

# The Gas Gain of a Proportional Tube

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In this paper the gas gain of a proportional tube is discussed. On the basis of analysing the existing calculation formulas, a new  $\alpha/p-E/p$  relation and a corresponding formula of gas gain are proposed. The theory is compared with the experimental data of  $\text{BF}_3$  and  $\text{CH}_4$  proportional tubes.

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Proportional chambers and multi-wire chambers of various types are widely used in the experimental area of particle physics. One of their important characteristics is gas gain. The calculation formulas are obtained from the extension of the formulas of the gas gain for a single wire proportional tube[1--3]. The characteristics of the gas gain in a proportional tube have been studied theoretically and experimentally by several authors with several main calculation formulas suitable for different ranges of electric field strength and gas pressure[4]. The crux lies in the research for the relation of  $\alpha/p$  to  $E/p$ . In this paper a new  $\alpha/p-E/p$  relation and corresponding formula of the gas gain is proposed. Based on the experimental data of the gas gain of home-made  $\text{BF}_3$  proportional tubes and methane ( $\text{CH}_4$ ) proportional tubes from publications, various theoretical formulas are analyzed and compared. As a result, the related parameters are obtained.

## 1. EXPERIMENTAL DATA

With the exception of the experiments for methane proportional tubes from publications, the experimental data used in this paper are taken from the experiments for the gas gain of home-made  $\text{BF}_3$  proportional tubes[5]. The experimental condition is briefed as follows. Various cylindrical  $\text{BF}_3$  proportional tubes are used with a gas pressure of 200 to 610 mmHg and central tungsten wires with radii of 0.00254 to 0.00765 cm, the radius and length of the copper cathodes being 1.03 cm and 25 cm respectively. The multiplication factor  $A$  is measured by two methods through measuring the output pulse height and the output current. The results of these two methods are compared with each

other. For the method of measuring the output pulse height,  $A = Q/ne$ . The  $\alpha$  particle yield in  $^{10}\text{B}(n, \alpha)^7\text{Li}$  caused by Po-Be neutron is used for creating the ionization in  $\text{BF}_3$  tubes. The energy of the  $\alpha$  particles is known as 2.795 MeV and the energy to create a pair of ions in  $\text{BF}_3$  gas is  $w = 33.8$  eV. So the value of the primary ionization is  $n = 8.27 \times 10^4$ . In order to measure the charge  $Q$  collected on anode wire after the gas multiplication, the output end of the anode wire is connected to the input of the electronic circuit through the central wire of the cylindrical capacitor  $C_1(0.76\text{pf})$ . The negative pulses of amplitude  $v$  provided by a pulser, the shape of which is similar to the output signal of the proportional tube, are applied to the capacitor. Consequently the negative pulses corresponding to charge  $vC_1$  are produced on the load and used for calibrating the mean output pulse height of the  $\text{BF}_3$  proportional tube. So  $Q$  is equal to  $vC_1$ . For the method of measuring the output current, a  $^{60}\text{Co}$   $\gamma$  source is used for radiating the proportional tube. The output current  $i$  is measured as a function of the operation voltage.  $A = i/i_0$  and  $i_0$  correspond to the saturation current of the ionization chamber region in  $A$  vs. the operation voltage curve. The curve of the amplification factor  $A$  is then obtained as a function of the operation voltage in the experimental measurements.

## 2. THEORETICAL FORMULAS, COMPARISON WITH EXPERIMENTS AND DETERMINATION OF PARAMETERS

During the movement of an electron to the central anode wire in the proportional tube, the total number  $N$  of electrons produced along its path is calculated according to the following equation

$$dN = -\alpha N dr,$$

thus it is derived that

$$\ln A = \int_a^b \alpha dr.$$

Here  $a$  and  $b$  are the radii of the central wire and the cathode wire respectively.  $r$  is the distance from the electron to the central wire.  $\alpha$  is the ionization coefficient or the first Townsend coefficient, i.e. the number of ionization or ion pairs produced by one electron in a 1 cm path.  $\alpha$  is a function of  $E/p$  and is generally expressed as

$$\alpha/p = f(E/p) = (E/p)\eta(E/p).$$

For the cylindrical proportional tube, the relation derived from calculation is

$$\frac{\ln A \ln(b/a)}{V} = \int_{E_b/p}^{E_a/p} \eta \frac{1}{E/p} d(E/p). \quad (1)$$

where  $V$  is the operation voltage applied to the proportional tube,  $E = V/[r \ln(b/a)]$  is the electric field strength,  $E_a = V/[a \ln(b/a)]$  and  $E_b$  are the electric field strength on the surface of the central wire and inner surface of the cathode respectively, and  $p$  is the pressure of  $\text{BF}_3$  gas in the tube.

Assuming that the ionization cross-section increases linearly with the energy of the electron and the energy distribution of the electron is decreasing monotonously for an energy higher than the ionization potential, Rose and Korff derived theoretically for the first time [6-8]

$$\alpha/p = k(E/p)^{1/2} \quad (\text{i.e., } \eta = k(E/p)^{-1/2}). \quad (2)$$

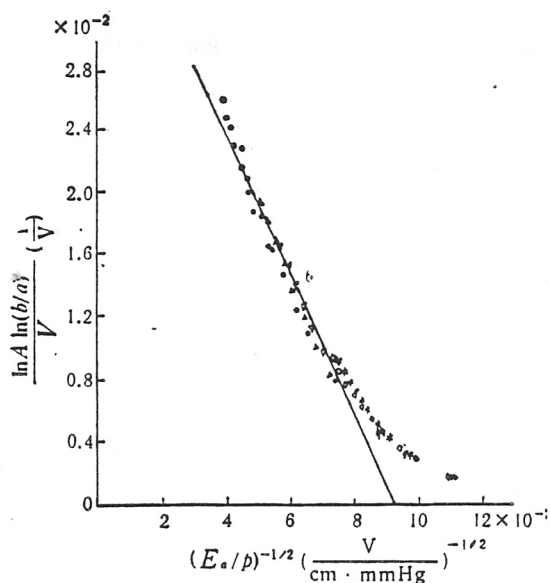


FIG.1

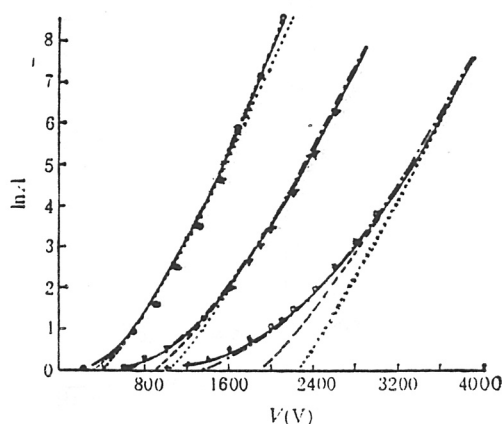


FIG.2

**FIG.1** The  $[\ln A \ln(b/a)]/V$  vs.  $(E_a/p)^{-1/2}$  plot of the experimental data of  $\text{BF}_3$  proportional tubes based on Korff Formula (3) (The experimental points  $\circ$ ,  $\Delta$  and  $\nabla$  correspond to  $p = 200, 250, 450$  and  $610$  mmHg respectively with  $a = 0.00254$  cm; and  $*$  to  $a = 0.00635$  and  $0.00763$  cm respectively with  $p = 610$  mmHg).  $2k = 0.45 [1/(V \cdot \text{cm} \cdot \text{mmHg})]^{1/2}$ ;  $(E_c/p)^{-1/2} = 9.3 \times 10^{-2} [V/(\text{cm} \cdot \text{mmHg})]^{1/2}$ .

**FIG.2** Several examples of the relation curve of  $\ln A - V$  for the  $\text{BF}_3$  proportional tubes. The curve ..... from Korff Formula (3); — according to Diethorn Formula (5); — from Formula (9); -- from the Formula (16) deduced in this paper (The experimental points  $\circ$  and  $\nabla$  correspond to  $p = 250$  and  $610$  mmHg respectively with  $a = 0.00254$  cm; to  $a = 0.00635$  cm,  $p = 610$  mmHg).

Substituting this Relation (2) for  $\eta$  in Eq.(1) and integrating Eq.(1) from  $E_c/p$  to  $E_a/p$  gives

$$\frac{\ln A \ln(b/a)}{V} = 2k[(E_c/p)^{-1/2} - (E_a/p)^{-1/2}]. \quad (3)$$

Here assuming that  $\alpha/p = 0$  as  $E/p \leq E_c/p$ , where  $E_c = V_c/[a \ln(b/a)] = V/[r_c \ln(b/a)]$ ,  $E_c$ ,  $V_c$  and  $r_c$  are the corresponding values of the threshold,  $k$  and  $(E_c/p)^{-1/2}$  are the characteristic constants of the gas. According to the Korff Formula (3) the plot of  $[\ln A \ln(b/a)]/V$  to  $(E_a/p)^{-1/2}$  for the experimental data of the  $\text{BF}_3$  proportional tube is shown in Fig.1. The gas constants  $k = 0.23 (\text{V} \cdot \text{cm} \cdot \text{mmHg})^{-1/2}$  and  $(E_c/p)^{-1/2} = 9.3 \times 10^{-2} [V/(\text{cm} \cdot \text{mmHg})]^{1/2}$  are obtained by drawing a straight line along the experimental points (the values of the methane given in Publication [9] are  $k = 0.33 (\text{V} \cdot \text{cm} \cdot \text{mmHg})^{-1/2}$  and  $(E_c/p)^{-1/2} = 9.1 \times 10^{-2} [V/(\text{cm} \cdot \text{mmHg})]^{1/2}$ ). In addition, several examples of  $\ln A - V$  curve calculated from Formula (3) are shown in Fig.2. In the range of  $E_a/p \leq 165 \text{ V}/(\text{cm} \cdot \text{mmHg})$ , obviously the theory is not consistent with the experiments, particularly for the cases of higher gas pressure and thick radius of the central wire (Figs.1 and 2). It is indicated by the experiments of some authors with other gases that Korff's theoretical formula could not describe the experiments perfectly in the range of  $E_a/p < 1000 \text{ V}/(\text{cm} \cdot \text{mmHg})$ . The experiment of Specht and Armbruster for a

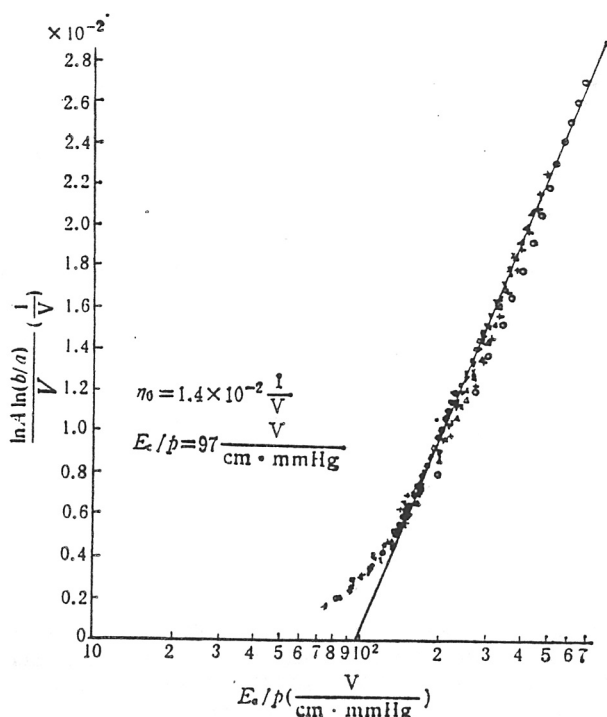


FIG.3 Check Diethorn Formula (5) with experiments (The experimental points  $\circ + \Delta \times \nabla$  correspond to  $p = 200, 300, 400, 500$ , and  $610$  mmHg respectively with  $a = 0.00254$  cm;  $\bullet$  \* to  $a = 0.003, 0.00508, 0.00635$  and  $0.00763$  cm respectively with  $p = 610$  mmHg).

Diethorn directly assumed a linear relation between  $\alpha$  and  $E$ [10], i.e.

$$\alpha/p = \eta_0 E/p \quad (\text{i.e., } \eta = \eta_0) \quad (4)$$

Integration of Eq.(1) from  $E_c/p$  to  $E_a/p$  with this relation gives

$$\frac{\ln A \ln(b/a)}{V} = \eta_0 [\ln(E_a/p) - \ln(E_c/p)] \quad (5)$$

methane proportional tube of very low gas pressure (0.33--10 mmHg) showed that for  $E_a/p$  from 1000 up to  $10^4$  V/(cm·mmHg) it could be consistent with experiments to a certain extent. where  $\eta_0 = \ln 2 / \Delta V$ ,  $\eta_0$  and  $E_c/p$  are the gas constants,  $\Delta V$  is the mean energy obtained by the electron from the electric field between successive ionization collisions. The relation plot of  $[\ln A \ln(b/a)/V]$  to  $E_a/p$  is drawn according to the Diethorn Formula (5) with the experimental data of gas gain in the  $\text{BF}_3$  proportional tubes [5] and is shown in Fig.3. The experimental points distribute within a range of certain width along a straight line. The values of the gas constant are  $\eta_0 = 1.4 \times 10^{-2} \text{ V}^{-1}$  (i.e.  $\Delta V = 49.5$  V) and  $E_c/p = 97 \text{ V}/(\text{cm} \cdot \text{mmHg})$  (the corresponding values for methane given by the experiments in publication[10] are  $\Delta V = 40.3$  V and  $E_c/p = 74.8 \text{ V}/(\text{cm} \cdot \text{mmHg})$ ). In

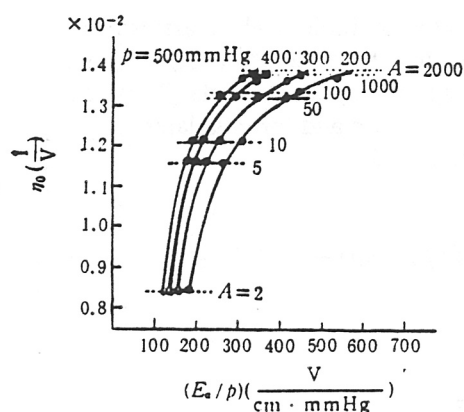


FIG.4

FIG.4  $\eta_0$  as a function of  $E_a/p$ .

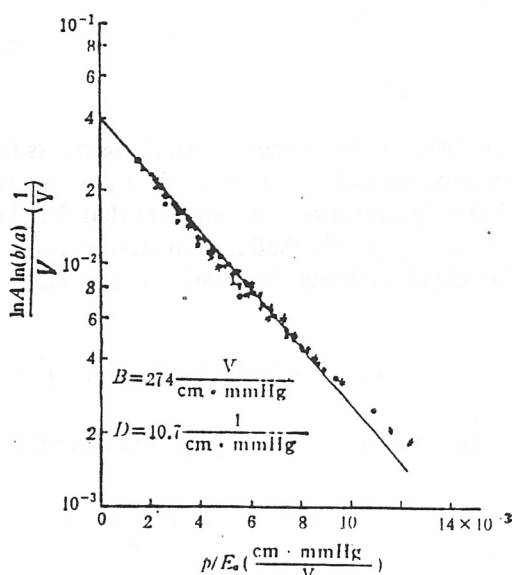


FIG.5

FIG.5 The comparison of Formula (9) with the experimental data of  $\text{BF}_3$  proportional tubes (The experimental points  $\circ + \Delta \times \nabla$  correspond to  $p = 200, 300, 400, 500$  and  $610$  mmHg respectively with  $a = 0.00254$  cm;  $\bullet$  to  $a = 0.00508, 0.00635$  and  $0.00763$  cm respectively with  $p = 610$  mmHg).

addition, several examples of  $\ln A - V$  curve are also shown in Fig.2. The experimental data used here belong to the range of  $E_a/p < 750$  V/(cm·mmHg), the coincidence of the Diethorn formula with experiments is better than that of the Korff formula but the deviation with experiments is still obvious in the range of  $E_a/p \sim 130$  V/(cm·mmHg). The Diethorn formula is applicable to the range of  $E_a/p \lesssim 800-1000$  V/(cm·mmHg) which is the usual range of proportional tubes. It is a formula generally used for the calculation of the gas gain[11-13], but it cannot be used for very low gas pressure, i.e.  $E_a/p > 1000$  V/(cm·mmHg), because of its disagreement with experiments. If we analyze Fig.3 carefully, it can be seen that the experimental points of different gas pressure  $p$  are not exactly on the same straight line, the gas constants  $\eta_0$  and  $E_c/p$  vary slightly with pressure and  $\eta_0$  is dependent on  $E_a/p$ . The relation curves of  $\eta_0$  as a function of  $E_a/p$  can be obtained from Fig.3 and are shown in Fig.4. The curves are similar to the form of  $k_1 \exp[-k_2/(E_a/p)]$ , i.e.  $\alpha/p \approx k_1(E/p) \exp[-k_2/(E/p)]$ . We shall theoretically deduce this relation later.

Zastawny proposed another linear relation based on the experimental data of  $\text{CO}_2$  proportional tubes with the gas pressures of 570 to 1250 mmHg[14]

$$\alpha/p = B_1[(E/p) - S_0], \quad (6)$$

and derived the corresponding formula of the gas gain

$$\begin{aligned} \frac{\ln A \ln(b/a)}{V} = & B_2 \\ & + B_1 \left[ \ln \left( \frac{E_a/p}{S_0} \right) + \frac{S_0}{E_a/p} - 1 \right]. \end{aligned} \quad (7)$$

$B_1$  and  $S_0 = (E/p)_0$  are the gas constants.  $B_2$  accounts for a small contribution, which arises from the difference between the linear approximation and the real function of  $\alpha/p$  for  $E_a/p \lesssim S_0$ . Relation (6) has only one more constant term than Diethorn's assumption, but Formula (7) can be applicable to lower  $E_a/p$ . We shall also theoretically deduce this relation and formula later.

Engel et al.[15] deduced the relation in the uniform electric field

$$\alpha/p = D \exp[-B/(E/p)] \quad (\text{i.e., } \eta = [D/(E/p)] \exp[-B/(E/p)]). \quad (8)$$

Integration of Eq.(1) from  $E_b/p$  to  $E_a/p$  with this relation yields

$$\frac{\ln A \ln(b/a)}{V} = \frac{D}{B} \exp[-B/(E_a/p)]. \quad (9)$$

Fig.5 shows the comparison between Formula (9) and the experimental data of  $\text{BF}_3$ . The straight line fit of experimental data gives the gas constant  $B = 274 \text{ V}/(\text{cm} \cdot \text{mmHg})$  and  $D = 10.7 (\text{mm} \cdot \text{mmHg})^{-1}$ . Several examples of  $\ln A - V$  curve are also shown in Fig.2 for comparison. It is noteworthy that Formula (9) is well consistent with the experimental data for the range of very low  $E_a/p$  and  $A$ . For the cases of thick central wire and higher gas pressure, this formula is superior to the Diethorn formula. Williams and Sara[16] used such an approach at first and got a formula similar to Eq.(9). Champion[17] et al. also showed this point from the experiments of methane and its mixture with Ar, and furthermore indicated consistence with experiments in the range of  $E_a/p \lesssim 250 \text{ V}/(\text{cm} \cdot \text{mmHg})$ . However, Fig.5 shows that for the  $\text{BF}_3$  gas Eq.(9) is still consistent with experiments up to  $E_a/p \approx 650 \text{ V}/(\text{cm} \cdot \text{mmHg})$ . Please note that when  $E/p$  increases  $\alpha/p$  tends to saturate early in Relation (8). So, Eq.(9) is not applicable for large  $E_a/p$ .

Khristov assumed that[18]

$$\alpha/p = C_1, \quad (10)$$

and derived from Eq.(1)

$$\frac{\ln A \ln(b/a)}{V} = C_2 - \frac{C_1}{E_a/p}. \quad (11)$$

Eq.(11) is applicable to the range of very large  $E_a/p$  ( $> 5 \times 10^3 \text{ V}/(\text{cm} \cdot \text{mmHg})$ ).

As stated above, up to now there are 5 types of  $\alpha/p - E/p$  relation and corresponding formulas of gas gain. They are applicable to different ranges of  $E_a/p$ . Below we are going to deduce a new type of  $\alpha/p - E/p$  relation and formula of gas gain, from which the substantial connection among the above various relations can be seen. It is assumed in discussion that the space charge effect, electron attachment effect and inner photon creation can be neglected. We will deduce the  $\alpha/p$  relation in a uniform electric field first and then derive the formula of the gas gain.

An electron moving in the uniform electric field gains energy continuously from the electric field and collides with gas molecules and atoms. Only in the non-elastic collision the electron loses its energy. Assume that  $\lambda_i$  is the mean free path of the electron. The electron gains an energy equal to the ionization potential  $V_i$  in path  $\lambda_i$ . The probability of the free path  $\geq l$  is  $\exp(-l/\lambda_i)$ . Thus, the probability of the free path between  $l \rightarrow l + dl$  is  $(l/\lambda_i) \exp(-l/\lambda_i) dl$ . The electron gains the energy mainly from the electric field. Therefore, the probability of the energy of this electron between  $V = lE \rightarrow lE + d(lE)$  is

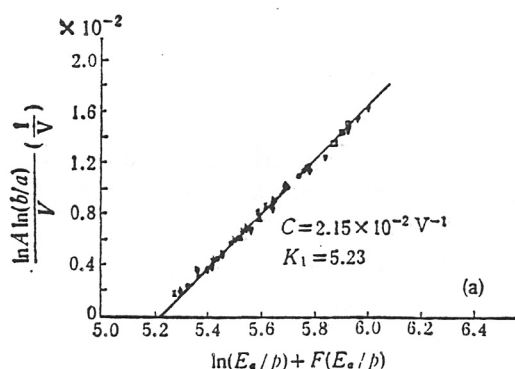


FIG. 6 (a)

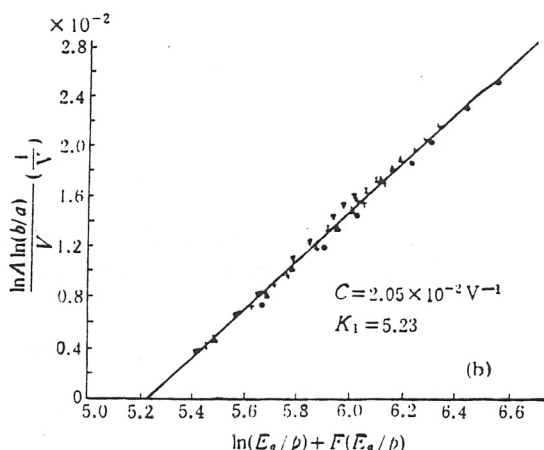


FIG. 6 (b)

FIG. 6 (a) The plot of the experimental data according to Formula (16) for the  $\text{BF}_3$  proportional tubes of various  $a$  (The experimental points  $\nabla$ ,  $\bullet$ ,  $*$  are attributed to  $a = 0.00254, 0.003, 0.0508, 0.00615$  and  $0.00763$  cm respectively with  $p = 610$  mmHg).

FIG. 6 (b) The plot of the experimental data according to Formula (16) for the  $\text{BF}_3$  proportional tubes (The experimental points  $\bullet$ ,  $+$ ,  $\times$  are attributed to  $p = 200, 300, 400, 500$  and  $610$  mmHg respectively with  $a = 0.00254$  cm).

$$dP = [1/(\lambda_i E)] \exp[-V/(\lambda_i E)] dV. \quad (12)$$

According to the relation curve of the ionization cross-section as a function of energy [2,19,20], the relation between the ionization cross-section and the electron energy is linear for the electron energy from ionization potential  $V_i$  to a certain upper limit  $V_m$ .  $V_m$  is dependent on the type of gas and is generally about 100 eV. So it can be assumed that the efficiency of ionization  $f_e(V)$  is

$$f_e(V) = C(V - V_i). \quad (13)$$

In a case of larger gas density, i.e. higher gas pressure, the  $\lambda_i$  is shorter and the energies of most electrons are low. So the probability that the electron moving in the electric field produces ionization can be expressed as

$$P_e = C \int_{V_i}^{\infty} \frac{1}{\lambda_i E} e^{-\frac{V}{\lambda_i E}} (V - V_i) dV. \quad (14)$$

The number of ionization or ion pairs by an electron per cm is  $\alpha = (1/\lambda_i)P_e$ , therefore

$$\alpha = \frac{C}{\lambda_i} \int_{V_i}^{\infty} \frac{1}{\lambda_i E} e^{-\frac{V}{\lambda_i E}} (V - V_i) dV = C E e^{-\frac{V}{\lambda_i E}},$$

thus

$$\frac{\alpha}{p} = C \frac{E}{p} e^{-\frac{g_i}{E/p}} \quad (\text{i.e., } \eta = C e^{-\frac{g_i}{E/p}}). \quad (15)$$

where  $g_i = E_i/p$ ,  $E_i = V_i/\lambda_i$ . This relation is derived from the uniform electric field. Interaction of Eq.(1) from  $E_i/p$  to  $E_a/p$  with this relation gives the corresponding formula of the gas gain

$$\begin{aligned} \frac{\ln A \ln(b/a)}{V} &= C \{ [\ln(E_a/p) + F(E_a/p)] - K_1 \}. \\ F(E_a/p) &= \frac{g_i}{E_a/p} - \frac{1}{2 \times 2!} \left( \frac{g_i}{E_a/p} \right)^2 + \frac{1}{3 \times 3!} \left( \frac{g_i}{E_a/p} \right)^3 - \dots, \\ K_1 &= \ln g_i + 1 - \frac{1}{2 \times 2!} + \frac{1}{3 \times 3!} - \dots. \end{aligned} \quad (16)$$

Here  $C$ ,  $g_i$  and  $K_1$  are the gas constants. Generally for  $F(E_a/p)$  it is accurate enough to take account of the terms up to the cube of  $g_i/(E_a/p)$ . Using the experimental data of  $\text{BF}_3$  proportional tubes to check this formula, the relation plot of  $\ln A \ln(b/a)/V$  to  $[\ln(E_a/p) + F(E_a/p)]$  is shown in Figs. 6(a) and 6(b). In calculation, as a first order approximation the  $g_i = E_i/p = 90 \text{ V}/(\text{cm} \cdot \text{mmHg})$  is abstracted first from Fig.3 and  $V_m = 100 \text{ eV}$  is taken. It can be seen from Fig.6(a) that the experimental points of different radii  $a$ 's of the central wires distribute well along a straight line up to very small  $E_a/p$ . The slope is  $C = 2.15 \times 10^{-2} \text{ V}^{-1}$  and the intersection point is  $K_1 = 5.23$ . It can be found out from Fig.6(b) that the experimental points of different pressures  $p$ 's distribute along the close straight line with slightly different slopes and intersection points. The straight line fitting gives

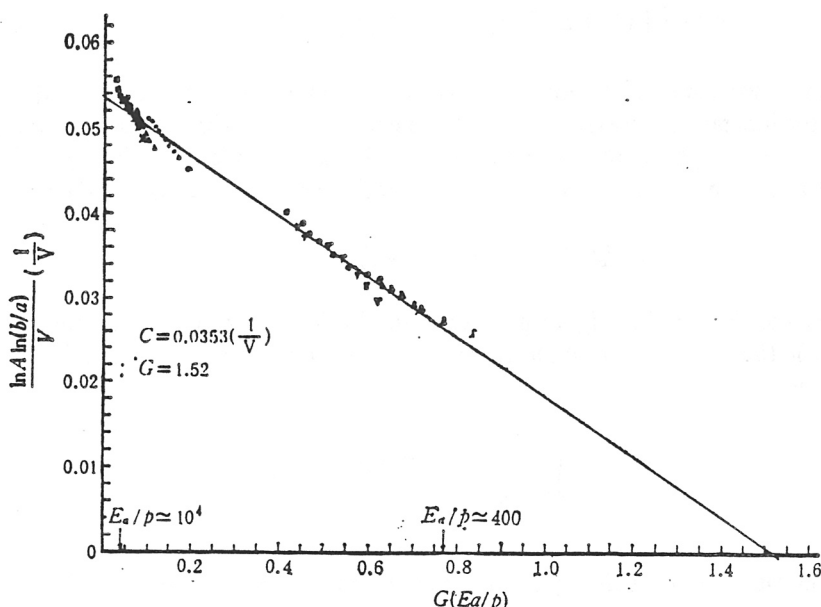


FIG.7 The comparison of Formula (18) with the experiments (The experimental data  $\times \bullet$  correspond to  $p = 1.3, 2.5, 5.0$  and  $10 \text{ mmHg}$  respectively [9];  $+$  [9]; to  $p = 100 \text{ mmHg}$  [6];  $\circ \Delta$  to  $p = 159, 301 \text{ mmHg}$  respectively [8]).

$C = 2.05 \times 10^{-2} \text{ V}^{-1}$  and  $K_1 = 5.23$ . The gas constants in all formulas of the gas gain listed above are slightly dependent on  $p$ ; this is because during the deduction of Eq.(1)  $dE$  is replaced by  $d(E/p)$  with  $p$  being regarded as a constant. The examples of the  $\ln A-V$  curve given by Formula (16) are shown in Fig.2 for comparison. The agreement of experiments with Formula (16) is as good as with the Williams and Sara Formula (9) in the range of low  $E_a/p$  and low gas gain, but with the Diethorn formula above that range (Figs.2 and 6).

Let us look into the  $\alpha/p$  Relation (15) and the Formula of the gas gain (16) deduced in this paper. At first, if we only take the first term in the expansion of Relation (15) and  $g_i/(E_a/p)$  term for  $F(E_a/p)$  in Formula (16), immediately we obtain the Diethorn assumption  $\alpha/p = C(E/p)$  and his Formula (5) of the gas gain. Secondly, if we only take the first and second terms in the expansion of Relation (15) and the terms of up to the square of  $g_i/(E_a/p)$  for  $F(E_a/p)$  in (16), then the Zastawny Relation (6) and his Formula of the gas gain (7) are derived. Consequently the assumption of a linear relation between  $\alpha$  and  $E$  made by Diethorn, and the  $\alpha/p$  Relation (6) assumed by Zastawny with fitting experimental data are theoretically deduced. In the range of low  $E_a/p$  the Relation (15) is close to the exponential relation of Eq.(8) whereas Formula (16) is close to the Williams and Sara Formula (9). Therefore the  $\alpha/p$  Relation (15) and the Formula of the gas gain (16) deduced in this paper embody the connection and unity of three relations or formulas. It is applicable to the range from very low  $E_a/p$  up to  $E_a/p \lesssim 800 \text{ V}/(\text{cm}\cdot\text{mmHg})$ .

In a case of small gas density i.e. low gas pressure, the  $\lambda_i$  is very long and an electron can gain a great deal of energy from the electric field in its free path. The electrons whose energies are above the upper limit  $V_m$  of the linear range of ionization cross-section possess a large portion. Then we have

$$\begin{aligned} \alpha - \frac{1}{\lambda_i} P_e &= \frac{C}{\lambda_i} \left[ \int_{V_i}^{\infty} \frac{1}{\lambda_i E} e^{-\frac{V}{\lambda_i E}} (V - V_i) dV - \int_{V_m \lambda_i E}^{\infty} \frac{1}{\lambda_i E} e^{-\frac{V}{\lambda_i E}} (V - V_m) dV \right] \\ &= CE \left( e^{-\frac{V_i}{\lambda_i E}} - e^{-\frac{V_m}{\lambda_i E}} \right), \\ \frac{\alpha}{p} &= C \frac{E}{p} \left( e^{-\frac{g_i}{E/p}} - e^{-\frac{mg_i}{E/p}} \right). \end{aligned} \quad (17)$$

Assume here that the ionization cross-section above the upper limit of linear range is approximately constant. Integration of Eq.(1) from  $E_c/p$  to  $E_a/p$  with this Relation (17) gives

$$\begin{aligned} \frac{\ln A \ln(b/a)}{V} &= C [G - G(E_a/p)], \\ G(E_a/p) &= (m-1) \frac{g_i}{E_a/p} - \frac{m^2-1}{2 \times 2!} \left( \frac{g_i}{E_a/p} \right)^2 + \frac{m^3-1}{3 \times 3!} \left( \frac{g_i}{E_a/p} \right)^3 - \dots \end{aligned} \quad (18)$$

Here  $m = V_m/V_i$ ; the  $C$  and  $G$  are the gas constants. This formula is applicable to the range of large  $E_a/p$  and generally only needs to take account of the terms up to the cube of  $g_i/(E_a/p)$ . For comparison with experiments we used the experimental data of various gas pressures of methane proportional tubes from publications[6,8,9]. Taking  $g_i = 83 \text{ V}/(\text{cm}\cdot\text{mmHg})$ [10,12] and  $m = 6$ , the relation plot of  $\ln A \ln(b/a)/V$  to  $G(E_a/p)$  is shown in Fig.7. Fitting the data with a straight line gives  $C = 0.0353 \text{ V}^{-1}$  and  $G = 1.52$ . For the range of large  $E_a/p$  from about 400 up to  $\sim 10^4 \text{ V}/(\text{cm}\cdot\text{mmHg})$ , the experimental points distribute along the straight line with the slopes of regular change. The Formula (18) is applicable.

In the case that the  $E_a/p$  is very large ( $4 \times 10^3 \lesssim E_a/p \lesssim 10^4 \text{ V}/(\text{cm}\cdot\text{mmHg})$ ), if we only take the first term in the expansion of Relation (17) and the terms up to the square of  $g_i/(E_a/p)$  for

$G(E_a/p)$  in the Formula (18), then the Kristov formulas  $\alpha/p = C_1$  and  $\ln A \ln(b/a)/V = C_2 - C_1/(E_a/p)$  are obtained.

Consequently, the Formulas of the gas gain (16) and (18) deduced in this paper are inclusive of the range of  $E_a/p$  from a very low pressure up to  $\sim 10^4$  V/(cm-mmHg), the former is applicable to the  $E_a/p$  range of a general proportional tube and the latter is applicable to the range of large  $E_a/p$ . This shows that the corresponding  $\alpha/p$  Relations (15) and (17) are effective.

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