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On the Electric Dipole Transition of ψ (3770)

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Various $E1$ transition rates for cc states are calculated in a QCD-motivated potential model. Relativistic corrections are found to be substantial. A good agreement between theory and experiment is achieved for the rates of $\psi(2S) \to \gamma\chi_{cJ}$ and $\chi_{cJ} \to \gamma\psi(1S)$ ($J = 0,1,2$), but the calculated rates for $\psi(3770) \to \gamma\chi_{cJ}$ are smaller by a factor of 2 than their experimental values obtained by the Mark III Collaboration. The effect of the $2^3S_1-1^3D_1$ mixing may be important and needs to be considered.

Recently the electric-dipole ($E1$) transitions for the charmonium $^3D_1$ state $\psi(3770)$ into ^3P_J states χ_0, χ_1 and χ_2 have been measured by the Mark III Collaboration [1] with the branching ratios of (2.0±0.8)%, (1.7±0.7)% and ≤ 0.2%(90%CL) obtained respectively. With the total width of (25±3)MeV for $\psi(3770)$ [2], we can get the partial widths for the above $E1$ decays, which are (500±200)keV, (430±180)keV and ≤ 500keV, respectively. In terms of these experimental results, this paper will discuss the $E1$ transition theory in the charmonium model.

It is well known that the potential model of the heavy quarkonium has been successful in describing the mass spectra and many transition processes for the charmonium and bottomonium families [3]. According to the generally accepted point of view, the interaction between the heavy quarks can be considered to be a short range one-gluon exchange potential (the Lorentz vector) plus a long range confining potential (the Lorentz scalar). We adopt the following scalar potential $S(r)$ and vector potential $V(r)$ [4]:

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$$S(r) = kr \tag{1}$$

$$V(r) = -\frac{8\pi}{25}\frac{1}{r\ln\Lambda r}\frac{1-\Lambda r}{1+\Lambda r} \tag{2}$$

This $V(r)$ has the following characteristics. When $r \to 0$, $V(r) \to (-8\pi/25)\cdot(1/r\ln\Lambda r)$, which represents a Coulomb potential with a running coupling constant, and when r increases up to the range comparable to the scale of the ψ family, the coefficient of the Coulomb potential approaches a constant, i.e. $V(r) \to -\beta/r$, and $\beta \approx 0.50$ (if taking 0.47GeV, $\beta \approx 0.50$ in the range $r \sim$ (1--5)GeV^{-1}).

Adopting such a potential is based on the following consideration. The calculations in the lattice QCD show that the interquark potential is indeed the sum of a Coulomb potential and a linear potential when $r \geq 0.5\mathrm{GeV}^{-1}$. Under the approximation which neglects the dynamical effects of the light quarks, the coefficient of the Coulomb potential is about $-\pi/12$ [5], which will increase after these effects are further considered [6]. It is possible that the vacuum polarization effects of the light quark pairs make the coefficient increase up to about 0.5 [7]. The Coulomb potential adopted by us embodies the characteristic that the Coulomb potential with a running coupling constant at short distances turns gradually into a Coulomb potential with a constant coefficient at long distances. When r increases up to $10\mathrm{GeV}^{-1}$, the coefficient of our Coulomb potential will reduce slightly and $\beta \approx 0.42$ (Λ = 0.47) GeV. Since the screening effects of the light quarks to the potential between the heavy quarks are important at such a long distance, the assumption of a Coulomb potential plus a linear one will itself be a very rough approximation.

Under the relativistic approximation, the Hamiltonian of the heavy quarkonium system can be expanded in powers of $\vec{p}^2/m^2$ (m and $\vec{p}$ are respectively the mass and the momentum of the quarks in the center-of-mass frame), which reads

$$H = H_0 + H_1 + \cdots \tag{3}$$

$$H_0 = \frac{\vec{p}^2}{m} + S(r) + V(r) \tag{4}$$

$$H_1 = H_{SD} + H_{SI} \tag{5}$$

$$H_{SD} = \frac{1}{2m^2 r}(3V' - S')(\vec{S}_1 + \vec{S}_2)\cdot\vec{L} + \frac{2}{3m^2}\vec{S}_1\cdot\vec{S}_2\nabla^2 V - \frac{1}{3m^2}[3(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r}) - \vec{S}_1\cdot\vec{S}_2]\left(V'' - \frac{V'}{r}\right) \tag{6}$$

$$H_{SI} = -\frac{\vec{p}^4}{4m^3} + \frac{1}{4m^2}\left\{\frac{2}{r}V'\vec{L}^2 + [\vec{p}^2, V - rV'] + 2(V - rV')\vec{p}^2 + \frac{1}{2}\left(\frac{8}{r}V' + V'' - rV'''\right)\right\} \tag{7}$$

where H_{SD} and H_{SI} represent respectively the spin-dependent and the spin-independent Hamiltonian up to the first order. We first use H_0 to solve the zeroth order Schrödinger equation and then treat H_1 as a perturbation to calculate the first order relativistic corrections to the energies and wave functions. When taking the following parameters

$$k = 0.222\text{GeV}^2, \qquad \Lambda = 0.47\text{GeV}, \tag{8}$$

$$m_c = 1.84\text{GeV}, \qquad m_b = 5.17\text{GeV}, \tag{9}$$

the mass spectra fitting to the data for $c\bar{c}$ and $b\bar{b}$ can be obtained, which have been briefly reported in the Ref. [4] and will be discussed in detail in another publication. This work will only concern the $E1$ transition processes for the charmonium.

Under the nonrelativistic approximation the $E1$ transition width and the matrix element are given by

$$\Gamma(E1) = \frac{4}{27} e_Q^2 \alpha |\langle f|r|i\rangle|^2 (2J_f + 1) S_{if} k^3, \tag{10}$$

$$\langle f|r|i\rangle = \int_0^\infty R_f(r) R_i(r) r^3 dr, \tag{11}$$

where k is the energy of the radioactive photon, $R(r)$ the radial wave function and S_{if} a statistic factor. The transition widths $\Gamma(E1)$ calculated in our model and the corresponding experimental values are shown in Table 1, where the widths without parentheses are the results calculated by using the zeroth order wave functions and the values in the parentheses are calculated by using the first order relativistically corrected wave functions. The experimental values for $\psi(2S) \to \gamma\chi_{cJ}$ and $\chi_{cJ} \to \gamma\psi(1S)$ ($J = 0, 1, 2$) are taken from the Ref. [2].

It can be seen from Table 1 that the widths for $\psi(2S) \to \gamma\chi_{cJ}$ and $\chi_{cJ} \to \gamma\psi(1S)$ with the relativistic corrections are in fairly good agreement with the data, which shows that the relativistic correction is not negligible for the cc system. Although the velocity of the quark in such a system is quite low ($v^2/c^2 \approx 0.2$), the rates of some $E1$ transitions are very sensitive to the small variation of the wave function. This is particularly evident for the transition $\psi \to \gamma\chi_{c0}$ ($2^3S_1 \to 1^3P_0$). In addition to an attractive force from the spin-independent part of the first order Hamiltonian H_1, the spin-orbit coupling term from the one-gluon exchange contributes a strong attractive force to the $^3P_0$ state, which makes the radial wave function contract towards the origin, and thus suppresses the matrix element $<2^3S_1|r|1^3P_0>$ and the width of the dipole transition. Since the matrix element is very sensitive to the overlap between the wave functions of the initial and the final states (in particular, when these wave functions have different numbers of nodes), the relativistic corrections to the widths could be substantial in some cases. Concerning the relativistic corrections for $2^3S_1 \to 1^3P_J$ and $1^3P_J \to 1^3S_1$, our results are qualitatively consistent with those of other authors [8]. However, for some processes, for example, $1^3P_0 \to 1^3S_1$, our result seems better. This may be due to the potential and the method we have used to deal with the relativistic correction. In fact, treating the H_1 as a perturbation seems to be more appropriate.

For the present interesting transitions $1^3D_J \to 1^3P_J$, the results in Table 1 show that the relativistic correction is again important, for which the following analysis can be given. For the $^3D_1$ state, there exist both attraction and repulsion in the first order Hamiltonian H_1. The spin-orbit coupling term from one-gluon exchange, $\sim 3/2m^2 \cdot (V'/r)$, is an attraction and the Thomas precession force from the scalar confining potential, $\sim -1/2m^2 \cdot (s'/r)$, is a repulsion. In the spin-

TABLE 1.
The electric dipole transitions for the charmonium.

Process	k(MeV)	S_{if}	$\Gamma(E1)$(keV)	Experimental Values
$2^3S_1 \to 1^3P_0$	261	1	42(25)	23 ± 4
$2^3S_1 \to 1^3P_1$	172	1	36(28)	21 ± 4
$2^3S_1 \to 1^3P_2$	128	1	25(22)	19 ± 4
$1^3P_0 \to 1^3S_1$	303	1	141(104)	95 ± 37
$1^3P_1 \to 1^3S_1$	389	1	299(216)	< 350
$1^3P_2 \to 1^3S_1$	429	1	401(282)	350^{+160}_{-120}
$1^3D_1 \to 1^3P_0$	338	2	312(183)	500 ± 200
$1^3D_1 \to 1^3P_1$	250	1/2	95(70)	430 ± 180
$1^3D_1 \to 1^3P_2$	208	1/50	3.6(3.0)	≤ 500

The states in the Table represent the following physical states respectively: 1^3S_1: $\psi(3097)$; 2^3S_1: $\psi(3686)$; 1^3P_0: $\chi_{c0}(3415)$; 1^3P_1: $\chi_{c1}(3510)$; 1^3P_2: $\chi_{c2}(3555)$; 1^3D_1: $\psi(3770)$. The widths without parentheses are the results calculated by using the zeroth order wave functions, and the values in the parentheses are calculated by using the first order relativistically corrected wave functions. The experimental values for $\psi(2S) \to \gamma\chi_{cJ}$ and $\chi_{cJ} \to \gamma\psi(1S)$ (J = 0, 1, 2) are taken from Ref. [2].

independent corrections, besides some attraction terms, the repulsion term $1/4m^2 \cdot (2/r) \cdot V' \vec{L}^2$ increases as the quantum number of the angular momentum increases. In addition, the relative proportion of the Thomas precession force also increases as the radius of the bound state increases. The relative enhancement of both repulsions in the $^3D_1$ state plays a part in balancing the attraction, so the total relativistic correction has less effect on the wave function of the $^3D_1$ state. However, for the $^3P_0$ state, the attraction is dominant in H_1 and evidently the relativistic correction makes its wave function contract towards the origin. Therefore, the overlap between the wave function $^3D_1$ and $^3P_0$ decreases after the relativistic correction is taken into account, which results in the suppression of the matrix element $<1^3D_1|r|1^3P_0>$ and the width. The calculated results show that the relativistically corrected widths for $1^3D_1 \to 1^3P_0$ and $1^3D_1 \to 1^3P_1$ are smaller by a factor of ~ 2 than their experimental values.

Of course, it should be noted that there exist some uncertainties in the theory. The first one is concerned with the quark mass. The quark mass adopted by us is the same as that adapted by the Cornell model [9] and by the authors of Ref. [8]. Only when we take such a large quark mass and then make the relativistic correction to get further suppressions, the obtained transition widths for $\psi(2S) \to \gamma\chi_{cJ}$ and $\chi_{cJ} \to \gamma\psi(1S)$ are compatible with the data. If a smaller charm quark mass is taken, although the transition widths for $1^3D \to 1^3P_J$ can increase, it would be difficult to make the widths for $2^3S_1 \to 1^3P_J$ and $1^3P_J \to 1^3S_1$ fit in the experiment. In fact, for any model [10,11] which adopts a smaller quark mass, the calculated widths for $\psi(2S) \to \gamma\chi_{cJ}$ and $\chi_{cJ} \to \gamma\psi(1S)$ are larger than the data.

The second uncertainty is concerned with the higher order relativistic corrections, for which to give a quantitative estimation is still very difficult at present. In any event, due to the rather low velocity ($v^2/c^2 \approx 0.2$) of the quark in the charmonium system, the first order relativistic correction should make sense and it is not expected that the higher order relativistic corrections could give an enhancing factor which is more than 2.

The third one is concerned with the coupling effects among the decay channels [9], which, in general, always reduce the proportion of $c\bar{c}$ and increase the proportion of the continuum states such as $D\bar{D}$, etc., in the physical state (if the $2S$—$1D$ mixing from the coupling-channel effects is not considered for the moment). Thus the decay channels only play a role to suppress the $E1$ transition widths.

The last one is concerned with the 2^3S_1—1^3D_1 mixing. The $\psi(3770)$ has an appreciable leptonic decay width $\Gamma_{ee} = (0.26\pm0.05)$keV [2]. This indicates that $\psi(3770)$ must contain a component of the $^3S_1$ state. Considering that $\psi(3686)$ and $\psi(3770)$ are two mixed states of 2^3S_1 and 1^3D_1, the interference between S and D states will strengthen some transition processes and weaken some others. These effects are not only dependent on the quantum numbers of the initial and final states, but depend also on the sign of the mixing angle. The degree of strengthening or weakening is dependent on the magnitude of the mixing angle. And the sign and the magnitude of the mixing angle are dependent on the mixing mechanism, e.g., by the coupling-channel (and) or by the tensor force. Evidently, the $E1$ transition processes for $\psi(3686)$ and $\psi(3770)$ will provide very useful information for the 2^3S_1—1^3D_1 mixing, which is worth further studying. The conclusions of the paper are as follows. In the framework of the charmonium model, if the relativistic effect is considered but the 2^3S_1—1^3D_1 mixing is neglected, it will be very difficult to make the experimentally observed transition widths for $\psi(2S)\to\gamma\chi_{cJ}$ and $\chi_{cJ}\to\gamma\psi(1S)$ to be consistent with the width for $\psi(3770)\to\gamma\chi_{cJ}$. If we require that the theory should give the transition widths for $\psi(2S)\to\gamma\chi_{cJ}$ and $\chi_{cJ}\to\gamma\psi(1S)$ compatible with the data, the predicted widths for $\psi(3770)\to\gamma\chi_{cJ}$ will be smaller by a factor of ~ 2 than their experimental values obtained by the Mark III Collaboration. However, there still exists a possibility to improve the theoretical result. That is to discuss the $2S$—$1D$ mixing. At the same time we suggest making more accurate measurements of the electro-magnetic transitions for $\psi(3770)$ on the BEPC (Beijing Electron Positron Collider).

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