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Two-Higgs-Doublet Model and the Splitting of $t\bar{t}$ p-States

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The existence of two Higgs-doublets alters the splittings of $1p$ and $2p$ $t\bar{t}$ states and reduces the ratios of the splittings, R. Their fine measurement will be helpful to clarify the existence of Higgs particles.

Since the Higgs particle, the key part of the standard model, has not been found so far, physicists are making continued efforts to extend and improve the model. There are quite a number of theoretical speculations, according to which there are at least two-Higgs-doublets which consist of two charged and two neutral physical Higgs bosons. Therefore, there are two vacuum expectation values. The phenomenological consequences of the two-Higgs-doublet model are extensively studied. Some authors, for example, G. Athanasiu, J. Frazini and J. Gilman[1], had discussed its effects on the $t\bar{t}$ spectrum in detail. They pointed out that if the two neutral Higgs-bosons had the same mass, the exchange of the scalar particle will correspond to a Yukawa-type attractive potential

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$$V_H(r) = -\left[\left(\frac{\xi}{\eta}\right)\frac{gm_t}{2M_W}\right]^2 \cdot \frac{1}{2\pi r} e^{-rM_H} \tag{1}$$

added to the spin-independent part of the nonrelativistic potential, where η and ξ are the vacuum expectation values of the unmixed Higgs fields coupled to charge 2/3 and —1/3 quarks, respectively. M_H, M_W and m_t are the masses of the neutral Higgs-boson, W boson and t quark, respectively. g is the coupling constant of the standard electroweak model. The restriction on the relation between the above parameters can be obtained from the K_S^0-K_L^0 mass difference and the B^0-$\bar{B}^0$ mixing. If M_H is not very large, ξ/η will be about 5 ~ 10. In that case, the presence of the Higgs-boson exchange potential will have observable effects on the splittings of energy levels, E_{2s}--E_{1s} and E_{2p}--E_{1p}, and the wave function at the origin $|\psi(0)|$.

However, it is unfortunate that in most cases the splittings of $t\bar{t}$ energy levels with the Higgs-boson exchange can be also obtained from changing the form or adjusting the parameters of the potential without the Higgs-boson exchange, except the particular cases in which the inversion of the $2s$ and $1p$ levels take place due to the presence of a Higgs-boson exchange of sufficient strength.

The reason for this is that the Yukawa-type coupling due to the scalar-Higgs-particle exchange can be considered as a quasi-Coulomb potential in the lowest-order nonrelativistic approximation. When it is added to the Coulomb potential of the vector-gluon exchange, the total effect amounts to enhancing the coupling constant of the Coulomb potential and changing it into a new effective form, $-\tilde{\alpha}/r$, where

$$\tilde{\alpha} = \frac{4}{3}\alpha_s + \frac{1}{4\pi}\left[\frac{gm_t}{2M_W}\right]^2\left[\frac{\xi}{\eta}\right]^2 e^{-M_H r} \tag{2}$$

On the right hand side of Eq.(2) the first term is from the one-gluon exchange and the other is from the Higgs-boson exchange. Therefore, the effect of adding Higgs-boson exchange in a sense is equivalent to no addition of Higgs-boson exchange and the taking of a larger α_s only.

How can the existence of the Higgs-boson be clarified more effectively? We think the key is that the scalar-Higgs-boson exchange and the vector-gluon exchange have different Lorentz transformation properties. Although these two exchanges have the same kind of contributions to the spin-independent potential in the lowest-order nonrelativistic approximation, they play contrary roles in the spin-dependent part of the first-order relativistic corrections. Therefore it is possible for us to clarify in principle the existence of the Higgs-boson to study carefully the spin-dependent relativistic corrections of the $t\bar{t}$ spectrum.

The strong interaction potential between quarks consists of a scalar confining potential and a potential from the vector-gluon exchange. If the Higgs-boson exchange exists also, the total potential will be the sum of the three potentials. By using the standard reducing procedure, to order (v^2/c^2) we have the Hamiltonian of $t\bar{t}$ system:

$$H = H_0 + H_1 \tag{3}$$

$$H_0 = \frac{p^2}{m} + S + V \tag{4}$$

$$H_1 = -\frac{p^4}{4m} + V_{SD} + V_{SI} \tag{5}$$

$$V_{SD} = \frac{1}{2m^2}\left(\frac{3}{r}V' - \frac{1}{r}S'\right)(S_1 + S_2)\cdot L + \frac{2}{3m^2}S_1\cdot S_2\nabla^2 V$$
$$-\frac{1}{3m^2}[3(S_1\cdot \hat{r})(S_2\cdot \hat{r}) - (S_1\cdot S_2)]\left(V'' - \frac{1}{r}V'\right) \tag{6}$$

$$V_{SI} = \frac{1}{4m^2}\left\{[p^2(V - rV')] + 2(V - rV')p^2\right.$$
$$\left. + \frac{1}{2}\left(\frac{8}{r}V' + V'' - rV'''\right) + \frac{2}{r}VL^2\right\}$$
$$-\frac{1}{4m^2}\left\{2[p^2, S] + 4Sp^2 + \frac{2}{r}S' + S''\right\} \tag{7}$$

where V is the vector potential and S is the scalar one. V_{SD} represents the spin-dependent terms and V_{SI} the spin-independent terms. We are chiefly concerned with the first term, the spin-orbit coupling term, of V_{SD}. The contribution of S to this term and that of V to it have opposite signs, while their contributions to the nonrelativistic potential have the same signs. Consequently, although the relativistic corrections are very small for the $t\bar{t}$ system, through a careful comparison of the 0-th order results with the first order values for the $t\bar{t}$ spectrum, it might be possible to clarify the existence of Higgs-boson exchange, provided that the experimental accuracy can reach a sufficient high level.

For the above-mentioned reasons, by using the following potential, of which the strong interaction part, i.e. the Λ-dependent part, was used by us as a one-parameter-potential with a running coupling constant to discuss the relativistic effects, $E1$ and $M1$ transitions for heavy quarkonia[2],

$$V(r) = -\frac{8\pi}{33 - 2n_f}\,\frac{1}{\ln \Lambda r}\,\frac{1 - \Lambda r}{1 + \Lambda r}\cdot\frac{1}{r} \tag{8}$$

$$S(r) = \frac{8\pi}{33 - 2n_f}\Lambda r^2 + V_H(r) \tag{9}$$

we calculated the splittings of the $t\bar{t}$ p-wave states, where QCD scale parameter Λ is taken to be 470 MeV by fitting the $c\bar{c}$ and $b\bar{b}$ spectrum, the flavor number n_f is taken to be 4 and m_t = 60 Gev. The values of ξ/η and M_H in V_H(r) are taken from Ref.[1]. The results are listed in Table 1, from which it can be seen that, when considering the contributions of the Higgs-boson exchange, the ratios R of the p-wave state splittings evidently reduce. With the same M_H, the larger ξ/η , the more the

TABLE 1. Calculated Values (All Units GeV) of the Splittings of the $t\bar{t}$ Low-Lying Levels for a Few Different Values of M_H and ξ/η and m_t = 60 GeV. α_s is Taken to be a Running Constant as Given in Eq.(8)

M_H	ξ/η	$E_{2s}-E_{1s}$	$E_{2s}-E_{1P}$	$E_{1P_1}-E_{1P_0}$	R_{1P}	$E_{2P_1}-E_{2P_0}$	R_{2P}
0	0	0.8384	0.0653	0.0080	0.841	0.0040	0.837
5	4	1.2866	0.0124	0.0084	0.799	0.0062	0.779
10	6	1.5873	−0.0165	0.0065	0.781	0.0044	0.724
10	8	2.4088	−0.1090	0.0060	0.727	0.0034	0.626
20	6	1.2585	0.0090	0.0086	0.815	0.0061	0.762
20	10	2.5213	−0.1452	0.0081	0.763	0.0048	0.648
20	14	6.9405	−0.3965	0.0077	0.674	0.0037	0.630
40	8	1.1933	0.0154	0.0094	0.802	0.0057	0.766

TABLE 2. Calculated Values (All Units GeV) of the Splittings of the $t\bar{t}$ Low-Lying Levels for a Few Different Values of M_H and ξ/η and m_t = 60 GeV. α_s is Taken to be a fix value, α_s = 0.38

M_H	ξ/η	$E_{2s}-E_{1s}$	$E_{2s}-E_{1P}$	$E_{1P_1}-E_{1P_0}$	R_{1P}	$E_{2P_1}-E_{2P_0}$	R_{2P}
5	4	1.2458	0.0243	0.0109	0.772	0.0144	0.770
10	3	1.0448	0.0586	0.0166	0.787	0.0087	0.785
20	10	2.3229	−0.1242	0.0166	0.754	0.0117	0.714
20	14	6.6360	−0.3672	0.0164	0.712	0.0083	0.696
40	8	1.1322	0.0292	0.0198	0.706	0.0132	0.764

reduction of R; and with the same ξ/η, the smaller M_H, the more the reduction of R. These results are interesting and can be easily understood.

It is well known that the splittings of ^3P_J states (i.e. the quark-antiquark bound states with L = 1 and J = 0, 1, 2.) are caused by the spin-orbit and tensor terms. Within the picture of vector and scalar exchanges the ratio R can written in the following[3]:

$$R = \frac{M(^3p_2) - M(^3p_1)}{M(^3p_1) - M(^3p_0)} \tag{10}$$

where a and b are the matrix elements

$$a = \frac{1}{2m^2}\left\langle \frac{3}{r}V' - \frac{1}{r}S' \right\rangle \tag{11}$$

$$b = \frac{1}{12m^2}\left\langle \frac{1}{r} V' - V'' \right\rangle \tag{12}$$

for a $1p$-wave state

$$R = \frac{2a - \frac{12}{5} b}{a + 6b} \tag{13}$$

Calculating the derivative of R with respect to a, we obtained

$$\frac{dR}{da} = \frac{72}{5} \frac{b}{(a + 6b)^2} \tag{14}$$

Since $b > 0$ for the Coulomb-type potential, we have

$$\frac{dR}{da} > 0 \tag{15}$$

Because the contribution of the scalar Yukawa-type potential $V_H(r)$ which is added to the S makes the a reduced, it is necessary that the R will decrease as the Higgs particles are added. The strength of $V_H(r)$ is proportional to $(\xi/\eta)^2$ and decreases as M_H increases. This can explain the variation in R with ξ/η and M_H.

However, as the shifts of the splittings are only a few MeV, to detect them will demand a very accurate experiment. Once the experimental technology is developed to such an excellent level, it will be possible to clarify the existence of the Higgs particle.

Moreover, it is worth noticing that if the measured value of $(E_{1P} - E_{1S})/(E_{2S} - E_{1S})$ approaches 1, then two possibilities can be expected. One is that α_s is a running and small coupling constant, but compensated by the Higgs-bosons exchanges. This is the case shown by Table 1. And the other possibility is that there are no Higgs particles and the α_s, as shown in Ref.[1], is not of asymptotic freedom but takes a larger fix value. In order to discuss the latter case, in accordance with Ref.[2], we assumed $\alpha_s = 0.38$ and calculated these splittings also. The results are shown in Table 2. From the table we can see that in this case the splittings are about twice the former and E_{1p}--E_{1p} is about 30 MeV. Therefore accurate measurement of the splittings of p-wave states will help to clarify the two possibilities.

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