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Exact Ground State and String Tension in Massive Lattice Schwinger Model

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A new Hamiltonian in the massive lattice Schwinger model is proposed and the exact ground state found. The string tension of the infinite string is exactly calculated in both the Naive and Susskind fermion schemes. The result shows that in both massless and massive lattice Schwinger models the string tension is equal to $1/2\, e^2$. This indicates that as in the massless Schwinger model, one can obtain the linear confinement potential in the massive Schwinger model and no deconfinement phase transition occurs when a approaches to 0. This result is in accord with that obtained in the continuum theory.

1. INTRODUCTION

As is well known, in the massless Schwinger model, physical quantities can be exactly solved and the quark confinement and the spectrum obtained. It can also be seen that these results are equivalent to those in the free boson field which has a mass $e/\sqrt{\pi}$ [1]. However, in the massive Schwinger model the exact solution still has not been found, despite the efforts made in the 1 + 1-dimensional gauge theory [2].

The lattice gauge theory is a promising low energy theory. Recently, physicists have intensively studied the massive lattice Schwinger model with the Monte Carlo method and made some progress [3], but the analytical results are still lacking.

In 1985, the Hamiltonian with exact solvable ground state in the pure lattice gauge theory was proposed by S. H. Guo et al. [4]. The string tension and glueball masses in the 2 + 1-dimensional lattice gauge theory were calculated by using the variational method and in this way good scaling behavior and universality were obtained [4]. Later, the Hamiltonian with solvable ground state in the massless lattice Schwinger model was found, the string tension of the infinite string exactly calculated

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[5] and the correct continuum limit obtained. In this paper, this method is further applied to the massive lattice Schwinger model.

2. EXACT GROUND STATE AND STRING TENSION

We first confine ourselves to the Naive fermion theory. In the massless lattice Schwinger model, the Hamiltonian with solvable exact ground state is [5]

$$H_0 = \frac{1}{2} e^2 a \sum_x \mathrm{e}^{-CR_1} E(x) \mathrm{e}^{2CR_1} E(x) \mathrm{e}^{-CR_1}, \tag{2.1}$$

where

$$R_1 = \sum_{\substack{x \\ k=\pm 1}} \bar{\psi}(x) r_k U(x,k) \psi(x+k), \tag{2.2}$$

with $r_{-1} = -r_1$, $\bar{\psi}(x)$ and $\psi(x)$ being fermion fields, $U(x, k)$ being the gauge field which satisfies

$$[U(x,1), E(x)] = U(x,1), \quad [U(x,-1), E(x-1)] = -U(x,-1), \tag{2.3}$$

and C satisfies

$$-2C + 3\int_0^C dC' I_0(-4C') - 2C I_0(-4C) = \frac{1}{(ae)^2}, \tag{2.4}$$

$I_0(z)$is the Bessel function of zeroth order. For convenience, we choose the representation

$$r_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad r_1 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \tag{2.5}$$

and denote

$$\psi(x) = \begin{pmatrix} \xi(x) \\ \eta^+(x) \end{pmatrix}, \quad \psi^+(x) = (\xi^+(x) \quad \eta(x)). \tag{2.6}$$

Assuming

$$H_m = m \sum_x \mathrm{e}^{-CR_1} \xi^+(x) \mathrm{e}^{2CR_1} \xi(x) \mathrm{e}^{-CR_1} + m \sum_x \mathrm{e}^{-CR_1} \eta^+(x) \mathrm{e}^{2CR_1} \eta(x) \mathrm{e}^{-CR_1}, \tag{2.7}$$

since

$$\mathrm{e}^{CR_1} \psi(x) \mathrm{e}^{-CR_1} = \sum_{n=0}^{\infty} \frac{1}{n!} C^n \psi_n(x),$$
$$\mathrm{e}^{-CR_1} \psi^+(x) \mathrm{e}^{CR_1} = \sum_{n=0}^{\infty} \frac{1}{n!} C^n \psi_n^+(x), \tag{2.8}$$

where

$$\begin{aligned}
\psi_0(x) &= \psi(x), \\
\psi_n(x) &= [R_1, \psi_{n-1}(x)] \\
&= (-1)^n r_0 r_{k1} U(x, k_1) \cdots r_0 r_{kn} U\left(x + \sum_{i=1}^{n-1} ki, kn\right) \psi\left(x + \sum_{i=1}^{n} ki\right), \\
\psi_0^+(x) &= \psi^+(x), \\
\psi_n^+(x) &= [-R_1, \psi_{n-1}^+(x)] \\
&= (-1)^n \bar{\psi}\left(x - \sum_{i=1}^{n} ki\right) r_{kn} U\left(x - \sum_{i=1}^{n} ki, kn\right) \cdots r_0 r_{k1} U(x - k1, k1).
\end{aligned} \tag{2.9}$$

it is easy to rewrite H_m as

$$\begin{aligned}
H_m = m \sum_x \bar{\psi}(x)\psi(x) &+ \sum_{n=0}^{\infty} \sum_{\substack{m=0 \\ n+m\neq 0}}^{\infty} \frac{1}{(2n)!(2m)!} C^{2n+2m} D_{2n+2m} \\
&- \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{1}{(2n+1)!(2m+1)!} C^{2n+2m+2} D_{2n+2m+2} \\
&- 2 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{1}{(2n)!(2m+1)!} C^{2n+2m+1} R_{2n+2m+1} + M_0,
\end{aligned} \tag{2.10}$$

where

$$\begin{aligned}
D_n &= \sum_x \bar{\psi}(x) r_0 r_{k1} U(x, k1) \cdots r_0 r_{kn} U\left(x + \sum_{i=1}^{n-1} ki, kn\right) \psi\left(x + \sum_{i=1}^{n} k_i\right), \\
R_n &= \sum_x \bar{\psi}(x) r_{k1} U(x, k1) \cdots r_0 r_{kn} U\left(x + \sum_{i=1}^{n-1} ki, kn\right) \psi\left(x + \sum_{i=1}^{n} ki\right),
\end{aligned} \tag{2.11}$$

and M_0 is a constant related to the normal ordering of operators. According to Eq.(2.4), when $a \to 0$, we have

$$C \sim \frac{1}{2} \ln(ae), \tag{2.12}$$

By further considering, in the classical continuum limit,

$$D_n \sim (ae)^n, \quad R_n \sim (ae)^n, \tag{2.13}$$

we obtain

$$H_m \to m \sum_x \bar{\psi}(x)\psi(x) + M_0, \text{ when } a \to 0. \tag{2.14}$$

namely, H_m approaches to the fermion mass term except for an irrelevant constant. Thus, we can take the Hamiltonian in the massive lattice Schwinger model as

$$H = H_0 + H_m. \tag{2.15}$$

Since

$$\begin{aligned}
(e^{CR_1}E(x)e^{-CR_1})^+ &= e^{-CR_1}E(x)e^{CR_1},\\
(e^{CR_1}\xi(x)e^{-CR_1})^+ &= e^{-CR_1}\xi^+(x)e^{CR_1},\\
(e^{CR_1}\eta(x)e^{-CR_1})^+ &= e^{-CR_1}\eta^+(x)e^{CR_1},
\end{aligned} \tag{2.16}$$

H is positive definite. Let

$$|\Omega\rangle = e^{CR_1}|0\rangle, \tag{2.17}$$

where $|0\rangle$ is defined as

$$E(x)|0\rangle = 0, \xi(x)|0\rangle = 0, \eta(x)|0\rangle = 0. \tag{2.18}$$

then

$$H|\Omega\rangle = 0, \tag{2.19}$$

This means that $|\Omega\rangle$ is an exact ground state of H.

We now calculate the string tension. Denote the state in which an pair of quark and anti-quark are connected by an n-string of the gauge field as $|M_n\rangle$,

$$\begin{aligned}
|M_n\rangle &= e^{CR_1}M_n^+|0\rangle,\\
M_n^+ &= \sum_{\substack{x\\ \Gamma=\pm n}} \xi^+(x)i^{\Gamma}U(x,\Gamma)\eta^+(x+\Gamma),\\
U(x, \pm n) &= \prod_{i=0}^{n-1} U(x\pm i, \pm 1).
\end{aligned} \tag{2.20}$$

the string tension of an infinite string is

$$\alpha = \lim_{n\to\infty} \frac{1}{na} \frac{\langle M_n|H|M_n\rangle}{\langle M_n|M_n\rangle}. \tag{2.21}$$

For arbitrary a, $\sum_{n=0}^{\infty}\langle 0|\, M_n \frac{1}{(2k)!}(2c)^{2k} R_1^{2k} M_n^+|\, 0\rangle$ converges uniformly for all n, therefore,

$$\begin{aligned}
\lim_{n\to\infty}\langle M_n|M_n\rangle &= \lim_{n\to\infty}\lim_{K\to\infty}\sum_{k=0}^{K}\left\langle 0\,\middle|\, M_n\frac{1}{(2k)!}(2c)^{2k}R_1^{2k}M_n^+\,\middle|\,0\right\rangle\\
&= \lim_{K\to\infty}\lim_{\substack{n\to\infty\\ n>2K}}\sum_{k=0}^{K}\left\langle 0\,\middle|\, M_n\frac{1}{(2k)!}(2c)^{2k}R_1^{2k}M_n^+\,\middle|\,0\right\rangle.
\end{aligned} \tag{2.22}$$

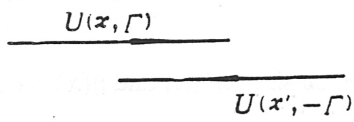

FIG.1

The constraint $n > 2K$ in the last line of Eq.(2.22) indicates that only the configuration shown in contributes to $\lim\limits_{n\to\infty}\langle M_n | M_n\rangle$

Analogously, by using

$$\lim_{n\to\infty}\frac{1}{na}\langle M_n|H_m|M_n\rangle = 0, \tag{2.23}$$

we get

$$\begin{aligned}\lim_{n\to\infty}\frac{1}{na}\langle M_n|H|M_n\rangle &= \frac{1}{2}e^2\lim_{n\to\infty}\langle 0|M_n E e^{2CR_1} E M_n^+|0\rangle \\ &= \frac{1}{2}e^2\lim_{K\to\infty}\lim_{\substack{n\to\infty\\ n>2K}}\frac{-1}{n}\sum_{k=0}^{K}\left\langle 0\left|[M_n,E]\frac{1}{(2k)!}(2C)^{2k}R_1^{2k}[M_n^+,E]\right|0\right\rangle,\end{aligned} \tag{2.24}$$

and again, contribution comes from the configuration as shown in Fig.1 only. For convenience, in the above equation, we have omitted the space index and the summation over x. By considering the effect of the electric field E, we have the following inequality

$$(n-2k) \leqslant \frac{-\langle 0|[M_n,E]R_1^{2k}[M_n^+,E]|0\rangle}{\langle 0|M_n R_1^{2k} M_n^+|0\rangle} \leqslant n, \tag{2.25}$$

consequently,

$$\lim_{n\to\infty}\frac{-1}{n}\langle 0|[M_n, E]R_1^{2k}[M_n^+, E]|0\rangle = \langle 0|M_n R_1^{2k} M_n^+|0\rangle, \tag{2.26}$$

Finally, we obtain

$$\alpha = \frac{1}{2}e^2. \tag{2.27}$$

For the Susskind fermion scheme, if we take the Hamiltonian as

$$\begin{aligned} H_s = \frac{1}{2}e^2 a\sum_x & e^{-CR_s}E(x)e^{2CR_s}E(x)e^{-CR_s} \\ &+ m\sum_x e^{-CR_s}\xi^+(2x)e^{2CR_s}\xi(2x)e^{-CR_s} \\ &+ m\sum_x e^{-CR_s}\eta^+(2x+1)e^{2CR_s}\eta(2x+1)e^{-CR_s},\end{aligned} \tag{2.28}$$

where $\xi^+(x)$ and $\xi(x)$ are defined on even sites, $\eta^+(x)$ and $\eta(x)$ on odd sites, and

$$R_t = \sum_{\substack{x \\ k=\pm 1}} \xi^+(2x)i^k U(2x,k)\eta^+(2x+k) + \eta(2x+1)i^k U(2x + 1,k)\xi(2x+1+k), \tag{2.29}$$

Results similar to those in the Naive fermion scheme can be obtained.

3.DISCUSSION

(1) We have proposed a Hamiltonian in the massive lattice Schwinger model and found its exact ground state. Because the fermion mass term breaks the chiral symmetry, the additional variable θ does not appear even in the Naive fermion theory [5].

(2) We have exactly calculated the string tension of the infinite string and the result is $1/2\, e^2$. This means that in the Massive lattice Schwinger model, one can obtain the same linear confinement potential as in the Massless lattice Schwinger model and no deconfinement phase transition occurs when a approaches to 0. This conclusion coincides with that obtained in the continuum theory [2].

(3) The Naive fermion theory is correct in some aspects, such as the string tension.

(4) It is possible to apply this method to the $SU(N)$ theory directly.

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