

# Ratios of the Yields of Charmed Baryon to Meson in $e^+e^-$ Annihilation

Chen Esheng and Xie Qubing

(Department of Physics, Shandong University, Jinan)

The ratios of the yields of charmed baryons to mesons in  $e^+e^-$  annihilations have been calculated by using the quark production rule and the quark combination rule. The results vary with energy  $\sqrt{s}$  and are in agreement with the available data.

---

## 1. INTRODUCTION

Recently people have paid much attention to the study of the properties of charmed particles produced in  $e^+e^-$  annihilations because these charmed particles result from the known primary partons and may contain more direct clues to the understanding of hadronization mechanisms which are as yet far from solved. The multihadron production in  $e^+e^-$  annihilation is a complex process: a primary quark-antiquark pair  $q_0\bar{q}_0$  is first produced by the electroweak interaction, then this primary  $q_0$  and  $\bar{q}_0$  pair creates many new quark-antiquark pairs by QCD vacuum excitation and all these quarks and antiquarks combine into hadrons through strong interactions. Using the theory of QED and taking the first order QCD corrections into account, the production of the primary quarks can be described correctly. But for the soft processes such as the QCD vacuum excitation of the new quark pairs induced by the primary quarks, and the combinations of all the quarks and antiquarks into hadrons, the description cannot be derived from the first principle since they involve non-perturbative treatment. Its realization requires a phenomenological model.

According to QED, the production weight of the primary quark  $q_0\bar{q}_0$  is proportional to the  $q_0^2$  charge squared, i.e. the ratio of production probabilities among different flavors  $u\bar{u}$ ,  $d\bar{d}$ ,  $s\bar{s}$ ,  $c\bar{c}$  and  $b\bar{b}$  is

$$P_u:P_d:P_s:P_c:P_b = 4:1:1:4:1 \quad (1)$$

For the QCD vacuum excitation one can estimate the relative probabilities of new quarks of various flavors, as follows. From the color flux tube model proposed by Casher *et al.* [1], the probability to produce a new quark pair with quark mass  $m_i$  by QCD vacuum excitation can be expressed as

$$p_i \propto e^{-km_i^2} \quad (2)$$

where  $k$  is a constant which is related to the properties of strong interaction among quarks. Eq.(2) is usually called the "tunneling effect". Taking  $m_u = m_d = 0.34$  GeV,  $m_s = 0.45$  GeV,  $m_c = 1.5$  GeV, and  $m_b = 4.73$  GeV and choosing the parameter  $k$  to fit the experimentally measured suppression factor of strange production  $\lambda \equiv P_s/P_u = 0.3$ , we get

$$P_u:P_d:P_s:P_c:P_b = 1:1:0.3:10^{-10}:10^{-35}. \quad (3)$$

Hence, in terms of the "tunneling effect", charm and heavier quarks are not expected to be produced in this soft process, and such a consequence has been confirmed by many experiments.

As mentioned above, the charm quark is only created by the electroweak interaction, and the fraction of c-jet events is  $4/10$  for  $2m_c \leq \sqrt{s} < 2m_b$  or  $4/11$  for  $\sqrt{s} \geq 2m_b$  in the process of  $e^+e^- \rightarrow$  two jets. For c-jets the primary  $c(\bar{c})$  must combine with the light quark  $q_i$  or  $\bar{q}_i$  produced by the QCD vacuum excitation to form a charmed particle. So the investigation of charm production can be used to study how the primary  $c(\bar{c})$  combines with light quarks and antiquarks to form various charm particles and give a sensitive test for different hadronization models. The ratio of the yields of charmed baryons to mesons,  $f \equiv B_c/M_c$  is a very significant quantity with respect to this problem. Anisovich *et al.* derived  $f = 1/2$  from their quark combination model. From the quark production rule in the process of  $e^+e^- \rightarrow$  two jets, proposed by the authors [2], we obtain that the ratio  $f$  depends on the total energy  $\sqrt{s}$ . When  $\sqrt{s}$  increases,  $f$  varies from 0 to  $1/3$ . In the following we will give a brief introduction to the calculation of  $f$  proposed by Anisovich *et al.* and some comments by us, then we will give a description of our work on this problem and finally the comparison between the calculated results using these two kinds of models and the experimental data.

## 2. ANISOVICH'S ALGORITHM

In the framework of the constituent quark model (CQM), Anisovich *et al.* [3] proposed a combination algorithm to describe how a pair of primary  $c\bar{c}$  quarks recombines with infinite pairs of light quarks  $q_i\bar{q}_i$  to form a charmed baryon and meson. They asserted that the combination is based on the nearest correlation in rapidity, i.e., they require the nearest neighbors from left to right in the rapidity axis to combine into a meson or a baryon. The probabilities of the light quarks  $q$  and  $\bar{q}$  adjacent to quark  $c(\bar{c})$  are  $1/2$ . For the  $c\bar{q}$  ..... type of arrangement in rapidity the  $c\bar{q}$  pair combines into a charmed meson  $M_c$ . For the  $c\bar{q}$  ..... type of arrangement the  $cq$  pair cannot form a hadron, and in these cases the consequence depends on what the next one is. When the number of light quark pairs  $N \rightarrow \infty$ , the probability of  $q$  or  $\bar{q}$  is still  $1/2$ . For the  $cq\bar{q}$ ..... type of arrangement the  $cq\bar{q}$  combines into a charmed  $B_c$ , and for  $cq\bar{q}$ ..... arrangement Anisovich's algorithms assume

that the  $q\bar{q}$  pair in these configurations combines into a light meson  $M$  and leaves the primary  $c$  alone. So their combination algorithm can be represented by

$$c \rightarrow c \left( \frac{1}{2} q + \frac{1}{2} \bar{q} \right) = \frac{1}{2} cq + \frac{1}{2} c\bar{q} = \frac{1}{2} cq + \frac{1}{2} M_c,$$

$$cq \rightarrow cq \left( \frac{1}{2} q + \frac{1}{2} \bar{q} \right) = \frac{1}{2} \underline{cqq} + \frac{1}{2} cq\bar{q} = \frac{1}{2} B_c + \frac{1}{2} cM,$$

Summing over these two steps they derived

$$c \rightarrow \frac{1}{4} B_c + \frac{1}{2} M_c + \frac{1}{4} cM, \quad (4)$$

The coefficients in the above equations express the probabilities for the appearance of the corresponding configurations. Eq.(4) shows that combining with light quarks twice, the primary  $c$  has a probability of  $1/4$  to become a charmed baryon,  $1/2$  to become a charmed meson, and another  $1/4$  to remain as what it is to repeat the above combination procedures. Continuing these algorithms an infinite number of times they obtained

$$c \rightarrow \frac{1}{4} B_c + \frac{1}{2} M_c + \frac{1}{4} \left\{ \frac{1}{4} B_c + \frac{1}{2} M_c + \frac{1}{4} \left[ \frac{1}{4} B_c + \frac{1}{2} M_c \right. \right. \\ \left. \left. + \frac{1}{4} \left( \frac{1}{4} B_c + \frac{1}{2} M_c + \dots \right) \right] \right\} \\ = \left[ \frac{1}{4} + \left( \frac{1}{4} \right)^2 + \left( \frac{1}{4} \right)^3 + \dots \right] B_c \\ + \frac{1}{2} \left[ 1 + \frac{1}{4} + \left( \frac{1}{4} \right)^2 + \dots \right] M_c = \frac{1}{3} B_c + \frac{2}{3} M_c. \quad (5)$$

Finally, Anisovich *et al.* derived their ratio of the yields of charmed baryon  $B_c$  to meson  $M_c$ ,  $f = 1/2$ .

### 3. AVERAGE NUMBER $\langle N \rangle$ OF THE LIGHT QUARK-ANTIQUARK PAIRS

The  $f = 1/2$  was derived by Anisovich *et al.* from the limit of the number of light quark pairs  $N \rightarrow \infty$ . But the average charged multiplicity measured by TASSO collaboration is  $\langle n_{ch} \rangle = 7.5 \pm 0.5 \pm 0.3$  for  $c$ -jets in  $e^+e^-$  annihilation at  $\sqrt{s} = 34$  GeV [4]. It can be seen that the average number of the light quark pairs,  $\langle N \rangle$ , is of the order of 10 around such energy and that for  $\sqrt{s} < 34$  GeV,  $N$  should be smaller than that. So for the data now available, the energy ranges from  $\sqrt{s} = 4$  GeV to 50 GeV, and the number of  $N$  is quite limited. We should at least derive the ratio  $f$  based on this practical number  $N$ .

In Ref. [2] we have investigated the quark production rule in the process of  $e^+e^- \rightarrow$  two jets, and obtained a formula to calculate the average number  $\langle N \rangle$  of the quark-antiquark pairs in the  $c$ -quark jets:

$$\langle N \rangle = \left\{ \alpha^2 + \beta \sqrt{s} \left[ 1 - 4 \left( 0.021 + \frac{0.664}{s} \right)^{\frac{1}{2}} \right] \right\}^{\frac{1}{2}} - \alpha + 1, \quad (6)$$

where  $\alpha = 1.44$ ,  $\beta = 4.7 \text{ GeV}^{-1}$ . For instance, the  $\langle N \rangle$  calculated by Eq.(6) gives 1.81 and 7.85 at  $\sqrt{s} = 5$  and  $34.4 \text{ GeV}$ , respectively.

#### 4. QUARK COMBINATION RULE

In considering combining a primary quark  $c$  (or  $\bar{c}$ ) with  $N$  ( $\geq 1$ ) light quark pairs into a charmed hadron, we still assume that the combination of quarks and antiquarks into hadron is required to obey the nearest correlation in rapidity. But just as is the case for what we have demonstrated in Ref. [5], not only can the nearest neighbours in the rapidity axis form a hadron, but also the quarks and/or antiquarks which are separated by one quark or antiquark can form a hadron in some cases. In fact, for  $cq\bar{q}\dots$  type of arrangement, the nearest correlation in rapidity requires combining  $c$  with  $\bar{q}$  to produce a charmed meson  $M_c$ , i.e. performing  $c\bar{q}$ , and rules out the  $cq\bar{q}$  combination choice which was adopted by Anisovich *et al.* (see Sec.2), since the latter choice might make the quark  $c$  wait for a light quark or antiquark which is separated by many quarks and/or antiquarks in the rapidity axis to form a hadron, which contradicts the requirement of the nearest correlation.

The experiments have shown that the mean momentum fraction of charmed mesons produced in  $e^+e^-$  annihilations is independent of  $\sqrt{s}$ , and  $\langle x \rangle \approx 0.58$  [6]. It is plausible to consider that the  $c$  and  $\bar{c}$  take the maximum rapidity in the two opposite directions of the  $c$ -quark jets, and the rapidities of all the light quarks and antiquarks then lie between the rapidity of  $c$  and  $\bar{c}$ , i.e. the configurations in the rapidity axis are always like  $cq\bar{q}\dots q\bar{c}$ . Now let us consider the combination between  $c$  (or  $\bar{c}$ ) and  $N$  pairs of light quark and antiquark. The probabilities of light quarks  $q$  and  $\bar{q}$  neighboring  $c$   $q$  are  $1/2$  respectively. For the  $cq\bar{q}\dots$  type of arrangement,  $cq$  combines into charmed meson  $M_c$ . For the  $cq\dots$  type of arrangement,  $cq$  cannot combine into a hadron, and in this case the consequence depends on the next neighbour. Since the number of light quark pairs,  $N$ , is finite, the probabilities of the appearance of  $q$  and  $\bar{q}$  in the position of the next neighbour are different, the former is  $(N-1)/(2N-1)$  while the latter is  $N/(2N-1)$ . For the  $cqq\dots$  configuration,  $cqq$  combines into a charmed baryon  $B_c$ ; for the  $cq\bar{q}$  one,  $c$  strides over to combine  $\bar{q}$  into a charmed meson  $M_c(cq\bar{q})$ . In term of this combination rule, the whole process of  $c$  (or  $\bar{c}$ ) combining with light quarks or antiquarks into charmed baryons or mesons is finished at most by two steps as mentioned above, and it can be expressed as

$$\begin{aligned} c &\rightarrow \frac{1}{2} \frac{N-1}{2N-1} B_c + \left( \frac{1}{2} + \frac{1}{2} \frac{N}{2N-1} \right) M_c \\ &= \frac{N-1}{2(2N-1)} B_c + \frac{3N-1}{2(2N-1)} M_c. \end{aligned} \quad (7)$$

Obviously, we have the same result for  $\bar{c}$

$$\bar{c} \rightarrow \frac{N-1}{2(2N-1)} \bar{B}_c + \frac{3N-1}{2(2N-1)} \bar{M}_c. \quad (8)$$



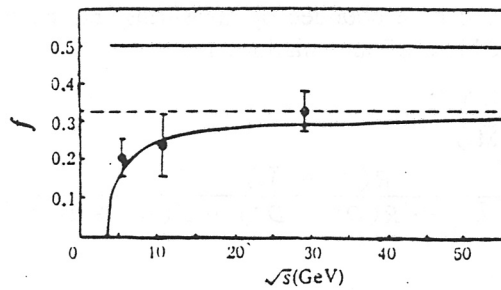


FIG.1

The comparison of the calculated  $f$  with the experimental data which are derived from Ref. [8] (see Sec.5). The curve represents our calculated results and the dashed line represents the limit at  $\sqrt{s} \rightarrow \infty$ . The solid straight line is the prediction given by Anisovich *et al.* in Ref. [3].

Finally, we get

$$f = \frac{N-1}{3N-1}. \quad (9)$$

## 5. AVERAGE VALUE OF $f$

For a given energy  $\sqrt{s}$ , one can get the average number of light quark pairs  $\langle N \rangle$  in the  $c$ -quark jet events using Eq.(6). In general  $\langle N \rangle$  is not an integer, but  $f$  is obtained by substituting  $N$  as integers 1, 2, ... into Eq.(9). In fact the measured  $f$  by the experiments are always the value on average over a large number of events. To calculate the corresponding average value of  $f$  we assume that the number  $N$  follows a Poisson distribution, i.e.

$$P(N) = \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!}. \quad (10)$$

So the average value of the ratio  $f$  is

$$\begin{aligned} \bar{f} &= \frac{\sum_{N=1}^{\infty} f(N) P(N)}{\sum_{N=1}^{\infty} P(N)} \\ &= \frac{\sum_{N=1}^{\infty} \frac{(N-1) \langle N \rangle^N e^{-\langle N \rangle}}{N! (3N-1)}}{\sum_{N=1}^{\infty} \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!}} \end{aligned} \quad (11)$$

where  $\langle N \rangle$  is determined by Eq.(6). Since  $\langle N \rangle$  is a function of  $\sqrt{s}$ ,  $\bar{f}$  is also a function of  $\sqrt{s}$ . Fig.1 shows the variation of the calculated  $\bar{f}$  with  $\sqrt{s}$ , and the limit at  $\sqrt{s} \rightarrow \infty$ ,  $\bar{f} \rightarrow 1/3$ . The solid straight line represents the prediction  $f = 1/2$  given by Ref. [3].

## 6. COMPARISON WITH DATA AND DISCUSSIONS

The experimental value  $f$  can be obtained by measuring the inclusive cross sections of various charmed particles produced in  $e^+e^-$  annihilations, i.e.

$$f = \frac{R(B_c + \bar{B}_c)}{R(M_c + \bar{M}_c)} = \frac{R(\Lambda_c + \bar{\Lambda}_c)}{R(D^0 + \bar{D}^0) + R(D^+ + D^-) + R(D_s^+ + D_s^-)}, \quad (12)$$

where  $R(B_c + \bar{B}_c)$  and  $R(M_c + \bar{M}_c)$  are the inclusive cross sections of all the charmed baryons and that of all the charmed mesons in units of the cross section for  $e^+e^-$  annihilating into  $\mu^+\mu^-$

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{86.856}{s(\text{GeV}^2)} (nb) \quad (13)$$

Since every charmed vector meson decays into a charmed pseudo-scalar meson with a very short lifetime, the inclusive cross sections measured by experiments contain the contribution from the former, so

$$R(B_c + \bar{B}_c) \equiv \frac{\sigma(e^+e^- \rightarrow B_c + \bar{B}_c)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}, \quad R(M_c + \bar{M}_c) \equiv \frac{\sigma(e^+e^- \rightarrow M_c + \bar{M}_c)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \quad (14)$$

$$R(M_c + \bar{M}_c) = R(D^0 + \bar{D}^0) + R(D^+ + D^-) + R(D_s^+ + D_s^-) \quad (15)$$

where the  $R(D^0 + \bar{D}^0)$ ,  $R(D^+ + D^-)$  and  $R(D_s^+ + D_s^-)$  denote the cross sections in unit of the  $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ . It is a common belief that all of the charmed baryons are to become a  $\Lambda_c(\bar{\Lambda}_c)$  or to decay into a  $\Lambda_c(\bar{\Lambda}_c)$  before they decay successively into stable particles, so

$$R(B_c + \bar{B}_c) = R(\Lambda_c + \bar{\Lambda}_c). \quad (16)$$

Unfortunately, the cross sections  $R(\Lambda_c + \bar{\Lambda}_c)$  and  $R(D_s^+ + D_s^-)$  cannot be determined because the  $\Lambda$ 's and  $D$ 's decay modes and branching ratios have not yet been exactly established, and so there is no way to get the experimental value  $f$  by Eq.(12) at present.

For comparison with the data, we use the total cross section  $R(h) = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ , the inclusive cross sections  $R(D^0 + \bar{D}^0)$  and  $R(D^+ + D^-)$  of the charmed meson  $D^0 + \bar{D}^0$  and  $D^+ + D^-$ , and the QCD fundamental formula to determine the experimental value of  $f$  as follows

$$f = \frac{2R(h)Q_c^2\theta(\sqrt{s} - 2m_c)}{\left(1 + \frac{\lambda}{2}\right)[R(D^+ + D^-) + R(D^0 + \bar{D}^0)]\left[\sum_{i=u,d,s,c} Q_i^2 + Q_b^2\theta(\sqrt{s} - 2m_b)\right]} - 1 \quad (17)$$

where  $Q_i$  and  $m_i$  denote the quark charge and mass of flavor  $i$ , and  $\lambda$  is the suppression factor for strange-quark production and takes the value of 0.3 from experiments. When  $\sqrt{s}$  nears the threshold of  $c$  or  $b$  production, i.e.  $\sqrt{s} \sim 2m_c$  or  $2m_b$ , the so-called mass effects should be taken into account. In such cases the formula is a little more complicated than Eq.(17). The method to determine has been discussed in detail by the authors in Ref. [7].

Using the data published by CLEO, HRS and MARK II collaborations [8], the following experimental values  $f$  are obtained:

$$\begin{aligned} f &= 0.21 \pm 0.05, \sqrt{s} = 5.2 \text{ GeV}; \\ f &= 0.24 \pm 0.08, \sqrt{s} = 10.55 \text{ GeV}; \\ f &= 0.33 \pm 0.05, \sqrt{s} = 29 \text{ GeV}. \end{aligned}$$

and have been plotted in Fig.1. From Fig.1 one can see that the data are in agreement with our theoretical curve but in disagreement with Anisovich's prediction.

## REFERENCES

- [1] A. Casher *et al.*, *Phys. Rev.* **D20**(1979)732.
- [2] Xie Qubing and Liu Ximing, *Phys. Rev.* **D38**(1988)2169.
- [3] V. V. Anisovich *et al.*, *Z. Phys. C-Particles and Fields*, **19**(1983)221.
- [4] TASSO Collab., M. Althoff *et al.*, *Phys. Lett.* **135B**(1984)243.
- [5] Xie Qubing, Proc. XIX th Int. Symp. of Multiparticle Dynamics, Arles 1988, eds D. Schiff *et al.* (Word Scientific, Singapore) p369.
- [6] P. Avery *et al.*, *Phys. Rev. Lett.* **51**(1983)1139.
- [7] Chen Esheng and Wu Qian, "The ratio of charmed baryon to meson in  $e^+e^-$  annihilation", *Chinese Phys. Lett.* to be published.
- [8] CLEO Collab., D. Bortoletto *et al.*, *Phys. Rev.* **D37**(1988)1719; HRS Collab., P. Baringer *et al.*, *Phys. Lett.* **206B**(1988)551; MARK II Collab., G. S. Abrams *et al.*, *Phys. Rev. Lett.* **44**(1980)10; CLEO Collab., R. Giles *et al.*, *Phys. Rev.* **D29**(1984)1285; J. L. Siegist *et al.*, *Phys. Rev.* **D26**(1982)969.