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Choice of Single Particle Potentials in a Many-Body calculation--Hartree-Fock and Mass Operator Potential

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A detailed comparison between the Hartree-Fock (HF) and the mass operator (M) potential is made by means of a solvable three-level model. It is found that in a quite large region of force parameters (FPR) the M potential gives a much better result than the HF potential and at certain points in FPR the HF result undergoes a sudden jump while the M result varies smoothly. The convergence behavior of the perturbation expansion of M has also been studied.

1. INTRODUCTION

An adequate choice of a single particle (SP) potential is important for a many-body calculation, as it affects the convergence behavior of the calculation significantly and is also needed to distinguish more clearly the SP from the collective mode of the particle motion.

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One of us has shown that from many aspects the SP potential $u_{ab} = M_{ab}(\epsilon_b^M)$ defined in terms of the mass operator (M) may be regarded as an optimal choice[1]. The Hartree-Fock (HF) potential is just its lowest order approximation. Using different realistic nuclear two-body potentials, such as the Paris, the Reid and the Hamada-Johnston potential, we have calculated the SP and the single hole (SH) spectra in the region of ^{16}O and ^{40}Ca[2--4]. Our results indicate that the M potential is a better choice than the Brueckner-Hartree-Fock (BHF) and the renormalized BHF potentials.

The solution of the solvable model provides a simple and reliable means to compare different approximate methods[5]. Using a solvable three-level model, Anastasio et al.[6] studied the convergence behavior of the linked-valence expansion of the effective interaction and its dependence on the choice of the SP potential. From their results it seems that the HF potential is a very good choice. However, this may be a special result related with the interaction Hamiltonian considered by them and may not be generally true. It is thus worthwhile to study this problem further and in more detail.

To enlarge the variety of states which the model may cover, we have considered various choices of the model Hamiltonian under the requirement that it preserve the SU(3) symmetry, contain the choice of Anastasio et al. as a special caseand yet still remain as simple as possible for the calculation. We have found that the Hamiltonian given in the next section suffices for our purpose. Just as expected from the theory[1], the SP and SH energies obtained from the M potential are exactly equal to the corresponding rigorous values, though the results of the HF calculation may deviate from them widely. Furthermore, the absolute value of the overlapping integral with the relevant rigorous eigenstate calculated with the M potential, is always larger than or equal to that calculated with the HF potential. Our calculation shows that in a quite large region of force parameters (FPR) the M potential gives a much better result than the HF potential and at certain points in FPR the HF result undergoes a sudden jump though the M result varies smoothly. Since in general the mass operator cannot be calculated rigorously, certain approximations are needed. In this paper results up to the third order in perturbation expansion are given. The convergence behavior of the perturbation expansion of the mass operator is studied for different parameters.

2. METHOD OF THE CALCULATION

2.1 Generalized Solvable Three-Level Model

As mentioned in the previous section, we have extended the model Hamiltonian considered by Anastasio et al. (MHA) so that the model can contain regions where the HF approximation shows different characteristics, but remains as simple as possible and include MHA as a special case. The three levels, each with a degeneracy Ω, will be labelled by z, y, x in the order of increasing energy. We shall use a pair ($p\alpha$) to denote a SP state, where the Greek letter α specifies the energy level and the Latin letter p the state within each level, i.e. $\alpha = z, y, x$ and $p = 1, 2, \cdots, \Omega$.

Let us introduce

$$H_0 = \sum_{p\alpha} \varepsilon_\alpha^0 a_{p\alpha}^+ a_{p\alpha}, \quad H_1 = V \sum_{pq\alpha\beta(\alpha\neq\beta)} a_{p\alpha}^+ a_{q\alpha}^+ a_{q\beta} a_{p\beta},$$

$$H_2 = X \sum_{pq\alpha\beta\gamma\delta} a_{p\alpha}^+ a_{q\beta}^+ a_{q\delta} a_{p\gamma}, \quad (\alpha, \beta \neq \gamma, \delta, \text{ when } \alpha = \beta, \gamma \neq \delta \text{ and when } \gamma = \delta, \alpha \neq \beta)$$

$$H_3 = W \sum_{pq\alpha\beta(\alpha\neq\beta)} a^+_{p\alpha}a^+_{q\beta}a_{q\alpha}a_{p\beta}, \quad H_4 = U \sum_{pq\alpha\beta\gamma(\beta\neq\gamma)} a^+_{p\alpha}a^+_{q\beta}a_{q\gamma}a_{p\alpha},$$
$$H_5 = T \sum_{pq\alpha\beta\gamma(\alpha\neq\beta,\gamma,\beta\neq\gamma)} a^+_{p\alpha}a^+_{q\beta}a_{q\alpha}a_{p\gamma}, \tag{1}$$

where ε^0_α is the energy of the single particle state α, while V, X, W, U and T are the force parameters. The Hamiltonian of the system is given by

$$H = H_0 + H_I, \quad H_I = \sum_{i=1}^{5} H_i. \tag{2}$$

The infinitesimal generators of the group $U(3)$ can be written as

$$A_{\alpha\beta} = \sum_p a^+_{p\alpha}a_{p\beta}.$$

Since the number of particles is fixed in the model, the total particle number operator $\hat{N} = A_{xx} + A_{yy} + A_{zz}$ can be excluded and we only need to consider the eight generators of $SU(3)$. In terms of $A_{\alpha\beta}$ the model Hamiltonian may be rewritten in the form

$$\begin{aligned}
H_0 &= \Sigma_\alpha \varepsilon^0_\alpha \hat{n}_\alpha, \quad H_1 = V(A^2_{xy} + A^2_{xz} + A^2_{yx} + A^2_{yz} + A^2_{zx} + A^2_{zy}),\\
H_2 &= 2X(A_{xy}A_{xz} + A_{yx}A_{yz} + A_{zx}A_{zy} + A_{yx}A_{zx} + A_{xy}A_{zy} + A_{xz}A_{yz}),\\
H_3 &= 2W(A_{xy}A_{yx} + A_{yz}A_{zy} + A_{zx}A_{xz} - \hat{N}),\\
H_4 &= U(\hat{N} - 1)(A_{xy} + A_{xz} + A_{yx} + A_{yz} + A_{zx} + A_{zy}),\\
H_5 &= T(A_{xy}A_{zx} + A_{xz}A_{yx} + A_{yx}A_{zy} + A_{yz}A_{xy} + A_{zx}A_{yz} + A_{zy}A_{xz}),
\end{aligned} \tag{3}$$

where $\hat{n}_\alpha = A_{\alpha\alpha}$ is the particle number operator of the level α. Eq.(3) shows that the eigensolutions of H can be classified according to the irreducible representations (IRs) of $SU(3)$ and H will not connect states belonging to different IRs. We shall use the same IRs $(\lambda\mu)$ and basis states $|(\lambda\mu)\epsilon\Lambda\upsilon>$ for $SU(3)$ as described in Ref.[6]. The matrix elements of H can be obtained easily by means of the relations given for $A_{\alpha\beta}$ in Ref.[7]. From the eigenvalue equation

$$\sum_{\varepsilon\Lambda\nu} [\langle(\lambda\mu)\varepsilon'\Lambda'\nu'|H|(\lambda\mu)\varepsilon\Lambda\nu\rangle - E\delta_{\varepsilon'\Lambda'\nu',\varepsilon\Lambda\nu}]C_{\varepsilon\Lambda\nu} = 0, \tag{4}$$

the exact eigenvalues and eigenfunctions of the system can be computed directly.

2.2 HF Basis

The three HF levels will be labelled by 1, 2 and 3. From Eq.(1) it can be easily seen that H_I will not mix different degenerate states. This means that the degeneracy of each HF level is also Ω and each HF eigenstate $|p\alpha>$ can be expanded in the unperturbed SP basis $|p\alpha>$ as $|p\alpha> = \Sigma_\alpha |p\alpha> C_{\alpha\alpha}$, $(\alpha = 1, 2, 3)$. The HF eigenvalue equation has the form

$$\sum_\alpha [(\varepsilon^0_\alpha - \varepsilon^{\mathrm{HF}}_\alpha)\delta_{\alpha\alpha'} + \langle p\beta|U^{\mathrm{HF}}|p\alpha\rangle]C_{\alpha\alpha} = 0, \tag{5}$$

where the matrix element of the HF potential is given by

$$\langle p\beta|U^{\mathrm{HF}}|p\alpha\rangle = \sum_{q,h,\gamma,\delta} C^*_{\gamma h}\langle p\beta q\gamma|H_I|p\alpha q\delta\rangle C_{\delta h}, \tag{6}$$

here h means summing over hole states only. Let N denote the number of particles of a closed-shell configuration. If $N = a$, we note that h can only be 3. For the matrix elements of H_I one can easily find

$$\begin{aligned}\langle p\beta q\gamma|H_I|p\alpha q\delta\rangle &= 2V(1-\delta_{pq})(1-\delta_{\tau\alpha})(1-\delta_{\beta\alpha})\delta_{\tau\beta}\delta_{\alpha\delta}\\ &+ 2X(1-\delta_{pq})(1-\delta_{\beta\alpha})(1-\delta_{\beta\delta})(1-\delta_{\tau\alpha})(1-\delta_{\tau\delta})[(1-\delta_{\alpha\delta})\delta_{\tau\beta}+(1-\delta_{\tau\beta})]\\ &+ U[\delta_{\beta\alpha}(1-\delta_{\tau\delta})+\delta_{\tau\delta}(1-\delta_{\beta\alpha})+\delta_{pq}\delta_{\beta\delta}(1-\delta_{\tau\alpha})-\delta_{pq}\delta_{\tau\alpha}(1-\delta_{\beta\delta})]\\ &+ 2W(\delta_{\beta\delta}\delta_{\tau\alpha}-\delta_{pq}\delta_{\tau\delta}\delta_{\beta\alpha})(1-\delta_{\alpha\delta})\\ &+ T(1-\delta_{\tau\beta})(1-\delta_{\alpha\delta})\{(1-\delta_{\beta\alpha})(1-\delta_{\tau\alpha})[(1-\delta_{\tau\delta})-\delta_{pq}(1-\delta_{\beta\delta})]\\ &+ (1-\delta_{\tau\delta})(1-\delta_{\beta\delta})[(1-\delta_{\beta\alpha})-\delta_{pq}(1-\delta_{\tau\alpha})]\}.\end{aligned} \tag{7}$$

Since the right hand side of Eq.(6) contains the unknowns $C_{\alpha\alpha'}$ as is well-known, Eq.(5) has to be solved self-consistently.

2.3 M Basis

Since no confusion will arise, we shall also use $|p\alpha>$ to denote the SP eigenstate determined by the M potential, where again α labels the energy level and p the state within each level. Let us expand $|p\alpha>$ in terms of the unperturbed states $|p\beta>$:

$$|p\alpha\rangle = \sum_{\beta} |p\beta\rangle d_{\beta\alpha},$$

By definition $<p\alpha|u|p\alpha> = <p\alpha|$ --- $|p\alpha>$, one finds easily that the SP Schrödinger equation can be written as

$$\sum_{\beta}[(\varepsilon^0_\beta-\varepsilon^M_\alpha)\delta_{\alpha\beta}+M_{\alpha\beta}(\varepsilon^M_\alpha)]d_{\beta\alpha}=0. \tag{8}$$

In the solvable model considered here, the SP Green function and its inverse can be obtained rigorously. Hence by means of the relation

$$M_{\alpha\beta}(\omega) = G^{-1}_{\alpha\beta}(\omega)+(\omega-\varepsilon^0_\alpha)\delta_{\alpha\beta},$$

the mass operator $M_{\alpha\beta}(\omega)$ can be calculated rigorously. Substituting the above relation into Eq.(8), we get

$$\sum_{\beta} G^{-1}_{\alpha\beta}(\varepsilon^M_\alpha)d_{\beta\alpha}=0. \tag{9}$$

Let $E_n(A)$ and $|\psi_n(A)>$ denote the exact eigenvalue and eigenfunction of a system of A particles, i.e. $H|\psi_n(A)> = E_n(A)|\psi_n(A)>$, where the ground state is specified by $n = 0$. From Eq.(9) we find that $\epsilon^M_\alpha = \pm[E_\alpha(N\pm1)-E_0(N)]$ holds rigorously, as it has been pointed out in Ref.[1]. We note that the solutions of Eq.(9) or (8) are overcomplete. Following the suggestion of Ref.[8], we have selected a complete set of eigensolutions according to the principle of maximum overlap. Our calculation indicates that this is a good choice.

2.4 Perturbation Expansion of M

For a real many-body system it is in general impossible to calculate M rigorously. For this reason, we have also studied the convergence behavior of the perturbation expansion of M by means of this solvable model. The perturbation series is calculated up to the third order in H_i and the importance of different vertices is investigated. For convenience of description, the relevant expressions are given below[9]:

$$M^{(1)}_{\alpha\beta}(\omega) = \sum_{\gamma} v_{\alpha\gamma,\beta\gamma}n_\gamma,$$

$$M^{(21)}_{\alpha\beta}(\omega) = \frac{1}{2}\sum_{\mu\nu\gamma} v_{\alpha\gamma,\mu\nu}v_{\mu\nu,\beta\gamma}\left(\frac{\bar{n}_\mu\bar{n}_\nu n_\gamma}{\omega+\varepsilon_{\gamma,\mu\nu}+i\eta}+bp.\right),$$

$$M^{(22)}_{\alpha\beta}(\omega) = \sum_{\mu\nu\gamma} v_{\alpha\nu,\mu\nu}v_{\mu\gamma,\beta\gamma}\left(\frac{\bar{n}_\mu n_\nu n_\gamma}{\omega-\varepsilon_\mu+i\eta}+bp.\right),$$

$$M^{(23)}_{\alpha\beta}(\omega) = \sum_{\mu\nu\gamma} v_{\alpha\nu,\beta\mu}v_{\mu\gamma,\nu\gamma}\frac{\bar{n}_\nu n_\mu n_\gamma-\bar{n}_\mu n_\nu n_\gamma}{\varepsilon_\mu-\varepsilon_\nu},$$

$$M^{(31)}_{\alpha\beta}(\omega) = \frac{1}{+}\Sigma_{\mu\nu\rho\lambda\gamma}v_{\alpha\nu,\rho\mu}v_{\rho\mu,\lambda\gamma}v_{\gamma\lambda,\nu\beta}$$
$$\times\left(\frac{\bar{n}_\rho\bar{n}_\gamma\bar{n}_\mu\bar{n}_\lambda n_\nu}{(\omega+\varepsilon_{\nu,\mu\rho}+i\eta)(\omega+\varepsilon_{\nu,\lambda\gamma}+i\eta)}+\frac{\bar{n}_\gamma\bar{n}_\lambda n_\rho n_\nu n_\mu}{(\omega+\varepsilon_{\nu,\lambda\gamma}+i\eta)\varepsilon_{\mu\rho,\gamma\lambda}}\right.$$
$$\left.+\frac{\bar{n}_\rho\bar{n}_\mu n_\gamma n_\nu n_\lambda}{(\omega+\varepsilon_{\nu,\rho\mu}+i\eta)\varepsilon_{\lambda\gamma,\rho\mu}}-bp.\right), \qquad (10)$$

$$M^{(32)}_{\alpha\beta}(\omega) = \Sigma_{\mu\nu\rho}v_{\alpha\mu,\rho\nu}v_{\rho\gamma,\lambda\mu}v_{\lambda\nu,\beta\gamma}$$
$$\times\left(-\frac{\bar{n}_\nu\bar{n}_\lambda\bar{n}_\rho n_\mu n_\gamma}{(\omega+\varepsilon_{\mu,\rho\nu}+i\eta)(\omega+\varepsilon_{\gamma,\lambda\nu}+i\eta)}-\frac{\bar{n}_\nu\bar{n}_\lambda\bar{n}_\mu n_\gamma n_\rho}{(\omega+\varepsilon_{\gamma,\lambda\nu}+i\eta)\varepsilon_{\gamma\rho,\mu\lambda}}\right.$$
$$\left.-\frac{\bar{n}_\nu\bar{n}_\gamma\bar{n}_\rho n_\mu n_\lambda}{(\omega+\varepsilon_{\mu,\rho\nu}+i\eta)\varepsilon_{\mu\lambda,\rho\gamma}}-bp.\right),$$

$$M^{(33)}_{\alpha\beta}(\omega) = \frac{1}{2}\Sigma_{\mu\nu\lambda\rho\gamma}v_{\alpha\lambda,\beta\rho}v_{\gamma\nu,\lambda\mu}v_{\rho\mu,\gamma\nu}$$
$$\times\left(\frac{\bar{n}_\rho\bar{n}_\lambda\bar{n}_\mu n_\nu n_\gamma-\bar{n}_\nu\bar{n}_\gamma n_\rho n_\lambda n_\mu}{\varepsilon_{\mu\rho,\nu\gamma}\varepsilon_{\mu\lambda,\nu\gamma}}+\frac{\bar{n}_\lambda\bar{n}_\mu n_\rho n_\nu n_\gamma-\bar{n}_\rho\bar{n}_\nu\bar{n}_\gamma n_\lambda n_\mu}{(\varepsilon_\rho-\varepsilon_\lambda)\varepsilon_{\mu\lambda,\gamma\nu}}\right.$$
$$\left.+\frac{\bar{n}_\rho\bar{n}_\mu n_\lambda n_\nu n_\gamma-\bar{n}_\lambda\bar{n}_\nu\bar{n}_\gamma n_\rho n_\mu}{(\varepsilon_\rho-\varepsilon_\lambda)\varepsilon_{\gamma\nu,\mu\rho}}\right).$$

where $\bar{n}_\mu = 1 - n_\mu$ and $n_\mu = 0$ or 1 refer to a particle or a hole state, ϵ_μ, $\upsilon_\lambda = \epsilon_\mu - \epsilon_\upsilon - \epsilon_\lambda$, $\epsilon_{\mu\upsilon,\lambda\rho} = \epsilon_\mu + \epsilon_\upsilon - \epsilon_\lambda - \epsilon\rho$ and bp (backward-propagating) designate the terms obtained from the previous one by changing n_μ, $\bar{n}_\mu$, $+i\eta$ to $\bar{n}_\mu$, n_μ, $-i\eta$, respectively.

3. RESULTS AND DISCUSSIONS

For simplicity we consider the case of $\Omega = 4$ in our calculation. Five sets of force and level distance parameters (FLPs) considered in the calculation are listed in Table 1, where set I and II are the same as those chosen in Ref.[6].

34 eigensolutions can be obtained from Eq.(9). In order to select an adequate complete set, we have calculated the square of the overlapping integrals (SOI), i.e. $|<\psi_n(N+1)\,|\,a_n^+\,|$

TABLE 1.

No. \ Parameter	U	V	X	W	T	ε_x^0	ε_y^0	ε_z^0
I	0.4	0.4	0.4	−0.5	0	12.5	10.0	0
II	−0.5	−0.5	−0.5	−1.0	0	2.5	2.0	0
III	−0.55	−1.80	−0.4	−0.45	2.10	2.5	2.0	0
IV	−0.1	−1.0	0.5	−0.2	0.5	2.5	2.0	0
V	0.4	5.0	6.0	−5.0	0.2	2.3	2.0	0

TABLE 2.

	1	2	3	4	5	...
SOI	0.9799	0.9880	0.9339	0.0284	0.0229	...

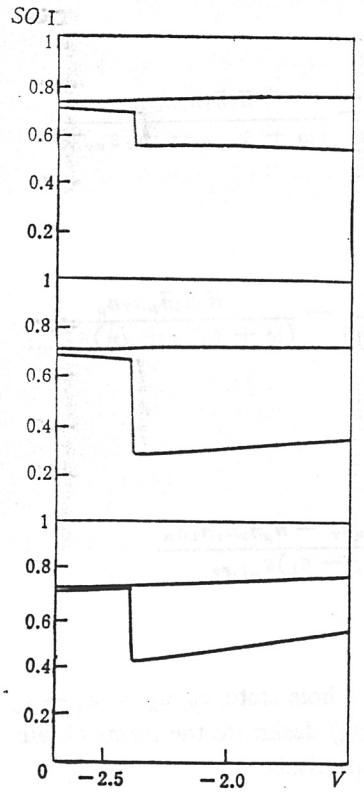

FIG. 1

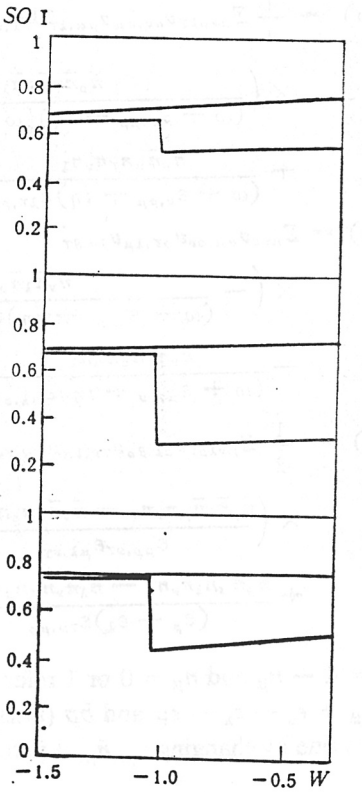

FIG. 2

TABLE 3.

Parameter	a	ε_a^{Γ}	ε_a^{M}	ε_a^{HF}	SOI value $(SOI)_a^{M}$	$(SOI)_a^{HF}$
I	1	−0.857	−0.857	−0.588	0.9799	0.9799
	2	10.046	10.046	10.068	0.9880	0.9871
	3	13.657	13.657	13.689	0.9339	0.9326
II	1	−14.465	−14.465	−14.498	0.9953	0.9953
	2	5.918	5.918	5.966	0.9961	0.9961
	3	6.991	6.991	7.092	0.9924	0.9924
III	1	−11.430	−11.430	−9.304	0.7553	0.4982
	2	1.823	1.823	0.369	0.7229	0.3149
	3	2.342	2.342	11.735	0.7508	0.5461
IV	1	−5.607	−5.607	−5.816	0.5489	0.3683
	2	2.589	2.589	3.573	0.7343	0.5915
	3	7.910	7.910	7.679	0.5129	0.4911
V	1	−49.124	−49.124	−21.304	0.4565	0.1436
	2	19.396	19.396	16.653	0.5679	0.1228
	3	19.953	19.953	28.228	0.5321	0.4128

$\psi 0(N)|^2$ and $|\psi_n(N-1)|\, a_n\, |\psi 0(N)|^2$ and selected a set of three linearly independent SP eigenstates by means of the three largest SOIs. As an example, we have listed the SOIs calculated with the first set of parameters in Table 2. Only 5 values are copied down explicitly, since the rest are all very small and some are almost zero. The value in the first column of Table 2 is that of SOI_h, while the other four are those of SOI_n. One can see that the three SP states can be selected satisfactorily by means of SOI, which also tells to what extent the eigenstate $|\psi n(N+1)>$ ($|\psi n(N-1)>$) may be described as a particle (hole) outside (inside) the core. The latter will be referred to as the SP (SH) mode of a system of $N+1$ ($N-1$) particles. We note that $E_n(N+1)$ and $E_h(N-1)$ can be calculated exactly from Eq.(4). Thus, ϵ_a can also be determined rigorously through $\epsilon_a = \pm[E_a(N\pm1) - E_0(N)]$. Our calculated results with the M and the HF potential are given in Table 3. It can be seen that we always have

$$(SOI)_a^{M} \geqslant (SOI)_a^{HF};\quad \varepsilon_a^{M} = \varepsilon_a^{\Gamma}.$$

but ε_a^{HF} may deviate from ε_a^{Γ} significantly. For parameter sets I and II the results obtained from the HF and the M potential are almost the same. They are in good agreement with the exact values. This is in conformity with what has been found in Ref.[6]. However, for the other parameter sets the HF results are poor. From the values calculated with parameter set III we note that even for those states of $(N\pm1)$ particles, which contain the SP (SH) mode as a principal component, the HF approximation may still be inadequate. It is worth-while to ask how good in this case the M and the HF basis will be for the calculation of a system of $(N\pm x)$ particles with $x \geq 2$. We have calculated the effective interaction between two valence particles as well as between two valence holes. The results

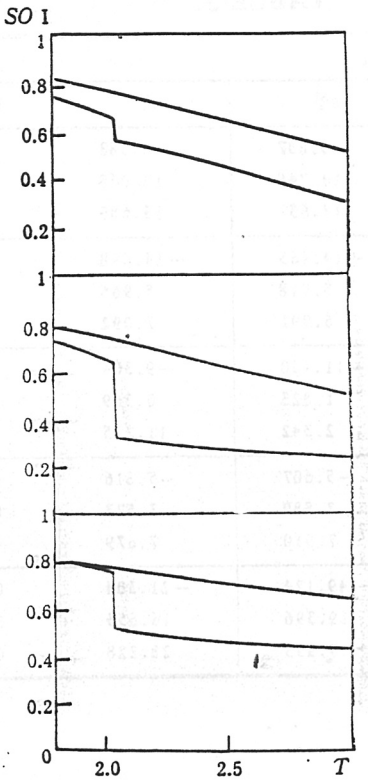

FIG. 3

will be reported elsewhere. Even though $\epsilon_a^M = \epsilon_a^r$ always holds, the SOI values listed in the fourth and fifth row of Table 3 show that for these sets of FLP the core polarization effect cannot be neglected if one wants to obtain a better approximation for the wavefunction.

In order to understand the difference of the HF and M basis we change the force parameters V, W and T, respectively to see the dependence of the M and HF results on the force parameters and list the $(SOI)_a^M$ and $(SOI)^{\mathrm{HF}}$, $(a = 1, 2, 3)$ result in Figs.1--3. From the figures one finds that $(SOI)_a^M$ varies smoothly within 0.7 to 0.8 for a given parameter region. This means that the M basis is a good choice. In contrast, each of the curves $(SOI)_a^{\mathrm{HF}}$ undergoes a sudden jump. It occurs, for instance, at $V = -2.4$ in Fig.1, $W = -1.1$ in Fig.2 and $T = 2.0$ in Fig.3. This is certainly very interesting since no such jump occurs for $(SOI)_a^M$. Its physical implication is being studied and will be discussed elsewhere.

However, it is still desirable to ask whether the M basis may make a perturbation expansion converge faster. Using the M basis, we have calculated the perturbation series of M through the third order and solved Eq.(8) without taking the wave-function (WF) self-consistency into account. The results are presented in Table 4. Consider, for instance, the column block under the heading III. Since we have not considered the WF self-consistency, the SP energies obtained from $M^{[1]}$ are, as expected, worse than the HF results, but the SOI values are even much better. If the higher order terms of M are taken into account, one can see that the results converge quite quickly. We have

TABLE 4.

Parameter		I			II			III		
Energy		1	2	3	1	2	3	1	2	3
g	$M^{(1)}$	13.792	10.091	−0.5858	7.0874	5.9694	−14.494	1.7118	−0.1847	−8.9855
	$M^{(1)} + M^{(2)}$	13.632	10.027	−0.7673	7.0120	5.9320	−14.490	2.4971	0.4864	−12.0170
	$M^{(1)} + M^{(2)} + M^{(3)}$	13.718	10.044	−0.8173	6.9928	5.9237	−14.480	2.5454	1.4069	−10.4270
	M	13.657	10.046	−0.8570	6.9908	5.9177	−14.465	2.3419	1.8226	−11.4300
SOI	$M^{(1)}$	0.9326	0.9875	0.9798	0.9924	0.9961	0.9953	0.7332	0.7114	0.7498
	$M^{(1)} + M^{(2)}$	0.9336	0.9879	0.9799	0.9924	0.9961	0.9953	0.7492	0.7227	0.7535
	$M^{(1)} + M^{(2)} + M^{(3)}$	0.9338	0.9880	0.9799	0.9924	0.9961	0.9953	0.7496	0.7224	0.7550
	M	0.9339	0.9880	0.9799	0.9924	0.9961	0.9953	0.7508	0.7229	0.7553

further studied the contribution of various low order irreducible vertices separately. Because of limitation, of space, we would only like to mention that in general the contribution of the forward-propagating terms is attractive whereas that of the *bp* terms is repulsive.

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