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D-Wave Components of the Glueball Candidate θ/f_2 (1720) and a Possible Experimental Test

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In this paper we discuss the production and decay of the glueball candidate $\theta/f_2(1720)$ in the J/ψ radiation decay. We find that in order to explain the ratios x and y of the helicity amplitudes of the process $J/\psi \to \gamma + \theta$, the contribution from two D-wave components ($l = 2, s = 0, 2$) must be considered in addition to the S-wave component. We also discuss the possible experimental test for the second D-wave component ($l = 2, s = 2$).

1. INTRODUCTION

The standard theory of color force (QCD) suggests that bound states consisting of gluons only (glueballs) should exist. The J/ψ radiation decay is expected to be an ideal process to look for glueballs. A 2^{++} meson $\theta/f_2(1720)$ has been observed in the $J/\psi \to \gamma\eta\eta$, $\eta K\bar{K}$, $\gamma\pi\pi$ reactions [1]. It is considered to be an acknowledged and likely expectant candidate for the 2^{++} glueball.

In Ref. [2] the helicity amplitudes of this process are discussed. The conclusion is that in order to explain the ratios x and y of the helicity amplitudes of the process $J/\psi \to \gamma + \theta$, the first D-wave component ($l = 2, s = 0$) is sufficient in addition to the S-wave glueball wave function of θ. But we do not find any special dynamic reason to suppress the second D-wave component ($l = 2, s = 2$). After making some correction for the first D-wave glueball wave function and reconsidering the dimension analysis, we find that the admixture of only one D-wave component is not sufficient. In general, the equations have no solution, and to get rid of this difficulty we have to consider the

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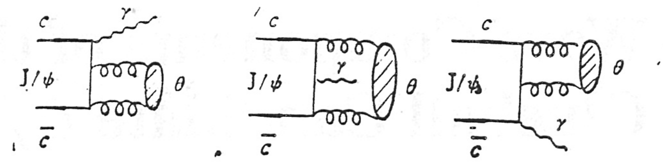

FIG.1 The lowest-order Feynman diagrams for the process J/ψ $\to \gamma + \theta(gg)$.

admixture of two D-wave components simultaneously. After choosing appropriate mixing parameters we can get the helicity amplitude ratios x and y, which are in agreement with the experimental data.

In the potential model [3], the gluon is a vector meson with an effective mass. To construct a 2^{++} glueball, there are four coupling patterns (l = 0, 2, 2, 4 corresponding to s = 2, 0, 2, 2 respectively).

We consider the process J/ψ $\to \gamma + \theta, \theta \to V_1 + V_2$, (here V denotes the vector meson). The (V_1V_2) system also has the same four coupling patterns. There is no special dynamic reason to suppress its second D-wave component (l = 2, s = 2). By using the angular distribution of this process, we can determine whether the contribution of the second D-wave component is negligible. Therefore we can test our arguments about the helicity amplitude ratios by comparison with the experimental data.

2. THE RATIOS x AND y OF THE HELICITY AMPLITUDES OF THE PROCESS J/ψ $\to \gamma+\theta$

To the lowest order in perturbative QCD, the process J/ψ $\to \gamma + \theta(gg)$ is described by the Feynman diagrams as shown in Fig.1. The S-matrix element corresponding to these diagrams can be written as

$$\langle \gamma_{\lambda_\gamma}\theta_A | T | J_{\lambda_J}\rangle = (2\pi)^4\delta^4(p_J - p_\gamma - p_\theta)\,\frac{eg^2}{3\sqrt{6\omega_\gamma}}\,e^{\lambda_\gamma *}_{\mu}(p_\gamma)\delta_{ab}$$
$$\cdot \int d^4x_1 d^4x_2 \mathrm{Tr}\{\chi_{\lambda_J}(0,x_1)\gamma^\alpha S_F(x_1-x_2)\gamma^\beta S_F(x_2)\gamma^\mu$$
$$+ \chi_{\lambda_J}(x_1,x_2)\gamma^\beta S_F(x_2)\gamma^\mu S_F(-x_1)\gamma^\alpha \tag{1}$$
$$+ \chi_{\lambda_J}(x_2,0)\gamma^\mu S_F(-x_1)\gamma^\alpha S_F(x_1-x_2)\gamma^\beta\}\cdot G^{ab}_{\alpha\beta}(x_1,x_2)_A,$$

where $\chi_{\lambda_J}(x_1,x_2)$ is the wave function of the J/ψ particle,

$$\chi_{\lambda_J}(x_1, x_2) = \frac{\sqrt{m_J}}{2\sqrt{2E_J}}\,\psi_J(0)e^{-ip_JX}\left[1+\frac{\not{p}_J}{m_J}\right]\not{e}^{\lambda_J}(p_J);$$
$$X = \frac{1}{2}(x_1+x_2),\quad x = x_1 - x_2, \tag{2}$$

and $G^{ab}_{\alpha\beta}(x_1,x_2)_A$ is the wave function of the glueball θ. In the rest frame of θ, the helicity amplitudes of the process J/ψ $\to \gamma + \theta$ have the form

$$\langle \gamma_{\lambda_\gamma}\theta_A | T | J_{\lambda_J}\rangle = (2\pi)^4\delta^4(p_J - p_\gamma - p_\theta)\,\frac{e}{(8\omega_\gamma m_\theta E_J)^{1/2}}\,T_A, \tag{3}$$

here λ_J, λ_γ and Λ are helicities of J, γ and θ, respectively. Because of parity conservation there are only three independent amplitudes $T_\Lambda : T_2, T_1$ and T_0. The helicity amplitude ratios are defined by

$$x = T_1/T_0, \quad y = T_2/T_0. \tag{4}$$

In Ref.[4] the experimental values of x and y for θ are given,

$$\begin{cases} x = -1.07 \pm 0.16 \\ y = -1.09 \pm 0.15. \end{cases} \tag{5}$$

The wave functions of the S-wave and two D-waves of the glueball θ (for simplicity, we do not consider the G-wave) have the following form:

S-wave:

$$G^{ab}_{\alpha\beta}(x_1, x_2)_\Lambda = e^{ip_\theta \cdot X} \frac{1}{\sqrt{2m_\theta}} \sum_{m_1 m_2} C^{2-\Lambda}_{1m_1 1m_2} e^{m_1^*}_{\alpha} e^{m_2^*}_{\beta} \delta^{ab} G_s(0), \tag{6}$$

D-wave ($l = 2$, $s = 0$):

$$G^{ab}_{\alpha\beta}(x_1, x_2)_\Lambda = e^{ip_\theta \cdot X} \frac{1}{\sqrt{2m_\theta}} \sum_{\substack{m_1, m_2, m_3 \\ m_4, M}} C^{2-\Lambda}_{002M} C^{00}_{1m_1 1m_2} C^{2M}_{1m_3 1m_4} \cdot e^{m_1^*}_{\alpha} e^{m_2^*}_{\beta} (x \cdot e^{m_3^*})(x \cdot e^{m_4^*}) m_\theta^2 \delta^{ab} G_d(0), \tag{7}$$

D'-wave ($l = 2$, $s = 0$):

$$G^{ab}_{\alpha\beta}(x_1, x_2)_\Lambda = e^{ip_\theta \cdot X} \frac{1}{\sqrt{2m_\theta}} \sum_{\substack{m_1, m_2, m_3 \\ m_4, M_1, M_2}} C^{2-\Lambda}_{2M_1 2M_2} C^{2M_1}_{1m_1 1m_2} \cdot C^{2M_2}_{1m_3 1m_4} e^{m_1^*}_{\alpha} e^{m_2^*}_{\beta} (x \cdot e^{m_3^*})(x \cdot e^{m_4^*}) m_\theta^2 \delta^{ab} G_{d'}(0). \tag{8}$$

As shown in Ref.[5], because the charm quark mass m_c is far larger than the internal momenta of the constituents in J/ψ and θ, we have approximated the internal wave functions of J/ψ and θ by their values at the origin. Here the spherical polarization vectors are

$$e^{+1} = \frac{-1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}, \quad e^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}, \quad e^{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{9}$$

It is obvious that the zero components of α and β vanish and only their 1, 2 and 3 components need to be considered. It is worthwhile to point out that the wave function (7) is different from Eq.(10) of Ref.[2]. From Eqs.(6), (7) and (8) one can see that $G_s(0)$, $G_d(0)$ and $G_d(0)$ defined in this paper have the same dimension. Substituting Eqs.(2), (6), (7) and (8) into Eq.(1) and comparing with Eq.(3), we can obtain various amplitudes for different wave functions:

For S-wave:

$$T_{2s}=\frac{32g^2\psi_J(0)}{3\sqrt{3}\,m_c^4}\sqrt{m_J}\left[\frac{m_\theta m_c}{m_J}\,p_J-m_c^2\right]G_s(0)=t_{2s}G_s(0),$$

$$T_{1s}=\frac{16\sqrt{2}\,g^2\psi_J(0)}{3\sqrt{3}\sqrt{m_J}\,m_c^2}\left[\frac{m_\theta}{m_J m_c}(E_J+p_J)p_J-E_J-\frac{m_\theta p_J^2}{\left(m_c^2+\frac{1}{4}m_J^2-\frac{1}{2}m_\theta^2\right)}\right]G_s(0)$$

$$=t_{1s}G_s(0),$$

$$T_{0s}=\frac{16\sqrt{m_J}}{9\sqrt{2}\,m_c^2}\,g^2\psi_J(0)\left[\frac{2}{m_c^2}\left(\frac{2m_c}{m_J}p_J^2+\frac{m_\theta m_c}{m_J}\,p_J-m_c^2\right)\right.$$

$$\left.-\frac{\left(4\frac{m_c}{m_J}+2\right)p_J^2}{\left(m_c^2+\frac{1}{4}m_J^2-\frac{1}{2}m_\theta^2\right)}\right]G_s(0)=t_{0s}G_s(0). \tag{10}$$

For D-wave:

$$T_{2d}=-\frac{64\sqrt{m_J}\,m_\theta^2}{9\,m_c^4}\,g^2\psi_J(0)G_d(0)=t_{2d}G_d(0),$$

$$T_{1d}=-\frac{64m_\theta^2}{9\sqrt{2}\,m_c^4\sqrt{m_J}}\,g^2\psi_J(0)\left(E_J+\frac{m_\theta p_J^2}{m_c^2+\frac{1}{4}m_J^2-\frac{1}{2}m_\theta^2}\right)G_d(0)$$

$$=t_{1d}G_d(0),$$

$$T_{0d}=\frac{16\sqrt{2}\sqrt{m_J}\,m_\theta^2}{9\sqrt{3}\,m_c^4}\,g^2\psi_J(0)\left\{\left[\frac{3}{m_c^2}\,p_J^2\left(m_\theta p_J-2\frac{m_c}{m_J}m_\theta E_J+4m_c^2\right)\right.\right.$$

$$\left.-\left(1+\frac{p_J^2}{m_c^2}\right)\left(4\frac{m_c}{m_J}+2\right)p_J^2\right]\Big/\left(m_c^2+\frac{1}{4}m_J^2-\frac{1}{2}m_\theta^2\right) \tag{11}$$

$$\left.+4\frac{p_J^2}{m_c^4}\left(\frac{m_c}{m_J}p_J^2+\frac{m_c}{2m_J}m_\theta p_J-\frac{m_c^2}{2}\right)-2\right\}G_d(0)$$

$$=t_{0d}G_d(0).$$

For D'-wave:

$$T_{2d'}=\frac{64\sqrt{m_J}\,m_\theta^2}{9\sqrt{7}\,m_c^4}\,g^2\psi_J(0)\left[7-\frac{2}{m_c^4}\,p_J^2\left(\frac{m_\theta m_c}{m_J}\,p_J-m_c^2\right)\right]G_{d'}(0)$$

$$=t_{2d'}G_{d'}(0),$$

$$T_{1d'}=\frac{64m_\theta^2}{9\sqrt{14}\,m_c^4\sqrt{m_J}}\,g^2\psi_J(0)\left\{7E_J+\frac{m_\theta p_J^2}{m_c^2+\frac{1}{4}m_J^2-\frac{1}{2}m_\theta^2}\left(7-\frac{p_J^2}{m_c^2}\right)\right.$$

$$+\frac{p_J^2}{m_c^2}\left[\frac{m_\theta}{m_J m_c}p_J(p_J+E_J)-E_J\right]\Big\}G_{d'}(0)=t_{1d'}G_{d'}(0),$$

$$T_{0d'}=\frac{64}{9}\sqrt{\frac{2}{21}}\frac{\sqrt{m_J m_\theta^2}}{m_c^4}g^2\psi_J(0)\left\{\frac{7}{2}+\frac{2p_J^4}{m_J m_c^3}+\frac{m_\theta p_J^3}{m_J m_c^3}-\frac{p_J^2}{m_c^2}\right.$$

$$\left.+\left(\frac{m_c}{m_J}+\frac{1}{2}\right)\left(7-\frac{2p_J^2}{m_c^2}\right)p_J^2\Big/\left(m_c^2+\frac{1}{4}m_J^2-\frac{1}{2}m_\theta^2\right)\right\}G_{d'}(0)$$

$$=t_{0d'}G_{d'}(0). \tag{12}$$

With the mixing between the S-wave and two D-waves taken into account, one finds,

$$\begin{aligned}T_2&=T_{2s}+aT_{2d}+bT_{2d'},\\T_1&=T_{1s}+aT_{1d}+bT_{1d'},\\T_0&=T_{0s}+aT_{0d}+bT_{0d'},\end{aligned}\tag{13}$$

where a and b are the mixing parameters. We define

$$G_d(0)=cG_s(0),\quad G_{d'}(0)=dG_s(0),\tag{14}$$

and introduce two new parameters $A = ac$ and $B = bd$. Then Eq.(13) becomes

$$\begin{cases}T_2=(t_{2s}+At_{2d}+Bt_{2d'})G_s(0),\\T_1=(t_{1s}+At_{1d}+Bt_{1d'})G_s(0),\\T_0=(t_{0s}+At_{0d}+Bt_{0d'})G_s(0).\end{cases}\tag{15}$$

From the definition (4) we obtain

$$\begin{cases}x=(t_{1s}+At_{1d}+Bt_{1d'})/(t_{0s}+At_{0d}+Bt_{0d'}),\\y=(t_{2s}+At_{2d}+Bt_{2d'})/(t_{0s}+At_{0d}+Bt_{0d'}).\end{cases}\tag{16}$$

Substituting the experimental values of x and y Eq.(5) into Eq.(16) and choosing a value of m_c in a reasonable range the equation can be solved and a set of solutions for A and B can be obtained. Some results are listed in Tables 1.1--1.3. If we only consider the mixing between the S-wave and one D-wave (that is $A = 0$ or $B = 0$), in general, Eq.(16) has no solution in the region of the experimental values of x and y. From Table.1, we can see that the change of the proportion of the D-wave with m_c is very small, but the proportion of D'-wave increases with m_c. In some cases, for example, $x = -0.91$, $y = -1.21$, $m_c = 1.6$ GeV, the proportions of the D-wave and D'-wave are nearly the same.

3. A POSSIBLE EXPERIMENTAL TEST FOR THE D'-WAVE COMPONENT ($l = 2, s = 2$)

We consider the processes $J/\psi \to \gamma + \theta$, $\theta \to V_1 + V_2$, $V_1 \to P_1 + P_2$, $V_2 \to P_3 + P_4$, where V and P symbolize the vector meson (e.g. ρ meson) and pseudoscalar meson (e.g. pion), respectively. So far we have not yet discovered these decay modes. We only have the upper limit of the branching ratio [4]: $B(J/\psi \to \gamma\theta)\cdot B(\theta \to \rho\rho) < 5.5 \times 10^{-4}$. Compared with the value of $B(J/\psi \to \gamma\theta)\cdot B(\theta \to KK) = (9.7 \pm 1.1) \times 10^{-4}$, we believe that it will be possible to observe this decay channel provided that the number of J/ψ events increases by one order of magnitude, as what is expected from the BEPC.

TABLE 1 The Relation Between the Two Parameters A and B and m_c for Different x and y

1.1 $x = -1.07$ $y = -1.09$

m_C(GeV)	1.2	1.3	1.4	1.5	1.6
$-A$	0.16	0.18	0.18	0.19	0.18
B	0.02	0.02	0.03	0.04	0.06

1.2 $x = -1.07$ $y = -1.21$

m_C(GeV)	1.2	1.3	1.4	1.5	1.6
$-A$	0.13	0.14	0.14	0.14	0.14
B	0.02	0.03	0.04	0.05	0.07

1.3 $x = -0.91$ $y = -1.21$

m_C(GeV)	1.2	1.3	1.4	1.5	1.6
$-A$	0.09	0.10	0.10	0.10	0.10
B	0.03	0.03	0.04	0.06	0.07

The matrix elements corresponding to this process can be written as

$$\begin{aligned}
\langle s_1\lambda_1, s_2\lambda_2 q\Omega | m | J\Lambda; \lambda_1\lambda_2\rangle &= a_{\lambda_1\lambda_2} D^{J*}_{-\Lambda\lambda_1-\lambda_2}(\phi, \theta, 0), \\
\langle k_1\Omega_1 | m | s_1\lambda_1\rangle &= F_1 D^{s_1*}_{\lambda_1,0}(\phi_1, \theta_1, 0), \\
\langle k_3\Omega_3 | m | s_2,\lambda_2\rangle &= F_2 D^{s_2*}_{\lambda_2,0}(\phi_3, \theta_3, 0),
\end{aligned} \tag{17}$$

where λ_1 and λ_2 are helicities of the vector mesons V_1 and V_2, respectively; $J = 2$, $s_1 = s_2 = 1$; q and Ω (θ, ϕ) describe the magnitude and direction of the V_1 momentum in the rest frame of θ. We choose the z axis to be parallel to the direction of the emitted photon in the rest frame of J/ψ; k_1 and $\Omega_1(\theta_1, \phi_1)$ (k_3 and $\Omega_3(\theta_3, \phi_3)$) are the magnitude and direction of the $P_1(P_3)$ momentum in the rest frame of $V_1(V_2)$. Again, the coordinate system for angles $\Omega_1(\Omega_3)$ has the z axis parallel to the direction of $V_1(V_2)$ in the rest frame of θ. a_{λ_1,λ_2}, F_1 and F_2 are the helicity amplitudes for the θ, V_1 and V_2 decay processes, respectively. F_1 and F_2 are constants. Because of the invariance of space reflection one finds the identity for the 2^{++} θ particle,

$$a_{-\lambda_1,-\lambda_2} = a_{\lambda_1,\lambda_2}. \tag{18}$$

If V_1 and V_2 are identical particles (e.g. $\rho^0\rho^0$, $\phi\phi$) we have

$$a_{\lambda_1,\lambda_2} = a_{\lambda_2,\lambda_1}. \tag{19}$$

TABLE 2 The Helicity Amplitudes and the Angular Distribution Parameters for the *S*-wave and the Two *D*-waves

J	l	s	a_{++}	a_{00}	a_{+0}	a_{0+}	a_{+-}	β	ζ
2	0	2	$\sqrt{\frac{1}{30}}$	$\sqrt{\frac{2}{15}}$	$\sqrt{\frac{1}{10}}$	$\sqrt{\frac{1}{10}}$	$\sqrt{\frac{1}{5}}$	1/15	0
2	2	0	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{3}}$	0	0	0	2/3	0
2	2	2	$-\sqrt{\frac{1}{21}}$	$-\frac{2}{\sqrt{21}}$	$-\sqrt{\frac{1}{28}}$	$-\sqrt{\frac{1}{28}}$	$\sqrt{\frac{2}{7}}$	2/21	−3/14

Thus there are four independent helicity amplitudes: a_{++}, a_{+-}, a_{+0}, a_{0+}, a_{00}. If V_1 and V_2 are not identical particles (e.g.), Eq.(19) will not hold and there will be five independent helicity amplitudes a_{++}, a_{+-}, a_{+0}, a_{0+}, a_{00}.

Define $\chi = \phi_1 + \phi_3$ to be the angle between the two decay planes of V_1 and V_2, in analogy to the formula of Ref.[6] the χ distribution function is

$$F(\chi) = 1 + \beta \cos 2\chi. \tag{20}$$

where

$$\beta = 2|a_{++}|^2/[2|a_{++}|^2 + |a_{00}|^2 + 2|a_{+0}|^2 + 2|a_{0+}|^2 + 2|a_{+-}|^2]. \tag{21}$$

The θ_1 distribution has the following form

$$G(\theta_1) = 1 + \zeta P_2(\cos\theta_1), \tag{22}$$

where

$$\zeta = \frac{2[|a_{00}|^2 - |a_{++}|^2 - |a_{+-}|^2 + 2|a_{0+}|^2 - |a_{+0}|^2]}{2|a_{++}|^2 + |a_{00}|^2 + 2|a_{+0}|^2 + 2|a_{0+}|^2 + 2|a_{+-}|^2}$$

$$P_2(\cos\theta_1) = \frac{1}{2}(3\cos\theta_1 - 1). \tag{23}$$

From Ref.[7] we have

$$\langle J\Lambda; ls | J\Lambda; \lambda_1\lambda_2\rangle = \left(\frac{2l+1}{2J+1}\right)^{1/2} (los\lambda | J\lambda)(s_1\lambda_1 s_2 - \lambda_2 | s\lambda), \tag{24}$$

This is the relation between the eigenvectors of the helicity representation and the spin-orbit coupling representation for the process $\theta \to V_1 + V_2$. l and s are the quantum numbers of the orbital angular momentum and the total spin of the (V_1V_2) system. From Eqs.(17) and (24) we can obtain the corresponding helicity amplitudes for the given (J, l, s) values. Then from Eqs.(21) and (23) we can calculate the distribution parameters ß and ζ. The results are shown in Table 2. We find that the θ_1

distribution of the process cannot be constant as long as the D'-wave component is not forbidden. It serves as an experimental criterion to determine if there exist some dynamical reasons to forbid the D'-wave. It provides also an indirect test for our discussions in the second section.

4. CONCLUSION

We have discussed the processes $J/\psi \rightarrow \gamma + \theta$, $\theta \rightarrow V_1 + V_2$, $V_1 \rightarrow P_1 + P_2$, and $V_2 \rightarrow P_3 + P_4$. Considering θ as a 2^{++} glueball, we are able to explain correctly the experimental data for the helicity amplitude ratios x and y only after the D-wave and D'-wave components in addition to the S-wave are taken into consideration. We also provide a possible experimental test to determine if there are some dynamic reasons forbidding the D'-wave component from the θ_1 distribution. As long as the data give a non-constant θ_1 distribution and the ζ parameter assumes approximately the value of -3/14, then we have to consider the contribution of the D'-wave component. In that case, though indirectly, it also provides a support for our discussion about the helicity amplitude ratios of the process.

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