different approximations[9--12] by means of this technique. In this paper, the BUU equation has been derived and the approximations and assumptions in the derivation discussed following this approach. In addition, the BUU equation has been developed both in the local and non-local approximations, respectively. The momentum-dependent mean field has been obtained naturally in the non-local approximation.

2. CLOSED TIME GREEN FUNCTION AND PERTURBATION

The closed time Green function technique allows one to study the time evaluation of many-particle quantum systems. As we know, the expectation value of an operator obtained by Gell-mann and Low theorem cannot be adopted for the non-stationary state[12]. Let us discuss the expectation value of an operator with respect to a state specified at t_0, we find

$$\langle \hat{O}_H(t) \rangle = \langle U(t_0, t) \hat{U}_I(t) U(t, t_0) \rangle, \tag{2.1}$$

Where the subscripts H and I stand for the Heisenberg and interaction pictures, respectively, and

$$U(t, t_0) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} T^c \left[\int_{t_0}^{t} dt_1 \cdots \int_{t_0}^{t} dt_n [H_I'(t_1) \cdot \cdots \cdot H_I'(t_n)] \right], \tag{2.2}$$

$$U(t_0, t) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} T^a \left[\int_{t}^{t_0} dt_1 \cdots \int_{t}^{t_0} dt_n [H_I'(t_1) \cdot \cdots \cdot H_I'(t_n)] \right]. \tag{2.2'}$$

where $H_I'(t)$ is the Hamiltonian in the interaction picture. Therefore

$$\langle \hat{O}_H(t) \rangle = \left\langle T^a \left[\exp\left(-i \int_{t}^{t_0} dt' H_I(t') \right) \right] \hat{O}_I(t) T^c \left[\exp\left(-i \int_{t_0}^{t} dt' H_I'(t') \right) \right] \right\rangle \tag{2.3}$$

where T^c and T^a represent the chronological and antichronological time ordering operators, respectively. We can identify whether the field operators belong to chronological or antichronological branch when the time ordering T is introduced and the exponential functions are all added to both the left side and right side of the operator. We introduce a closed contour, as shown in Fig.1, which runs forwards from t_0 to t and backwards from t to t_0. The Green functions are defined along the contour. Since the time variable is along the contour, this is called the closed time Green function

(CTGF). In this way, four Green functions are defined by the positions of the field operators in the contour.

i) The single particle Green function

$$-iG^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \langle \hat{\phi}_H^+(\vec{x}_2, t_2)\hat{\varphi}_H(\vec{x}_1, t_1)\rangle, \tag{2.4}$$

where

$$\langle \cdots\cdots \rangle = \mathrm{Tr}\,(\hat{\rho}\cdots)/\mathrm{Tr}\,(\hat{\rho}), \tag{2.5}$$

The r. h. s. of (2.4) is the single particle density matrix for $t_2 = t_1$ and $\vec{x}_2 = \vec{x}_1$, namely,

$$n(\vec{x}_1, t_1) = \langle \hat{n}_H(\vec{x}_1, t_1)\rangle = -iG^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2)\Big|_{\substack{x_1=x_2 \\ t_1=t_2}}, \tag{2.6}$$

After the Fourier transformation for the relative variable, the so-called Wigner function is obtained as

$$f(\vec{P}\vec{R}T) = \int d\vec{r}\, e^{-i\vec{P}\cdot\vec{r}} \left\langle \hat{\phi}_H^+\left(\vec{R} - \frac{\vec{r}}{2}, T\right)\hat{\phi}_H\left(\vec{R} + \frac{\vec{r}}{2}, T\right)\right\rangle, \tag{2.7}$$

which corresponds to a classical particle density in the phase space. We will use the BUU equation to study its variation with time.

ii) $$iG^{+-}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \langle \hat{\phi}_H(x_1, t_1)\hat{\phi}_H^+(\vec{x}_2, t_2)\rangle, \tag{2.8}$$

Its Wigner function corresponds to the hole density.

iii) $$iG^{--}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \langle T^c|\,\hat{\phi}_H(\vec{x}_1, t_1)\hat{\phi}_H^+(\vec{x}_2, t_2)\,|\rangle, \tag{2.9}$$

which is the chronological Green function.

iv) $$iG^{++}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \langle T^a|\,\hat{\phi}_H^+(\vec{x}_2, t_2)\hat{\phi}_H(\vec{x}_1, t_1)\,|\rangle, \tag{2.10}$$

which is the antichronological Green function.

It is easy to demonstrate that the four Green functions satisfy the following relations:

$$G^{--}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \theta(t_1 - t_2)G^{+-}(\vec{x}_1, t_1, \vec{x}_2, t_2) + \theta(t_2 - t_1)G^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2), \tag{2.11}$$

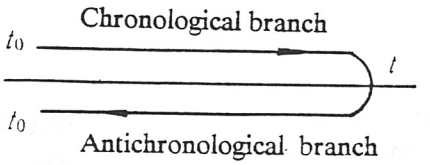

FIG.1 The "closed time path" contour.

$$G^{++}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \theta(t_1 - t_2)G^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) + \theta(t_2 - t_1)G^{+-}(\vec{x}_1, t_1, \vec{x}_2, t_2), \tag{2.12}$$

where $\Theta(t)$ is the step function. Eqs.(2.11) and (2.12) can be rewritten in a compact form as

$$G^{--} + G^{++} = G^{+-} + G^{-+}, \tag{2.13}$$

Using Eq.(2.13), the four Green functions can be redefined as,

$$iG^{--}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \left\langle \left| T^a \left[\exp\left(-i \int_t^{t_0} dt' H_I'(t') \right) \right] T^c \left[\exp\left(-i \right.\right.\right.$$
$$\left.\left.\left. \times \int_{t_0}^{t} H_I'(t') dt' \hat{\phi}_I(x_1, t_1) \times \hat{\phi}_I^+(x_2, t_2) \right) \right] \right| \right\rangle, \tag{2.14.1}$$

$$iG^{++}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \left\langle \left| T^a \left[\exp\left(-i \int_t^{t_0} dt' H_I'(t') \right) \hat{\phi}_I(\vec{x}_1, t_1) \hat{\phi}_I^+(\vec{x}_2, t_2) \right] \right.\right.$$
$$\left.\left. \times T^c \left[\exp\left(-i \int_{t_0}^{t} dt' H_I'(t') \right) \right] \right| \right\rangle, \tag{2.14.2}$$

$$iG^{+-}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \left\langle \left| T^a \left[\exp\left(-i \int_t^{t_0} dt' H_I'(t') \right) \hat{\phi}_I(\vec{x}_1, t_1) \right] \right.\right.$$
$$\left.\left. \times T^c \left[\exp\left(-i \int_{t_0}^{t} dt' H_I'(t') \right) \hat{\phi}_I^+(\vec{x}_2, t_2) \right] \right| \right\rangle, \tag{2.14.3}$$

$$-iG^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \left\langle \left| T^a \left[\exp\left(-i \int_t^{t_0} dt' H_I'(t') \right) \hat{\phi}_I^+(\vec{x}_2, t_2) \right] \right.\right.$$
$$\left.\left. \times T^c \left[\exp\left(-i \int_{t_0}^{t} dt' H_I(t') \right) \hat{\phi}_I(\vec{x}_1, t_1) \right] \right| \right\rangle, \tag{2.14.4}$$

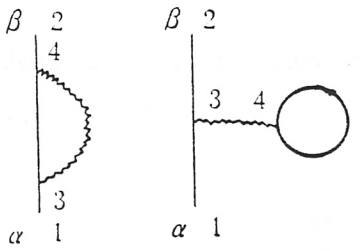

FIG.2 First-order Feynman diagrams in H-F approximation.

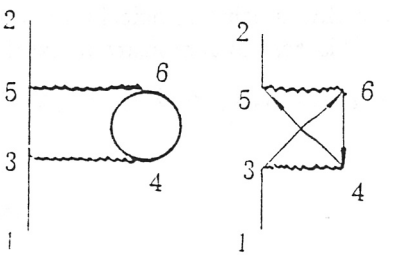

FIG.3 Second-order Feynman diagrams in Born approximation.

On the basis of (2.14), we can make a perturbation expansion according to the Feynman rules and Wick theorem.

First, let us discuss the first-order perturbation in the two-body interaction. We shall consider a nonrelativistic system of fermions. The Hamiltonian can be expressed as the following form in the interaction representation,

$$H_I' = \frac{1}{2}\hat{\psi}_\mu^+(\vec{x}_3, t_3)\hat{\psi}_\nu^+(\vec{x}_4, t_4)V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4)\hat{\psi}_{\nu'}(\vec{x}_4, t_4)\hat{\psi}_{\mu'}(\vec{x}_3, t_3). \quad (2.15)$$

We can draw all topologically distinctive connected and direct diagrams. Fig.2 shows the Feynman diagrams in the first-order approximation, in which each diagram has four topological equivalent ones, respectively.

One can prove that all the four Green functions satisfy the Dyson equation, i.e.

$$\begin{aligned} G_{\alpha\beta}^{(1)-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) &= G_{\alpha\beta}^{(0)-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) + \iint G_{\alpha\mu}^{(0)-+}(\vec{x}_1, t_1, \vec{x}_3, t_3) \\ &\times \Sigma_{\mu\nu'}^{(1),++}(\vec{x}_3, t_3, \vec{x}_4, t_4) \times G_{\nu'\beta}^{(0)-+}(\vec{x}_4, t_4, \vec{x}_2, t_2)d\vec{x}_3dt_3d\vec{x}_4dt_4 \\ &+ \iint G_{\alpha\mu}^{(0)--}(\vec{x}_1, t_1, \vec{x}_3, t_3)\Sigma_{\mu\nu'}^{(1),--}(\vec{x}_3, t_3, \vec{x}_4, t_4) \\ &\times G_{\nu'\beta}^{(0)-+}(\vec{x}_4, t_4, \vec{x}_2, t_2)d\vec{x}_3dt_3d\vec{x}_4dt_4, \end{aligned} \quad (2.16)$$

where

$$\begin{aligned} \Sigma_{\mu\nu'}^{(1),++}(\vec{x}_3, t_3, \vec{x}_4, t_4) &= -iG_{\mu'\nu}^{(0)++}(\vec{x}_3, t_3, \vec{x}_4, t_4)V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4) \\ &+ i\delta(\vec{x}_3 - \vec{x}_4) \times \delta(t_3 - t_4)\delta(\nu' - \mu')\int d\vec{x}_4'dt_4'G_{\nu''\nu}^{(0)--}(\vec{x}_4', t_4', \vec{x}_4', t_4') \\ &\times V_{\mu\nu\mu'\nu''}(\vec{x}_3, t_3, \vec{x}_4', t_4'), \\ \Sigma_{\mu\nu'}^{(1)--}(\vec{x}_3, t_3, \vec{x}_4, t_4) &= iG_{\mu'\nu}^{(0)--}(\vec{x}_3, t_3, \vec{x}_4, t_4)V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4) \\ &- i\delta(\vec{x}_3 - \vec{x}_4) \times \delta(t_3 - t_4)\delta(\nu' - \mu')\int d\vec{x}_4'dt_4'G_{\nu''\nu}^{0--}(\vec{x}_4', t_4', \vec{x}_4', t_4') \\ &\times V_{\mu\nu\mu'\nu''}(\vec{x}_3, t_3, \vec{x}_4', t_4'). \end{aligned} \quad (2.17)$$

If we write the four Green functions in a matrix form (2.18)

$$G^{(1)} = \begin{pmatrix} G^{(1)--} & G^{(1)-+} \\ G^{(1)+-} & G^{(1)++} \end{pmatrix}, \quad G^{(0)} = \begin{pmatrix} G^{(0)--} & G^{(0)-+} \\ G^{(0)+-} & G^{(0)++} \end{pmatrix},$$
$$\Sigma^{(1)} = \begin{pmatrix} \Sigma^{(1)--} & 0 \\ 0 & \Sigma^{(1)++} \end{pmatrix}, \quad (2.19)$$

the Dyson equation can be put in a more compact form, (2.20)

$$G^{(1)} = G^{(0)} + G^{(0)}\Sigma^{(1)}G^{(0)}. \quad (2.21)$$

We only take the Born approximation in the second-order Feynman diagrams as shown in Fig.3. From Eq.(2.14), we can also obtain four Green functions which satisfy the Dyson equation

$$G^{(2)} = G^0\Sigma^{(2)}G^0, \quad (2.22)$$

Here $\Sigma^{(2)}$ are the self-energies in the second-order perturbation and can be expressed as the following,

$$\begin{aligned}\Sigma^{(2)++}_{\mu i'}(\vec{x}_3, t_3, \vec{x}_5, t_5) &= \iint d\vec{x}_4 dt_4 d\vec{x}_6 dt_6 V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4) V_{iji'j'}(\vec{x}_5, t_5, \vec{x}_6, t_6) \\ &\times G^{(0)++}_{\mu' i}(\vec{x}_3, t_3, \vec{x}_5, t_5) G^{(0)++}_{\nu' j}(\vec{x}_4, t_4, \vec{x}_6, t_6) G^{(0)++}_{j\nu'}(\vec{x}_6, t_6, \vec{x}_4, t_4) \\ &- \iint d\vec{x}_4 dt_4 d\vec{x}_6 dt_6 V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4) V_{iji'j'}(\vec{x}_5, t_5, \vec{x}_6, t_6) \\ &\times G^{(0)++}_{\mu' j}(\vec{x}_3, t_3, \vec{x}_6, t_6) \times G^{(0)++}_{j'\nu}(\vec{x}_6, t_6, \vec{x}_4, t_4) G^{(0)++}_{\nu' i}(\vec{x}_4, t_4, \vec{x}_3, t_3)\end{aligned} \tag{2.23}$$

$$\begin{aligned}\Sigma^{(2)--}_{\mu i'}(\vec{x}_3, t_3, \vec{x}_5, t_5) &= \iint d\vec{x}_4 dt_4 d\vec{x}_6 dt_6 V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4) V_{iji'j'}(\vec{x}_5, t_5, \vec{x}_6, t_6) \\ &\times G^{(0)--}_{\mu' i}(\vec{x}_3, t_3, \vec{x}_5, t_5) G^{(0)--}_{\nu' j}(\vec{x}_4, t_4, \vec{x}_6, t_6) G^{(0)--}_{j'\nu}(\vec{x}_6, t_6, \vec{x}_4, t_4) \\ &- \iint d\vec{x}_4 dt_4 d\vec{x}_6 dt_6 V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4) V_{iji'j'}(\vec{x}_5, t_5, \vec{x}_6, t_6) \\ &\times G^{(0)--}_{\mu' j}(\vec{x}_3, t_3, \vec{x}_6, t_6) G^{(0)--}_{j'\nu}(\vec{x}_6, t_6, \vec{x}_4, t_4) G^{(0)--}_{\nu' i}(x_4, t_4, x_5, t_5),\end{aligned} \tag{2.24}$$

$$\begin{aligned}\Sigma^{(2)+-}_{\mu i'}(\vec{x}_3, t_3, \vec{x}_5, t_5) &= -\iint d\vec{x}_4 dt_4 d\vec{x}_6 dt_6 V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4) V_{iji'j'}(\vec{x}_5, t_5, \vec{x}_6, t_6) \\ &\times G^{(0)++}_{\mu' i}(\vec{x}_3, t_3, \vec{x}_5, t_5) G^{(0)-+}_{j'\nu}(\vec{x}_6, t_6, \vec{x}_4, t_4) G^{(0)+-}_{\nu' j}(\vec{x}_4, t_4, \vec{x}_6, t_6) \\ &+ \iint d\vec{x}_4 dt_4 d\vec{x}_6 dt_6 V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4) V_{iji'j'}(\vec{x}_5, t_5, \vec{x}_6, t_6) G^{(0)+-}_{\mu' j}(\vec{x}_3, t_3, \vec{x}_6, t_6) \\ &\times G^{(0)-+}_{j'\nu}(\vec{x}_6, t_6, \vec{x}_4, t_4) G^{(0)+-}_{\nu' i}(\vec{x}_4, t_4, \vec{x}_5, t_5),\end{aligned} \tag{2.25}$$

$$\begin{aligned}\Sigma^{(2)-+}_{\mu i'}(\vec{x}_3, t_3, \vec{x}_5, t_5) &= -\iint d\vec{x}_4 dt_4 d\vec{x}_6 dt_6 V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4) V_{iji'j'}(\vec{x}_5, t_5, \vec{x}_6, t_6) \\ &\times G^{(0)-+}_{\mu' i}(\vec{x}_3, t_3, \vec{x}_5, t_5) G^{(0)-+}_{j'\nu}(\vec{x}_6, t_6, \vec{x}_4, t_4) G^{(0)+-}_{\nu' j}(\vec{x}_4, t_4, \vec{x}_6, t_6) \\ &+ \iint d\vec{x}_4 dt_4 d\vec{x}_6 dt_6 V_{\mu\nu\mu'\nu'}(\vec{x}_3, t_3, \vec{x}_4, t_4) V_{iji'j'}(\vec{x}_5, t_5, \vec{x}_6, t_6) G^{(0)-+}_{\mu' j}(\vec{x}_3, t_3, \vec{x}_6, t_6) \\ &\times G^{(0)+-}_{j'\nu}(\vec{x}_6, t_6, \vec{x}_4, t_4) G^{(0)-+}_{\nu' i}(\vec{x}_4, t_4, \vec{x}_5, t_5),\end{aligned} \tag{2.26}$$

In general, the four self-energies in Eqs.(2.23--2.26) satisfy the following relation

$$\Sigma^{++} + \Sigma^{--} = -(\Sigma^{+-} + \Sigma^{-+}), \tag{2.27}$$

While in the first-order approximation (2.20), one finds

$$\Sigma^{++} + \Sigma^{--} = 0. \tag{2.28}$$

3. THE VLASOV EQUATION

In general, the two-body interaction can be expressed as

$$V(\vec{x}_3, t_3, \vec{x}_4, t_4) = V(\vec{x}_3, \vec{x}_4)\delta(t_3 - t_4), \tag{3.1}$$

If an approximation similar to

$$iG^{(0)++}(\vec{x}_3, t_3, \vec{x}_4, t_4)\big|_{t_3 = t_4 + 0^+} \Longrightarrow iG^{(0)-+}(\vec{x}_3, t_3, \vec{x}_4, t_4), \tag{3.2}$$

which is made for the chronological and antichronological Green functions in Eqs.(2.17) and (2.18) Eq.(2.17) becomes:

$$\Sigma^{(1)++}(\vec{x}_3, t_3, \vec{x}_4, t_4) = \delta(t_3 - t_4)\left[\delta(\vec{x}_3 - \vec{x}_4)\int d\vec{x}_4 V(\vec{x}_3, \vec{x}_4')\right.$$
$$\left.\times\, G^{(0)-+}(\vec{x}_4, \vec{x}_4') - V(\vec{x}_3, \vec{x}_4)G^{(0)-+}(\vec{x}_3, \vec{x}_4)\right], \tag{3.3}$$

This is just the Hartree-Fock self-energy. The first and second terms in Eq.(3.3) are the direct (Hartree) and exchange terms respectively, namely,

$$\Sigma^{\mathrm{HF}}(\vec{x}_3, \vec{x}_4, t_3) = \Sigma^{(1)++}(\vec{x}_3, \vec{x}_4, t_3), \tag{3.4}$$

We define the following notations:

$$G_{01}^{0-1} = i\frac{\partial}{\partial t_1} - \frac{\nabla_1^2}{2m}, \quad G_{02}^{*0-1} = -i\frac{\partial}{\partial t_2} - \frac{\nabla_2^2}{2m},$$

From the Dyson equation (2.16), the Kadanoff-Baym equation[10] can be obtained directly by using the properties of the different time-ordering Green functions:

$$G_{01}^{0-1}G_{\alpha\beta}^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \int \Sigma_{\alpha\mu}^{--}(\vec{x}_1, t_1, \vec{x}_3, t_3)G_{\mu\beta}^{-+}(\vec{x}_3, t_3, \vec{x}_2, t_2)d\vec{x}_3 dt_3, \tag{3.5}$$

$$G_{02}^{*0-1}G_{\alpha\beta}^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \int G_{\alpha\mu}^{-+}(\vec{x}_1, t_1, \vec{x}_3, t_3)\Sigma_{\mu\beta}^{++}(\vec{x}_3, t_3, \vec{x}_2, t_2)d\vec{x}_3 dt_3, \tag{3.6}$$

Using Eqs.(3.1), (3.5) and (3.6) in the H-F approximation, after performing the integration over t_3 one has

$$(G_{02}^{0-1*} - G_{01}^{0-1})G_{\alpha\beta}^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \int [G_{\alpha\mu}^{-+}(\vec{x}_1, \vec{x}_3, t_1)\Sigma_{\mu\beta}^{\mathrm{HF}}(\vec{x}_3, t_1, \vec{x}_2, t_2)$$
$$\cdot\ - \Sigma_{\alpha\mu}^{\mathrm{HF}}(\vec{x}_1, \vec{x}_3, t_1)G_{\mu\beta}^{-+}(\vec{x}_3, t_1, \vec{x}_2, t_2)]d\vec{x}_3, \tag{3.7}$$

Making Fourier transformation of Eq.(3.7), taking $t_1 = t_2 = t$ and introducing the new coordinates,

$$\vec{x} = \frac{1}{2}(\vec{x}_1 + \vec{x}_2), \quad \vec{r} = \vec{x}_1 - \vec{x}_2, \tag{3.8}$$

replacing the l. h. s. of (3.7) by the Wigner function (2.7), one arrives at

$$(G_{02}^{*0-1} - G_{01}^{0-1})G_{\alpha\beta}^{-+}(\vec{x}, \vec{p}, t) = \left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\nabla_x\right)f(\vec{x}, \vec{p}, t). \tag{3.9}$$

In the following, we will discuss the r. h. s. of (3.7) both in the local and non-local approximations. The interaction in the local case is denoted as:

$$V(\vec{x}_3, \vec{x}_2) = V_0\,\delta(\vec{x}_3 - \vec{x}_2)\delta(t_3 - t_2) \tag{3.10}$$

In practice, only the diagonal terms are involved. Thus, we can take the diagonal form for the spin and isospin indexes in (3.7). Therefore, the r. h. s. of (3.7) becomes

$$\text{Right side} = \int e^{-i\vec{p}\cdot\vec{r}}\left[\Sigma^{HF}_{\alpha\alpha}\left(\vec{x}-\frac{1}{2}\vec{r},\ t\right)-\Sigma^{HF}_{\alpha\alpha}\left(\vec{x}+\frac{1}{2}\vec{r},\ t\right)\right]G^{-+}\left(\vec{x}-\frac{1}{2}\vec{r},\right.$$

$$\left.\vec{x}+\frac{1}{2}\vec{r},\ t\right)d\vec{r}, \tag{3.11}$$

We expand the Σ^{HF} in Taylor series and find

$$\text{Right side} = 2\sinh\frac{\nabla_r\nabla_p}{2}\Sigma^{HF}_{\alpha\alpha}(\vec{x},\ t)f(\vec{x},\ \vec{p},\ t). \tag{3.12}$$

Substituting the δ-function of Eq.(3.10) by a non-local interaction, the r. h. s. of (3.7) will become a product of two functions with double variables, i.e.,

$$\int d\vec{x}_3 f(\vec{x}_1,\ \vec{x}_3)u(\vec{x}_3,\ \vec{x}_2). \tag{3.13}$$

To simplify the notation, we neglect the indexes of the spin and isospin, make the coordinate transformation Eq.(3.8) and introduce into Eq.(3.13) the new coordinates,

$$\vec{x}' = \frac{1}{2}(\vec{x}_3-\vec{x}_2) \qquad \vec{r}' = \vec{x}_3-\vec{x}_2 \tag{3.14}$$

Then, the following equation is obtained:

$$\int d\vec{x}_3 f(\vec{x}_1,\ \vec{x}_3)u(\vec{x}_3,\ \vec{x}_2) = \int d\vec{r}' f\left(\vec{r}-\vec{r}',\ \vec{x}+\frac{\vec{r}'}{2},\ t\right)u\left(\vec{r}',\ \vec{x}+\frac{\vec{r}'-\vec{r}}{2},\ t\right) \tag{3.15}$$

Making the Taylor expansions over $\vec{x}$ for the double variable functions of f and u, and keeping only the first order terms for $\vec{r}$ ' and $\vec{r}$ ' - $\vec{r}$, then the product of f and u is

$$f(\vec{x}_1,\ \vec{x}_3)u(\vec{x}_3,\ \vec{x}_2) = f(\vec{r}-\vec{r}',\ \vec{x},\ t)u(\vec{r}',\vec{x},t)+\frac{1}{2}\vec{r}'\frac{\partial f(\vec{r}-\vec{r}',\ x,\ t)}{\partial x}$$

$$\times u(\vec{r}',\ \vec{x},\ t)+\frac{1}{2}(\vec{r}'-\vec{r})f(\vec{r}-\vec{r}',\ x,\ t)\frac{\partial}{\partial x}u(\vec{r}',\ \vec{x},\ t). \tag{3.16}$$

Taking the Fourier transformation of (3.16) and using the relation:

$$\vec{r}'e^{i\vec{p}_2\cdot\vec{r}'} = -i\frac{\partial}{\partial p_1}e^{i\vec{p}_2\cdot\vec{r}}, \tag{3.17}$$

the integration over the space and momentum variables $\vec{r}$, $\vec{r}$', $\vec{p}_1$, and $\vec{p}_2$ can be performed, Eq.(3.16) becomes:

$$f(\vec{x}_1, \vec{x}_3)u(\vec{x}_3, \vec{x}_2) = f(\vec{x}_1, \vec{p}, t)u(\vec{x}, \vec{p}, t) + \frac{i}{2}\left[\frac{\partial}{\partial \vec{x}} f(\vec{x}, \vec{p}, t) + \frac{\partial}{\partial \vec{p}} u(\vec{x}, \vec{p}, t)\right.$$
$$\left. - \frac{\partial}{\partial \vec{p}} f(\vec{x}, \vec{p}, t) \frac{\partial}{\partial \vec{x}} u(\vec{x}, \vec{p}, t)\right], \tag{3.18}$$

Substituting (3.18) into Eq.(3.7) and using the Wigner function of Eq(2.7), the r. h. s. of (3.7) can be finally expressed as

$$\text{Right side} = \left(\frac{\partial}{\partial \vec{x}} \Sigma^{\text{HF}}(\vec{x}, \vec{p}, t) \frac{\partial}{\partial \vec{p}} - \frac{\partial}{\partial \vec{p}} \Sigma^{\text{HF}}(\vec{x}, \vec{p}, t) \frac{\partial}{\partial \vec{x}}\right) f(\vec{x}, \vec{p}, t). \tag{3.19}$$

We find that in the local approximation, when (3.12) is expanded up to the first-order approximation and Eq.(3.9) is used, the commonly used Vlasov equation has been obtained:

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m} \nabla_x - \nabla_x \Sigma^{\text{HF}} \nabla_p\right) f(\vec{x}, \vec{p}, t) = 0. \tag{3.20}$$

While in the non-local approximation (3.20) is modified and becomes

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m} \nabla_x - \nabla_x \Sigma^{\text{HF}} \nabla_p + \nabla_p \Sigma^{\text{HF}} \nabla_x\right) f(\vec{x}, \vec{p}, t) = 0 \tag{3.21}$$

Comparing (3.21) with (3.20), the Vlasov equation now acquires an additional term $\nabla_p \Sigma^{\text{HF}} \nabla$, because of the non-locality of the interaction for the momentum-dependent H-F mean field.

4.COLLISION TERM

Within the first-order and the second-order perturbation approximation, the Kadanoff-Baym equation can be derived from the Dyson equation,

$$G_{01}^{0-1} G^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \int d\vec{x}_3 dt_3 \Sigma^{\text{HF}}(\vec{x}_1, \vec{x}_3, t_1) G^{-+}(\vec{x}_3, t_1, \vec{x}_2, t_2)$$
$$- \int_{t_0}^{t_1} [\Sigma^{(2)+-}(\vec{x}_1, \vec{x}_3, t_1) - \Sigma^{(2)-+}(\vec{x}_1, \vec{x}_3, t_1)] G^{-+}(\vec{x}_3, t_1, \vec{x}_2, t_2) d\vec{x}_3 dt_3$$
$$+ \int_{t_0}^{t_2} \Sigma^{(2)-+}(\vec{x}_1, \vec{x}_3, t_1)[G^{+-}(\vec{x}_3, t_1, \vec{x}_2, t_2) - G^{-+}(\vec{x}_3, t_1, \vec{x}_2, t_2)] d\vec{x}_3 dt_3, \tag{4.1}$$

$$G_{02}^{*-01} G^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) = \int d\vec{x}_3 dt_3 G^{-+}(\vec{x}_1, t_1, \vec{x}_3, t_2) \Sigma^{\text{HF}}(\vec{x}_3, \vec{x}_2, t_2)$$
$$- \int_{t_0}^{t_1} [G^{+-}(\vec{x}_1, t_1, \vec{x}_3, t_2) - G^{-+}(\vec{x}_1, t_1, \vec{x}_3, t_2)] \Sigma^{(2)-+}(\vec{x}_3, \vec{x}_2, t_2) d\vec{x}_3 dt_3$$
$$+ \int_{t_0}^{t_2} G^{-+}(\vec{x}_1, t_1, \vec{x}_3, t_2)[\Sigma^{(2)+-}(\vec{x}_3, \vec{x}_2, t_2) - \Sigma^{(2)-+}(\vec{x}_3, \vec{x}_2, t_2)] d\vec{x}_3 dt_3, \tag{4.2}$$

The first terms in Eqs.(4.1) and (4.2) are the results in the first-order perturbation. As demonstrated in Section 3, it gives rise to the Vlasov equation in combination with l. h. s. of (3.7). In the following we will mainly discuss the contributions of the second and the third terms in (4.1) and (4.2). Let C stand for the contribution from the second-order perturbation:

$$\begin{aligned} C = -\int [\,&\Sigma^{(2)--}(\vec{x}_1, \vec{x}_3, t_1) G^{-+}(\vec{x}_3, t_1, \vec{x}_2, t_2) + \Sigma^{(2)-+}(\vec{x}_1, \vec{x}_3, t_1) \\ &\times G^{++}(\vec{x}_3, t_1, \vec{x}_2, t_2) + G^{-+}(\vec{x}_1, t_1, \vec{x}_3, t_2)\Sigma^{(2)++}(\vec{x}_3, \vec{x}_2, t_2) \\ &+ G^{-+}(\vec{x}_1, t_1, \vec{x}_3, t_2) \times \Sigma^{(2)--}(\vec{x}_3, \vec{x}_2, t_2)\,], d^4\vec{x}_3 \end{aligned} \tag{4.3}$$

Eq.(4.3) possesses the nature of the product of functions of double variables as in (3.15). Similar to the case for Σ^{HF} we only keep zero-order term in the Taylor expansion and make the Fourier transformation. Furthermore by approximately introducing $\delta(t_1 - t_2)$ which implies an integration over $d\omega/2\pi$ (4.3) can be simplified as,

$$\begin{aligned} C = \int \{ &-\Sigma^{(2)-+}(\vec{x}, \omega, \vec{p}, t) G^{+-}(\vec{x}, \omega, \vec{p}, t) + \Sigma^{(2)+-}(\vec{x}, \omega, \vec{p}, t) \\ &\times G^{-+}(\vec{x}, \omega, \vec{p}, t)\} \frac{d\omega}{2\pi} \end{aligned} \tag{4.4}$$

In nuclear matter, the field operator can be written as[11]

$$\hat{\phi}(\vec{x}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{p}} \hat{a}_{\vec{p}} \exp[i[\vec{p} \cdot \vec{x} - (\varepsilon_p - \mu)t]] \tag{4.5}$$

where the operator $\hat{a}_{\vec{p}}$ has the properties:

$$\langle a^+_{\vec{p}} a_{\vec{p}} \rangle = n_{\vec{p}}, \tag{4.6}$$

$$\langle a_{\vec{p}} a^+_{\vec{p}} \rangle = 1 - \vec{n}_p. \tag{4.7}$$

Then the single-particle Green function can be represented by (4.5), (4.6) and (4.7) as

$$G^{(0)-+}(\omega, \vec{p}) = 2\pi i n_{\vec{p}} \delta(\omega - \varepsilon_{\vec{p}} + \mu) = 2\pi i n_{\vec{p}} \delta(\omega - \omega_p), \tag{4.8}$$

$$G^{(0)+-}(\omega, \vec{p}) = -2\pi i(1 - n_{\vec{p}})\delta(\omega - \omega_{\vec{p}}). \tag{4.9}$$

We assume that $G(\vec{x},t)$ varies with $\vec{x}$ rather slowly, which means that the coordinates do not change rapidly before and after the collisions. In this situation, the local density approximation can be applied, i.e.,

$$n_{\vec{p}} \approx f(\vec{x}, \vec{p}, t), \tag{4.10}$$

By substituting (4.8), (4.9) and (4.10) into (4.4), we find

$$\begin{aligned} C = \int \{ &-\Sigma^{(2)-+}(\vec{x}, \omega, \vec{p}, t)[-2\pi i(1 - f(\vec{x}, p, t))\delta(\omega - \omega_{\vec{p}})] \\ &+ \Sigma^{(2)+-}(\vec{x}, \omega, \vec{p}, t)[2\pi i f(\vec{x}, \vec{p}, t)\delta(\omega - \omega_{\vec{p}})]\} \frac{d\omega}{2\pi}. \end{aligned} \tag{4.11}$$

If we take the two-body interaction to be the form:

$$V(\vec{x}_1, t_1, \vec{x}_2, t_3) = V(\vec{x}_1 - \vec{x}_3)\delta(t_1 - t_3),$$
$$V(\vec{x}_2, t_2, \vec{x}_4, t_4) = V(\vec{x}_2 - \vec{x}_4)\delta(t_2 - t_4), \tag{4.12}$$

then Eq.(2.25) can be rewritten as,

$$\begin{aligned}\Sigma^{(2)-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) = &-\int d\vec{x}_3 d\vec{x}_4 V(\vec{x}_1 - \vec{x}_3) V(\vec{x}_2 - \vec{x}_4) G^{(0)-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) \\ &\times G^{(0)+-}(\vec{x}_4, t_2, \vec{x}_3, t_1) G^{(0)-+}(\vec{x}_3, t_1, \vec{x}_4, t_2) + \int d\vec{x}_2 d\vec{x}_4 V(\vec{x}_1 - \vec{x}_3) \\ &\times V(\vec{x}_2 - \vec{x}_4) G^{(0)-+}(\vec{x}_1, t_1, \vec{x}_4, t_2) G^{(0)+-}(\vec{x}_4, t_2, \vec{x}_3, t_1) G^{(0)-+}(\vec{x}_3, t_1, \vec{x}_2, t_2) \\ = &-{}^d\Sigma^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) + {}^e\Sigma^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2)\end{aligned} \tag{4.13}$$

where "d" means the direct term, "e" the exchange term. First of all, let us evaluate the direct term. Like Eqs.(3.10) and (3.16), we make the coordinate transformation and Fourier transformation, and integrate over t, then we get the expression

$$\begin{aligned}{}^d\Sigma^{(2)-+}(\vec{x}, \omega, \vec{p}, t) = &-\int d\vec{r}' d\vec{x}' \int d\vec{r} e^{-i\vec{p}\cdot\vec{r}} \int \frac{d^3p_3}{(2\pi)^3} e^{i\vec{p}_3(\vec{x}-\vec{x}') + \frac{i}{2}\vec{p}_3(\vec{r}-\vec{r}')} \\ &\times V(p_3) \int \frac{d^3p_4}{(2\pi)^3} e^{i\vec{p}_4(\vec{x}-\vec{x}') - i\frac{\vec{p}_4}{2}(\vec{r}-\vec{r}')} V(\vec{p}_4) \int \frac{d\vec{p}'d\omega'}{(2\pi)^4} e^{i\vec{p}'\vec{r}} G^{-+}(\vec{x}, \vec{p}', \vec{\omega}', t) \\ &\times \int \frac{d\vec{p}_1' d\omega_1'}{(2\pi)^4} e^{i\vec{p}_1'\vec{r}'} G^{-+}(\vec{x}', \vec{p}_1', \omega_1', t) \int \frac{d\vec{p}_1 d\omega_1}{(2\pi)^4} e^{-i\vec{p}_1\vec{r}'} G^{+-}(\vec{x}', \vec{p}_1, \omega_1, t) \\ &\times 2\pi\delta(\omega + \omega_1 - \omega' - \omega_1').\end{aligned} \tag{4.14}$$

The integrand of (4.14) does not depend on the $\vec{x}'$ because we assume that $G(\vec{x}, t)$varies with $\vec{x}$ slowly in analogy to the approximation of (4.10). Therefore, $(2\pi)^3\delta(\vec{p}_3 \times \vec{p}_4)$ will appear after integrating over $\vec{x}'$. After performing the integrations over $\vec{r}$, $\vec{r}'$, $\vec{p}_3$ and $\vec{p}_4$, respectively, we arrive at the expression

$$\begin{aligned}{}^d\Sigma^{(2)-+}(\vec{x}, \omega, \vec{p}, t) = &-\int \frac{d\vec{p}'d\omega'}{(2\pi)^4} \int \frac{d\vec{p}_1' d\omega_1'}{(2\pi)^4} \int \frac{d\vec{p}_1 d\omega_1}{(2\pi)^4} (2\pi)^4 \delta(\vec{p} + \vec{p}_1 - \vec{p}_1' - \vec{p}') \\ &\times \delta(\omega + \omega_1 - \omega_1' - \omega') |V(\vec{p} - \vec{p}')|^2 G^{-+}(\vec{x}, \omega', \vec{p}', t) \\ &\times G^{-+}(\vec{x}, \omega_1', \vec{p}_1', t) G^{+-}(\vec{x}, \omega_1, \vec{p}_1, t).\end{aligned} \tag{4.15}$$

Due to the approximation that G^{-+} and G^{+-}vary slowly with $\vec{x}$, $G^{+-}$and G^{-+}can also be approximated by the free particle Green function which does not explicitly depend on $\vec{x}$. Then, we have

$$
{}^{d}\Sigma^{(2)-+}(\vec{p},\omega,t) = -\int \frac{d\vec{p}'d\omega'}{(2\pi)^4}\int \frac{d\vec{p}_1'd\omega_1'}{(2\pi)^4}\int \frac{d\vec{p}_1 d\omega_1}{(2\pi)^4}(2\omega)^4\delta(\omega_1+\omega-\omega_1'-\omega') \tag{4.16}
$$
$$
\times \delta(\vec{p}_1+\vec{p}-\vec{p}_1'-\vec{p}')\,|V(\vec{p}-\vec{p}')|^2 G^{(0)-+}(\vec{p}',\omega',t)
$$
$$
\times G^{(0)-+}(\vec{p}_1',\omega_1',t)G^{(0)+-}(\vec{p}_1,\omega_1,t).
$$

The exchange term can be given using the same method and approximation:

$$
{}^{e}\Sigma^{(2)-+}(\vec{p},\omega,t) = \int \frac{d\vec{p}'d\omega'}{(2\pi)^4}\int \frac{d\vec{p}_1'd\omega_1'}{(2\pi)^4}\int \frac{d\vec{p}_1 d\omega_1}{(2\pi)^4}(2\pi)^4\delta(\omega_1+\omega-\omega_1'-\omega') \tag{4.17}
$$
$$
\times \delta(\vec{p}_1+\vec{p}-\vec{p}_1'-\vec{p}')\,|V(p_1'-p)V(p-p')|
$$
$$
\times G^{(0)-+}(\vec{p}',\omega,t)G^{(0)-+}(\vec{p}_1',\omega_1',t)G^{(0)+-}(\vec{p}_1,\omega_1,t),
$$

Substituting (4.8) and (4.9) into (4.16) and (4.17), the total $\Sigma^{(2)-+}(\vec{x},\omega,\vec{p},t)$ is obtained from (4.13). Integrating over ω', ω_1' and ω_1, we obtain an equation

$$
i\Sigma^{(2)-+}(\vec{p},\omega_p,x,t) = \int \frac{d\vec{p}_1}{(2\pi)^3}\int \frac{d\vec{p}_1'}{(2\pi)^3}\int \frac{d\vec{p}'}{(2\pi)^3}\,(2\pi)^4\delta(\omega_p+\omega_{p_1}-\omega_{p'}-\omega_{p_1'})
$$
$$
\times \delta(\vec{p}+\vec{p}_1-\vec{p}'-\vec{p}_1')[V^2(\vec{p}-\vec{p}')+V^2(p-p_1')][1-f(\vec{p}_1,x,t)] \tag{4.18}
$$
$$
\times f(\vec{p}_1',x,t)f(\vec{p}',x,t),
$$

In the same way, $-i\Sigma^{(2)+-}$ can be written as follows:

$$
-i\Sigma^{(2)+-}(\vec{p},\omega_p,x,t) = \int \frac{d\vec{p}_1}{(2\pi)^3}\int \frac{d\vec{p}_1'}{(2\pi)^3}\int \frac{d\vec{p}'}{(2\pi)^3}(2\pi)^4\delta(\omega_p+\omega_{p_1}-\omega_{p'}-\omega_{p_1'})
$$
$$
\times \delta(\vec{p}+\vec{p}_1-\vec{p}'-\vec{p}_1')[V^2(\vec{p}-\vec{p}')+V^2(\vec{p}-\vec{p}_1')] \tag{4.19}
$$
$$
\times [1-f(\vec{p}_1',x,t)][1-f(\vec{p}',x,t)]f(\vec{p}_1,x,t).
$$

Inserting the equations (4.18) and (4.19) into (4.11) we obtain the expression of the collision term in the non-local approximation:

$$
C = \int \frac{d\vec{p}_1}{(2\pi)^3}\int \frac{d\vec{p}_1'}{(2\pi)^3}\int \frac{d\vec{p}'}{(2\pi)^3}\,(2\pi)^4\delta(\omega_p+\omega_{p_1}-\omega_{p'}-\omega_{p_1'})\delta(\vec{p}+\vec{p}_1-\vec{p}_1'-\vec{p}')
$$
$$
\times [V^2(\vec{p}-\vec{p}')+V^2(\vec{p}-\vec{p}_1')]\{[1-f(\vec{p},x,t)][1-f(\vec{p}_1,x,t)]f(\vec{p}_1',x,t) \tag{4.20}
$$
$$
\times f(\vec{p}_1',x,t)-f(\vec{p},x,t)f(\vec{p}_1,x,t)[1-f(\vec{p}',x,t)][1-f(\vec{p}',x,t)]\}.
$$

In the local approximation of the interaction, i.e., $V(\vec{x}_1,\vec{x}_3)\approx V_0\delta(\vec{x}_1-\vec{x}_3)\times\delta(t_1-t_3)$, in which V_0 does not depend on the momentum, the calculated results show that the contributions of two diagrams in Fig.3 to the self-energy cancel each other and only the exchange term is left in C:

$$
C = V_0\int \frac{d\vec{p}_1}{(2\pi)^3}\int \frac{d\vec{p}_1'}{(2\pi)^3}\int \frac{d\vec{p}'}{(2\pi)^3}\,(2\pi)^4\delta(\omega_p+\omega_{p_1}-\omega_{p'}-\omega_{p_1'})\delta(\vec{p}+\vec{p}_1-\vec{p}'-\vec{p}_1')
$$
$$
\times \{[1-f(\vec{p},\vec{x},t)][1-f(\vec{p}_1,x,t)]f(\vec{p}',x,t)f(\vec{p}_1',x,t) \tag{4.21}
$$
$$
-f(\vec{p},x,t)f(\vec{p}_1,x,t)[1-f(\vec{p}',x,t)][1-f(\vec{p}_1',x,t)]\}.
$$

5. SUMMARY AND DISCUSSION

In the local approximation the BUU equation reads

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\nabla_x-\nabla_x\Sigma^{\mathrm{HF}}\nabla_p\right)f(\vec{x},\vec{p},t)=C, \tag{5.1}$$

where C is given by Eq.(4.21), which comes from the exchange term. In the non-local approximation, the BUU equation is written as

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\nabla_x-\nabla_x U^{\mathrm{HF}}\nabla_p+\nabla_p U^{\mathrm{HF}}\nabla_x\right)f(\vec{x},\vec{p},t)=C, \tag{5.2}$$

where C is given by Eq.(4.20). In order to obtain the BUU equation, the following necessary assumptions have to be made in both the local and the non-local approximations:

1) The contributions to the self-energy come from the narrow domain of $\vec{r}\,'$ and $\vec{r}-\vec{r}\,'$ around $\vec{x}$ in both the H-F and Born approximations and the lowest order Taylor expansion is valid.

2) The Green function slowly varies with coordinate $\vec{x}$. This means that the change of coordinate is small after and before two-body collisions. Following this assumption the particle density at momentum $\vec{p}$ is replaced by that with the classical coordinate and momentum, and the local density approximation like (4.10) can be used. In the same way, integration of (4.14) is simplified because $\vec{x}\,' = \frac{1}{2}(\vec{x}_3 + \vec{x}_4)$ can be substituted by $\vec{x} = \frac{1}{2}(\vec{x}_1 + \vec{x}_2)$.

3) In connection with the approximation (2), we assume that both G^{+-} and G^{-+} take the forms of the free particle Green function as (4.8) and (4.9). Because of the small variation of the coordinates after and before the collisions, the Green function and the self-energies are independent of $\vec{x}$ and $\vec{x}\,'$ approximately but depend on the energy ω and momentum p. We note that the integration of the quantum-kinetic equation with respect to the energy ω leads to the classical BUU equation.

It should be noted that the BUU equation is derived under various approximations and that the BUU equation suits the dilute density and is obtained by only considering the two-body collisions. In analogy to the assumption of the Boltzmann equation, the molecule chaos assumption is clearly included in the above three approximations.

The Boltzmann-like equation has been derived by some authors based on the non-equilibrium Green function method. Discussions have recently been given by J. Rammer et al.[13] and P. Danielewicz[12]. In comparison with our results, the result of Ref.[13] is the same as ours in the local case because it takes $V(\vec{x}) = V_0$, while in Ref.[12] the equation obtained is similar to our result in the non-local approximation. We note that it is necessary to take some proper approximations in the study of collision term by different methods. In this paper we have systematically studied the BUU equation in two kinds of approximations. The appearance of the $\nabla_p\Sigma^{\mathrm{HF}}$ and ∇_xterms in the Vlasov equation is the nature of the lowest order approximation. If Σ^{HF} does not depend on the momentum, Eq.(3.21) becomes a local form which is commonly used.

The calculated H-F mean field shows that only the density dependence of the mean field is not enough because the effect of the effective mass should be considered. Therefore, it is important to study the density and momentum dependence of the mean field. Of course, it is related to the physical system. We believe that the discussion on the approximation of the BUU from the view of quantum mechanics is of significance.

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