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Two-pion Interferometry for Multi-pion Events in Relativistic Heavy Ion Collisions

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The effect of the multi-pion correlations on two pion interferometry in a multi-pion event is studied. A new general two-pion interferometry method is developed, taking into consideration the effect of the multi-pion correlations. The data for 1.8 A GeV Ar + Pb central collisions at the Bevalac streamer chamber are analyzed using this new method.

1. INTRODUCTION

Pion interferometry is a very valuable way of studying the space-time structure, including the time evolution, of a pion-emitting source in relativistic heavy ion collisions [1,2]. Two-pion interferometry investigates only the two-body correlation between identical pions in a collision event. Since the multiplicity of identical pions is usually greater than 2 in an event, there are many kinds of multi-pion correlations with various orders among the identical pions. How do these multi-pion correlations affect the results of two-pion interferometry analyses? What kind of relation exists between the effect of multi-pion correlations and the properties of a pion-emitting source? How can

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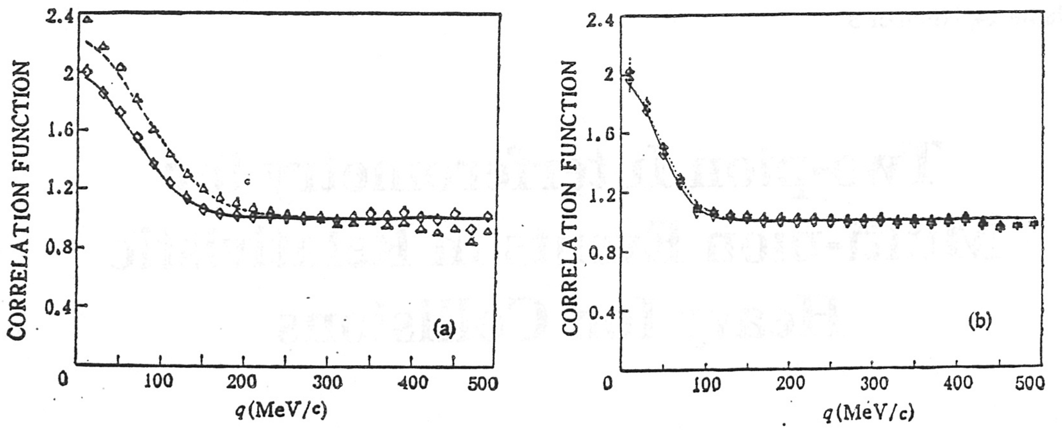

(a) T = 30 MeV, R = 3 fm. (b) T = 50 MeV, R = 5 fm.

Fig.1
Comparison of the fitted curves corresponding to the ideal background and the multi-pion correlated background for three-negative-pion event generated by Monte Carlo.

Δ and solid line are for the multi-pi correlated background
◊ and dotted line are for the multi-pi ideal background

the effect of multi-pion correlations be excluded from two-pion interferometry analyses? These are very important problems in pion interferometry [3--6]. On the other hand, because of the complexity of the relativistic heavy ion collisions, many effects can influence the interpretation of the pion interferometry results. Therefore, by comparing and analyzing the results obtained from different pion interferometry methods, we can test the assumption that the enhancement of the correlation function in the low relative momentum region comes solely from Bose-Einstein correlations of identical pions, and verify the adequacy of the phenomenological Gaussian model used to describe the pion-emitting source in relativistic heavy ion collisions [3,4,6]. In Sec.2 of this paper, the multi-pion correlation functions in multi-pion events are discussed, and the effect of multi-pion correlations on the two-pion correlation function in a multi-pion event is studied on the basis of the modified Gaussian model [3]. In Sec.3, the effect of the multi-pion correlations on two-pion interferometry in three-negative-pion and four-negative-pion events generated by Monte Carlo are analyzed; a general two-pion interferometry method that allows for the effect of multi-pion correlations is proposed, and using this method, the data for 1.8 A GeV Ar + Pb central collisions at the Bevalac streamer chamber are analyzed. Finally, the conclusions are given in Sec.4.

2. MULTI-PION CORRELATIONS AND THE TWO-PION CORRELATION FUNCTION

2.1 Multi-pion Correlation Functions in Multi-pion Events

Assuming that the multiplicity of negative pions in an event is n, the n-pion correlation function in the event can be defined as

$$C_n(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_n) = P_n(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_n)/[P(\vec{p}_1)P(\vec{p}_2)\cdots P(\vec{p}_n)]. \tag{1}$$

Table 1
Results of two-pion interferometry analyses for multi-negative-pion events generated by Monte Carlo

	π-parameters of pion surface	Analyzed type	Fitted results
three-negative-pion event ($3\pi-$)	T=30MeV R=3fm	(a)	R=2.53±0.02fm λ=1.21±0.01
	$\lambda=\xi=1$	(b)	R=3.03±0.02fm λ=0.97±0.01
	T=50MeV R=5fm	(a)	R=4.68±0.08fm λ=0.98±0.08fm
	$\lambda=\xi=1$	(b)	R=5.03±0.08fm λ=0.98±0.03
four-negative-pion event ($4\pi-$)	T=30MeV R=3fm	(a)	R=2.22±0.03fm λ=1.55±0.01
	$\lambda=\xi=1$	(b)	R=2.96±0.02fm λ=0.99±0.01
	T=50MeV R=5fm	(a)	R=4.31±0.08fm λ=0.90±0.03
	$\lambda=\xi=1$	(b)	R=4.95±0.09fm λ=0.90±0.03

where $P_n(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_n)$ is the probability of observing n negative pions of momenta $\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_n$ in the event, and $P(\vec{p}_n)$ is the single-negative-pion-inclusive distribution.

Assuming that the probability of observing m negative pions of momenta $\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m$ in a n-negative-pion event ($2 \le m < n$) is $P_{m/n}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m)$, the m-pion correlation function in the event is defined as

$$C_{m/n}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m) = P_{m/n}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m)/[P(\vec{p}_1)P(\vec{p}_2) \cdots P(\vec{p}_m)], \qquad (2 \leqslant m < n), \tag{2}$$

where $P_{m/n}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m)$ is related to $P_n(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_n)$ by

$$P_{m/n}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m) = \int \cdots \int d\vec{p}_{m+1} \cdots d\vec{p}_n P_n(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_n). \tag{3}$$

From Eq.(1)--Eq.(3), $C_{m/n}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m)$ can be written as

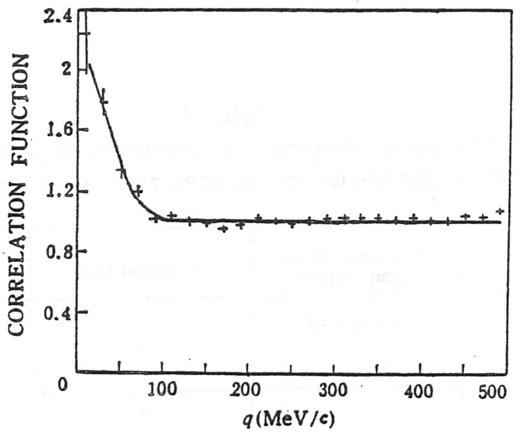

Fig.2
The fitted two-pion correlation function corrected for higher order pion correlations for 1.8 A GeV Ar + Pb, integrated over q_0.

$$C_{m/n}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m) = C_m(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m)\bar{C}_{n-m}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m), \tag{4}$$

$$\bar{C}_{n-m}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m) = \int\cdots\int d\vec{p}_{m+1}\cdots d\vec{p}_n[C_n(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_n) P(\vec{p}_{m+1})\cdots P(\vec{p}_n)/C_m(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m)]. \tag{5}$$

Here $C_m(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m)$ is the m-pion correlation function in a m-negative-pion event, and $C_{n-m}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m)$ represents the difference between the m-pion correlation functions in the m-negative-pion event and in the n-negative-pion event. When considering the correlation of m negative pions in a n-negative-pion event, if we neglect the correlations included the other negative pions, we have

$$C_{m/n}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m) \doteq C_m(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m),$$
$$\bar{C}_{n-m}(\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_m) = 1.$$

2.2 The Effect of Multi-pion Correlations on the Two-pion Correlation Function

From Eq.(2) and Eq.(4), the two-pion correlation function in a n-negative-pion event can be written as

$$C_{2/n}(\vec{p}_1, \vec{p}_2) = P_{2/n}(\vec{p}_1, \vec{p}_2)/[P(\vec{p}_1)P(\vec{p}_2)] = C_2(\vec{p}_1, \vec{p}_2)\bar{C}_{n-2}(\vec{p}_1, \vec{p}_2), \tag{6}$$

where $C_2(\vec{p}_1, \vec{p}_2)$ the two-pion correlation function in a two-negative-pion event, and $C_{n-2}(\vec{p}_1, \vec{p}_2)$ is the effect of multi-pion correlations on the two-pion correlation function. For the modified Gaussian model, from Eq.(5), and with the graphical method [3,4], we can obtain the expressions for the effects of multi-pion correlations on the two-pion correlation functions in three-negative-pion and four-negative-pion events:

$$\bar{C}_{3-2}(q, q_0) = 1 + \{2\lambda I_1(q, q_0) + 2\xi I_2(q, q_0)\exp(-q^2R^2/4 - q_0^2\tau^2/4)\}/[I_0(q, q_0)], \tag{7}$$

$$\bar{C}_{4-2}(q, q_0) \approx 1 + \{4\lambda I_1(q, q_0) + \lambda I_1'(q, q_0)[1 + \lambda\exp(-q^2R^2/2 - q_0^2\tau^2/2)] + 4\xi I_2(q, q_0)\exp(-q^2R^2/4 - q_0^2\tau^2/4)\}/[I_0(q, q_0)]. \tag{8}$$

Here, q and q_0 are the values of the relative momentum and energy of the considered negative pion pair; R and τ are the space and time parameters of the pion source described by the Gaussian model; λ and ξ are the coherence factors for two-pion correlations and three-pion correlations, respectively.

$$I_0(q, q_0) = [1 + \lambda\exp(-q^2R^2/2 - q_0^2\tau^2/2)]\left[\int d\vec{g}P(\vec{g})\int d\vec{g}'P(\vec{g}') \cdot \delta(|\vec{g} - \vec{g}'| - q)\delta(|E(\vec{g}) - E(\vec{g}')| - q_0)\right], \tag{9}$$

$$I_1(q, q_0) = \int d\vec{g}P(\vec{g})\int d\vec{g}'P(\vec{g}')\delta(|\vec{g} - \vec{g}'| - q)\delta(|E(\vec{g}) - E(\vec{g}')| - q_0)\int d\vec{p}P(\vec{p})[\exp(-|\vec{g} - \vec{p}|^2R^2/2 - |E(\vec{g}) - E(\vec{p})|^2\tau^2/2)], \tag{10}$$

$$I_1'(q, q_0) = \int d\vec{g}P(\vec{g})\int d\vec{g}'P(\vec{g}')\delta(|\vec{g} - \vec{g}'| - q)\delta(|E(\vec{g}) - E(\vec{g}')| - q_0)\int d\vec{p}P(\vec{p})\int d\vec{p}'P(\vec{p}')[\exp(-|\vec{p} - \vec{p}'|^2R^2/2 - |E(\vec{p}) - E(\vec{p}')|^2\tau^2/2)], \tag{11}$$

$$I_2(q, q_0) = \int d\vec{g}P(\vec{g})\int d\vec{g}'P(\vec{g}')\delta(|\vec{g} - \vec{g}'| - q)\delta(|E(\vec{g}) - E(\vec{g}')| - q_0)\int d\vec{p}P(\vec{p})\{\exp[-(|\vec{g} - \vec{p}|^2 + |\vec{g}' - \vec{p}|^2)R^2/4 - (|E(\vec{g}) - E(\vec{p})|^2 + |E(\vec{g}') - E(\vec{p})|^2)\tau^2/4]\}. \tag{12}$$

In Eq.(7) and Eq.(8), the $\lambda I_1(q, q_0)$ terms reflect the average effects on the two-pion correlation functions of the two pion correlations between one pion of the considered negative pion pair and one other negative pion; the $\lambda I'_1(q, q_0)$ term reflects the average effect on the two-pion correlation function of the two pion correlation, which excludes the negative pions of the considered negative pion pair; the $2\xi I_2(q, q_0)\exp(-q^2R^2/4 - q_0^2\tau^2/4)$ terms represent the average effects on the two-pion correlation functions of the pure triplet correlations among the two pions of the considered negative pion pair combined with one other pion; and the $\lambda^2 I'_1(q, q_0)\exp(-q^2R^2/2 - q^2\tau^2/2)$ terms reflect the average effect on the two-pion correlation function of the correlations between the considered negative pion pair and other possible pion pairs from the remaining pions. In Eq.(8), the smaller terms corresponding to the pure quadruplet correlation of four negative pions, and the pure triplet correlations and the two pion-pair correlations which exclude the considered negative pion pair, are neglected.

We assume that the single-negative-pion-inclusive distribution has the form

$$P(\vec{p}) \propto \exp[-E(\vec{p})/T], \tag{13}$$

where T is the temperature of the pion source. In the non-relativistic approximation, integrating $I_1(q, q_0)$, $I'_1(q, q_0)$, and $I_2(q, q_0)$ over (q, q_0), we have

$$I_1 \equiv \int dq \int dq_0 I_1(q, q_0) = \int dq \int dq_0 I'_1(q, q_0) \approx 1/(1 + 2R^2 m_\pi T)^{3/2}, \tag{14}$$

$$I_2 \equiv \int dq \int dq_0 I_2(q, q_0) \approx 1/(1 + 2R^2 m_\pi T + 3R^4 m_\pi^2 T^2/4)^{3/2}. \tag{15}$$

where m_π is the mass of a pion, and the parameter τ was taken to be zero for simplicity.

Eq.(14) and Eq.(15) indicate that I_1 and I_2 increase when T and R decrease. This effect is caused by the contraction of the phase space when T decreases, and the increased coherence effects when R decreases. In either of these situations, the correlations among the identical pions are enhanced. For T = 30 MeV, R = 3 fm and T = 50 MeV, R = 5 fm, it can be calculated that $I_1 \approx 0.199$, $I_2 \approx 0.144$ and $I_1 \approx 0.032$, $I_2 \approx 0.008$ respectively.

For a n-negative-pion event, if only the two-pion correlations, the pure triplet correlations and the two-pion-pair correlations included the investigated negative pion pair are taken into account, we have

$$\bar{C}_{n-2}(q, q_0) \approx 1 + \{2n_1\lambda I_1(q, q_0) + n_2\lambda I'_1(q, q_0)[1 + \lambda\exp(-q^2R^2/2 - q_0^2\tau^2/2)] + 2n_1\xi I_2(q, q_0)\exp(-q^2R^2/4 - q_0^2\tau^2/4)\}/[I_0(q, q_0)]. \tag{16}$$

where

$$n_1 = n-2,\quad n_2 = (n-2)(n-3)/2,\quad (n > 2) \tag{17}$$

3. TWO-PION INTERFEROMETRY ANALYSES FOR MULTI-PION EVENTS

It is well known that the form of the two-pion correlation function

$$C_2(q, q_0) = 1 + \lambda\exp(-q^2R^2/2 - q_0^2\tau^2/2), \tag{18}$$

is derived from the symmetric wave function of a two-identical-pion system, on the assumption of a Gaussian source distribution. It does not include the effect of multi-pion correlations. Therefore, it is only valid for a two-negative-pion event [3], or when the effect of multi-pion correlations can be neglected for a multi-negative-pion event. In carrying out two-pion interferometry for multi-negative-pion events, the correlation function is obtained from the ratios of the correlated negative-pion-numbers COR(q, q_0) to the uncorrelated negative-pion-pair-numbers UNCOR(q, q_0) (background) in the given regions around (q, q_0), that is

$$C(q, q_0) = \kappa\mathrm{COR}(q, q_0)/\mathrm{UNCOR}(q, q_0).$$

where κ is a normalization factor. Since the pions of each correlated pion pair are selected from the same multi-negative-pion event, the multi-pion correlations will certainly affect COR(q, q_0).

Table 2
The fitted pion source parameters for 1.8 A GeV Ar+ Pb

	R(fm)	λ
Two-pion interferometry results corrected for multi-pion correlation	5.72±0.36	1.08±0.15
Previous two-pion interferometry results	5.53±0.45	$0.99^{+0.01}_{-0.24}$
Previous three-pin interferometry results	5.65±0.49	$0.98^{+0.02}_{-0.26}$

Therefore, the two-pion correlation function can be correctly fitted in the form of Eq.(18) only if the effect of multi-pion correlations is also included in the background. Otherwise, the fitted results will be distorted by the effect of multi-pion correlations. Based on the discussion in Sec.2, the effect of multi-pion correlations on the two-pion correlation function in a multi-negative-pion event, which is represented by a factor $C_{n-2}(q,q_0)$, can be included in weighted background and excluded from the fitted two-pion correlation function in the form of Eq.(18).

The results of our two-pion interferometry analyses for the three-negative-pion and four-negative-pion events generated by Monte Carlo with different pion source parameters T and R are given in Table 1. The correlated pion-pair-number for each type of Monte Carlo calculation is 500,000. In Table 1, the results labelled type (a) are for the ideal background which is constructed by the pions generated directly from the distribution of Eq.(13). In this background, there is no effect of remaining correlations from multi-pion interference, which exists in the background constructed by selecting uncorrelated pions from different events with the same π^- multiplicity [3,7]. In Table 1, the results labelled type (b) are for the multi-pion correlated background which is constructed by weighing the effect of multi-pion correlations in the ideal background. Fig.1 shows the comparison of the fitted curves corresponding to the two backgrounds for three-negative-pion events generated by Monte Carlo. The results for the Monte Carlo events indicate that the multi-pion correlations can directly affect the results of two-pion interferometry analysis for multi-negative-pion events. It makes the fitted result for R decrease and distorts the fitted result for λ. This effect is important for the cases in which the pion source has lower temperature and a smaller radius.

When carrying out two-pion interferometry analyses while taking into consideration the effect of multi-pion correlations, the temperature value T needed in generating the ideal background can be determined by fitting the single-negative-pion-inclusive distribution given by the experiment. It was indicated by Monte Carlo simulations that the tail of the fitted correlation function will slope up or down when T is smaller or greater than the real value of the temperature. If this is the case, the fitted results for the pion source parameters will be distorted. In weighing the background to incorporate the multi-pion interference effect, two independent parameters R_0 and λ_0 are required. They can be determined by an iterative calculation, starting with the results corresponding to the ideal background. For three-negative-pion and four-negative-pion events generated by Monte Carlo with the pion source parameters T = 30 MeV, R = 3 fm, and λ = 1 (500,000 pion pairs), the differences ΔR and $\Delta\lambda$ between fitted results and the real pion source parameters are less than 0.05 fm and 0.04 respectively, after two iterations. Since the real physical value of λ can not be greater than 1, this condition was imposed during the iterative calculation.

Using the above method, the data for 1.8 A GeV Ar + Pb central collisions [2] at the Bevalac streamer chamber are analysed. In order to remove the effect of electron contamination of the π^- sample [2], a momentum cut $p_{lab} \geq$ 100 MeV/c has been imposed [2,4]. With this condition, there are 3500 events with negative pion multiplicity $M_\pi \geq 2$, and the average observed π^- multiplicity is $< M_{\pi^-} > = 9.0$. In this analysis, the correlated negative pion pairs include all the possible negative pion pairs within each event (the total correlated negative-pion-pair-number is 98,520), and the background is the multi-pion correlated background generated by Monte Carlo. The temperature T is taken to be 70 MeV, and is determined by fitting the experimental single-negative-pion-inclusive distribution. Based on the results of two iterations with the ideal background, the two independent parameters R_0 and λ_0 are taken to be 5.38 fm and 0.99 respectively. The uncorrelated negative-pion-pair-number before weighing is taken to be 30 times of the correlated negative-pion-pair-number for each multiplicity, then the weighing factor for each multiplicity is added. Table 2 provides our fitted results and the previous results of two-pion and three-pion interferometry analyses [4]. It can be seen that the fitted results are consistent with the values of R_0 and λ_0 within statistical errors, which indicates that the iterative results for R and λ have converged, and that the selection of the temperature T in this analysis is also appropriate. From Table 2, it also can be seen that within the experimental accuracy, all the results obtained from different pion interferometry methods are consistent with each other. Fig.2 shows our fitted curve integrated over q_0.

4. CONCLUSIONS

In a multi-negative-pion event, the multi-pion correlations can affect the results of two-pion interferometry analysis. In particular, this effect becomes more significant for a pion source with lower temperature and smaller radius. The method proposed in this paper is a general two-pion interferometry method. In this method, the main effect of multi-pion correlations on the two-pion correlation function can be factored out in the background, and removed from the two-pion interferometry results through an iterative calculation. For the three-negative-pion and four-negative-pion events generated from the pion source with lower temperature and smaller radius (T = 30 MeV, R = 3 fm) by Monte Carlo simulations, the analyses point out that the effects of multi-pion correlations can be almost completely excluded after just a few iterations. Using the method to analyse the data for 1.8 A GeV Ar + Pb central collisions at the Bevalac streamer chamber, we obtain fitted results for pion source parameters that are consistent with the previous results of two-pion and three-pion interferometry analyses. This indicates, for pion sources with higher temperature and greater radii ($T \sim 70$ MeV, $R \sim 5.6$ fm), although the average negative pion multiplicity is quite high ($<M_{\pi^-}> \sim 9.0$), the level of distortion introduced by the previous methods can be neglected within present statistical error. Moreover, it is indicated that the enhancement of the two-pion correlation function in the low relative momentum region comes solely from the Bose-Einstein correlation between two identical pions, and the Gaussian model is a self-consistent phenomenological model which describes the pion-emitting source in relativistic heavy ion collisions at the present level of experimental accuracy.

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