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The Fragmentation Mechanism in e^+e^- Annihilation

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A comparison of two different fragmentation schemes in e^+e^- annihilation is presented in this paper. After further calculations of the average number of various final hadrons and comparison with experiments, we find that the independent fragmentation scheme is in contradiction with the data whereas the string fragmentation assumption fits the data well.

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1. INTRODUCTION

Hadronic events in e^+e^- annihilation have been studied in detail in the past decade. All the available data support such a simple physical picture: after the annihilation of the e^+ and e^-, a qq pair is produced with flavor fractions proportional to the charges squared, and then this primary qq fragments into the observed hadrons. Although the details of this process are not yet totally understood, it is now generally accepted that the hard processes involved can be calculated by the theory of quantum chromodynamics (QCD). But the hadronization process by which the partons fragment into hadrons is an unsolved nonperturbative problem. In recent years, many models have been developed and gradually improved. However, the methods of the QCD calculations in these models are different. There are mainly two groups: those partons produced according to the QCD matrix elements of the second order approximation in α_s and those produced based on the leading-logarithmic parton-shower evolution. On the other hand, the treatment of the evolution process, in

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which the quarks and antiquarks are produced and combined into hadrons, can also be divided into two groups. In one group the production of the quarks and antiquarks and their combination into hadrons are treated as two manifestly separated stages, such as the QCD cluster model by Webber[3], the CALTECH II model[4] and SHANDONG model[5], etc; in the other group the hadronization is treated as a recursive cascade process in which the hadrons are produced immediately after the production of the quarks and the antiquarks. These include the F.F. model[1], Lund model[2] and so on. For the fragmentation of the partons into hadrons, there are three main schemes available: independent fragmentation (IF), string fragmentation (SF), and cluster fragmentation (CF). The independent fragmentation scheme[1] assumes that quarks and gluons fragment independently into hadron jets. In contrast, the string fragmentation assumes[3] that the hadrons are formed by the production of qq par is during the break-up of the color string between the partons. In the cluster fragmentation model, the colorless clusters are formed by the partons in the parton showers, and these clusters decay into hadrons independently through two-body decays.

These models might help us to understand the long-range behavior of the strong interaction and explain most of the experimental results. Experiments give the direct tests of the fundamental assumptions of the models and make them develop further. Recently, these models have been tested by the MARKII collab.[6] using their abundant experimental materials. They put the parton shower or hadronization scheme of the string model into other models and compare the obtained result with the data. In this way, they test part of the model assumptions independently. This is a useful method. In this paper, we use this method to compare the independent fragmentation and string fragmentation schemes. Some of the differences in the predictions of these schemes are given in Section II. In Section III, they are combined with a hadronization model to calculate the average number of the final hadrons. The comparison with the data shows that the independent fragmentation scheme clearly deviates from the experimental results, clearly whereas the string fragmentation is in agreement with the data.

2. DIRECT TEST OF THE FRAGMENTATION SCHEMES

There are great differences between the independent fragmentation scheme, which assumes that the primary partons in e^+e^- annihilations fragment independently, and the string fragmentation scheme, which assumes that the partons fragment coherently. There must also be many differences in their predictions on the properties of the events, which should reveal themselves in comparisons with data. Here, we will not go into details of these models, but just compare the fundamental hypotheses of these two kinds of fragmentation schemes.

The forward-backward multiplicity correlation in the two-jet events in e^+e^- annihilation is usually described by $<n_F> = a + bn_B$ which gives the relation between the averaged charged multiplicity $<n_F>$ in the forward hemisphere (forward jet) and the multiplicity n_B in the backward hemisphere (backward jet). If the quark and antiquark which create the forward and backward jets fragment independently, the F-B correlation should be very small, i.e, $b \approx 0$. Recently experimental measurements [7] have been made in different rapidity regions and give $b \approx 0$. In the experiments by TASSO [8], $<n_F>$ rises smoothly with n_B but if the contributions from the decay of the heavy particles (containing c or b quark) are excluded, there will be no correlation between these two hemispheres. It is interesting that the multiplicity distributions given by HRS collaboration [7] for two jet events and single jets in these events can all be well fitted by Poissonian. They also obtain that the dispersion $D_2 = (<n^2>-<n>^2)^{\frac{1}{2}}$ for the whole events is $\surd 2$ times that for single jets and that

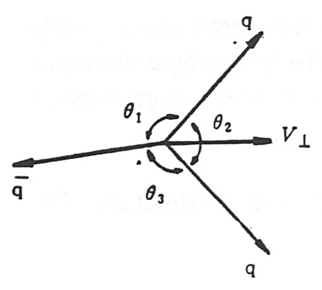

FIG. 1

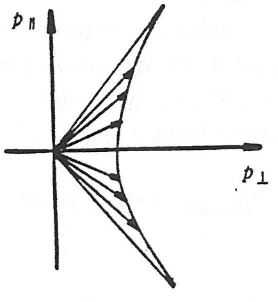

FIG. 2

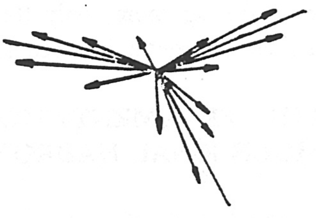

FIG. 3

at a fixed $n = n_F + n_B$, the n_F and n_B satisfy a binomial distribution. All these results support the simple picture that the two jets fragment independently.

In recent years, many studies have been made of the $e^+e^- \to$ 3jet events. One of the most important discoveries from experiments is the 'string effect' which tells us that the energy flow $dE/Nd\theta$, and the angular distribution $dN/Nd\theta$ between the quark and the gluon jets or the antiquark and the gluon jets are much higher than those between the quark and the antiquark jets. This is also the major property which reveals the differences between the 3jet and the 2jet events. Such a phenomenon reflects the differences between the gluon and the quark fragmentations. The 'string effect' should exist in the C.M.S. of e^+e^-, i.e., in the Lab. frame as long as we assume that the gq and g$\bar{q}$ form two systems and each of these two systems fragments independently. This can be seen in Fig.1, where θ_i are the angles between the directions of quarks and the gluon. First, let us see the system formed by the quark and the gluon. For convenience, the transverse momenta of the quarks produced in the fragmentation are taken as zero. The momentum of a produced hadron is then $\boldsymbol{p}' = (0, p'_{/\!/}, 0)$. After the Lorentz transformation into the C.M.S. of e^+e^-, one gets.

$$p_{/\!/} = p'_{/\!/}, \quad p_\perp = \gamma\beta(p^{2\prime} + m^2)^{1/2},$$

where $\beta = \cos(\theta_2/2)/c$, $\gamma = 1/(1-\beta^2)^{\frac{1}{2}}$. The relation between $P_\perp$ and $p_{/\!/}$ is

$$\frac{p_\perp^2}{\beta^2\gamma^2 m^2} - \frac{p_{/\!/}^2}{m^2} = 1, \tag{1}$$

which shows that in the Lab. frame the momenta of the hadrons distribute along a hyperbola as shown in Fig.2. Its asymptotes $p_{/\!/} = \pm p_\perp/\beta\gamma$ are along the directions of the gluon and the quark momenta, respectively. In the same way, one can show that the momenta of the hadrons produced in the g$\bar{q}$ system distribute along another hyperbola with the asymptotes being along the directions of the momenta of the gluon and the antiquark. If the very small transverse momenta of the new created quarks and antiquarks are considered, there will be a small amount of hadrons lying between the quark and antiquark, but most of them lie between the quark and the gluon or the antiquark and the gluon. This is the 'string effect'. But if the quark, the antiquark and the gluon fragment independently into three hadron jets with axes along the directions of their momenta as shown in

Fig.3, apparently, there will be no 'string effect'.

A large amount of experimental data, such as those on the shape of the events, single particle spectra, correlations and energy behaviors, etc. have been compared with the predications of various models. Among them, only the 'string effect' clearly excludes the independent fragmentation hypothesis. Therefore, it is necessary to make a further test of this hypothesis.

3. THE FRAGMENTATION SCHEMES AND THE AVERAGE NUMBER OF VARIOUS FINAL HADRONS

The investigation of the average number of various final hadrons is the simplest way to study the interaction mechanism, and it also provides the most fundamental test of the models. To test two different fragmentation schemes it is clearly better to start only from their fundamental hypotheses but to avoid using the details of the hadronization schemes in the corresponding models. For this purpose, we use the hadronization method given in [5] to calculate the average number of the final hadrons. Let us take the complex case, $e^+e^- \to 3$ jets. Using $E_i (i=1,2,3)$ to denote the energies of the quark, antiquark and gluon, and

$$x_i = 2E_i/\sqrt{s},$$

being the energy fractions. From the conservation of energy one has

$$x_1 + x_2 + x_3 = 2,$$

Neglecting the quark and gluon masses, which is a good approximation in high energy e^+e^- annihilation, using the conservation of momentum from Fig.1, one has

$$x_i = \frac{2\sin\theta_i}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}. \tag{2}$$

The corresponding energy E_i is the energy of the portion i under the assumption of the independent fragmentation. After the coordinate transformation, one gets the C.M.S energies of the gq and g$\bar{q}$ systems respectively[9].

$$E_i = \sqrt{s}\,\frac{(2\sin\theta_3\sin\theta_i)^{1/2}\sin(\theta_i/2)}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}, \quad (i = 1,2). \tag{3}$$

For a given energy, we can calculate the average number of the quark-antiquark pairs and then the average number of the final hadrons using the method in Ref.5. Under the assumption of independent fragmentation, the quark and the gluon fragment in the same way into the hadrons. Using the method given in Ref.5, the average number of various final hadrons produced in the fragmentation of every parton can be calculated. The sum of the results over the two partons is the total result of the events. Under the assumption of string fragmentation, the gq and the g$\bar{q}$ systems fragment into the hadrons in the same way as an e^+e^- 2jet events. Using Eq.(3), one can calculate the energies of these two systems and then the average number of the final hadrons produced in each system. The sum gives the total result of the 3jet events. In Table 1 the results with the two method

TABLE 1. The Comparison of the Calculated Average Charged Multiplicity for the 3jet Events at 29 GeV with Data [12]

Ex. results	Results of this paper		Lund Model predict
	Independent fragmentation	String fragmentation	
16.3±0.3±0.7	21.88	16.86	16.7

TABLE 2. Comparison of the Calculated Average Number of Various Final Hadrons for the 2jet Events at 29 GeV with the Data [10]

Particle	Ex. results	Results of this paper		Model predict[11]		
		Independent fragmentation	String fragmentation	Lund	Webber	Caltech II
$\pi^\pm$	10.7±0.6	14.91	10.67	10.5	11.3	10.3
π^0	5.3±0.7	6.4	4.2	6.1	6.5	5.7
$K^\pm$	1.35±0.13	1.83	1.40	1.48	1.24	1.35
$K^0,\bar{K}^0$	1.22±0.18	1.17	0.76	1.40	1.21	1.38
η	0.58±0.10	0.39	0.25	0.73	0.72	0.31
ρ^0	0.95±0.09	1.45	0.94	0.91	0.66	0.37
$K^{*\pm}$	0.62±0.085	0.81	0.53	0.76	0.48	0.39
K^{*0}	0.49±0.11	0.81	0.53	0.71	0.48	0.36
ϕ	0.086±0.013	0.120	0.079	0.12	0.07	0.09
p	0.60±0.08	1.04	0.77	0.64	0.45	0.71
λ	0.220±0.027	0.46	0.34	0.22	0.23	0.24
Ξ	0.020±0.012	0.050	0.037	0.028	0.069	0.041

are given, calculated for the completely symmetrical three jet events and compared with the only available experimental data[12].

The calculation of the average number of final hadrons in $e^+e^- \rightarrow$ 2jet events is simpler. Under the assumption of independent fragmentation, the quark and antiquark each take half of the total energy and fragment in the same way. As long as the average number of the final hadrons produced in the quark or antiqurak fragmentation is calculated (which can be done using the method given in Ref.[5] to calculate the average number of various final hadrons corresponding to half of the total energy), twice the obtained value gives the result of the total event. Under the assumption of string fragmentation, the C.M.S energy of the $q\bar{q}$ system is just the total e^+e^- energy. We calculate the average number of final hadrons corresponding to the total energy using the method given in Ref.[5], which are just the results of the 2jet events. Although the energy in one case is two times that in the other, the average number has no simple linear relations since the energy dependence given in Ref.5 is not linear. The comparison of the calculated results with the data is given in Table 2.

From Tables 1 and 2 we can see that the results obtained under the independent fragmentation assumption deviate clearly from the data, especially in the three jet events whereas those under the string fragmentation assumption are in agreement with the data. From Table.2 we can see that under the string fragmentation assumption both of our results based on the method in

Ref.[5] and those predicted in the Lund model are in good agreement with the data. This shows that the string fragmentation scheme does not contradict with the variable experimental data up to now. But under the assumption of the independent fragmentation, the average number of the final hadrons calculated using the same hadronization formula is much higher than the data. Furthermore, under this assumption we cannot get the 'string effect', which has been observed in many experiments. All these show that the independent fragmentation scheme is far from perfect. But to rule it out completely in e^+e^- annihilations, more experimental evidences and theoretical analyses are still needed.

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