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Azimuthal Correlation Function and the Nuclear Equation of State

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Collisions of 1.2 A GeV Ar on KCl in the Bevalac Streamer Chamber are studied using two methods: azimuthal distribution analysis and azimuthal correlation function analysis. Comparing with VUU (Vlasov-Uehling-Uhlenbeck) model predictions for different nuclear equations of state, the stiffness of the high density nuclear matter is estimated. The maximum azimuthal anisotropy inferred from the azimuthal distribution method is decreased by about 30% because of dispersion of the azimuthal angle of the reaction plane. This distortion can be eliminated by using the azimuthal correlation function analysis proposed in this paper.

A major goal of high energy heavy ion collisions is to extract information about properties of nuclear matter at high density and high temperature and to obtain the nuclear equation of state (EOS). The momentum distribution of particles in the final state of high energy nucleus-nucleus collisions shows promise of providing valuable information about the EOS of compressed nuclear matter. In order to study the EOS using the momentum distribution of particles, several parameters have been chosen in the past, including flow angle [1,2], transverse momentum [3-7], and the maximum azimuthal anisotropy [8,9] proposed recently. However those parameters can be calculated only after the reaction plane in each event has been determined, which is affected by many factors, such as the observational losses in measuring the tracks of charged particles, the finite-multiplicity fluctuation in each event, and the elimination of "self correlation". So it is a very interesting problem to find an observable which is sensitive to the EOS and independent of the dispersion of the

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azimuthal angle of the reaction plane.

In this paper, we first study the EOS by means of the method of azimuthal distribution analysis [9] proposed by Welke *et al*. Then the azimuthal correlation function is derived from the distribution function of the azimuthal angle and applied to study the EOS. Being independent of any explicit determination of the reaction plane, the azimuthal correlation function analysis gives more precise information about the maximum azimuthal anisotropy than the azimuthal distribution analysis does.

The model used in this study is a microscopic simulation [10-12] which can be considered a solution of the Vlasov-Uehling-Uhlenbeck [13] (VUU) equation. In fact, it is not a direct numerical solution from the VUU equation at all, but is instead a Monte Carlo simulation. At its core, the VUU model is an intranuclear cascade. On top of this has been added a potential field and a Pauli blocking prescription. The mean field potential used in the VUU model is $U(\rho) = a\rho + b\rho^c$, in which the momentum-dependence interactions [14,15] are not incorporated. ρ is the density of the nucleons. a, b, c are three constants. $c = 2$ corresponds to $K = 380$ MeV, and implies a "stiff" EOS, while $c = 7/6$ corresponds to $K = 200$ MeV, usually characterized as a "soft" EOS.

The data for this investigation come from a streamer chamber experiment at the Lawrence Berkeley Laboratory with a 1.2 A GeV Ar beam incident on a KCl target. The samples under consideration contain 571 events with charged multiplicity $M \geq 30$. Assuming a simple geometrical picture, the impact parameters are between 0 and 3.6 fm. Further experimental details can be found elsewhere [16-18].

For each of the two values of EOS stiffness mentioned above, we have generated model statistics amounting to typically five times the experimental samples. The VUU model predictions must be filtered [19] before being compared with experimental data. The filtering process includes observational losses, energy loss and absorption in the target, and distortions resulting from particle misidentification.

For a non-zero impact parameter, the beam direction (z) and the line joining the centers of the nuclei determine the reaction plane, i.e., the x-z plane. The azimuthal angle of a nucleon is

$$\phi = \cos^{-1}(p_x/\sqrt{p_x^2 + p_y^2}) \tag{1}$$

The distribution function of ϕ in a certain region of rapidity (y) can be fitted by the form [9]

$$d\sigma/d\phi = A(1 + \lambda\cos\phi) \tag{2}$$

and the maximum azimuthal anisotropy is defined as

$$R = (1 + \nu)/(1 - \nu) \tag{3}$$

which is strongly influenced by the EOS and is a useful quantity to measure [9,20].

The reaction plane, unknown for each event in an experiment, is estimated using

$$\mathbf{Q} = \sum \mathbf{p}^t(\mu)\omega(\mu), \tag{4}$$

where $\mathbf{p}^t(\mu)$ is the transverse momentum per nucleon for μth track. y_{CM} is the rapidity of the center of mass of the system. The azimuthal angle of the μth track is calculated as follows

$$\phi(\nu) = \cos^{-1}(p_x(\nu)/|\mathbf{p}^t(\nu)|) \tag{5}$$

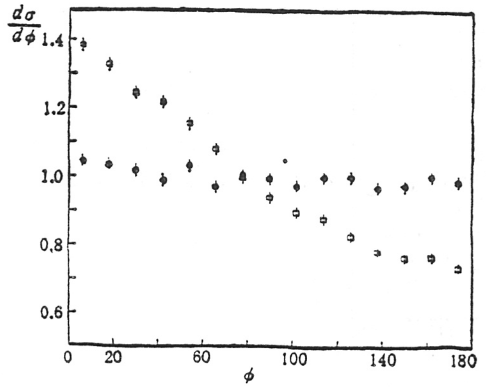

Fig. 1
The azimuthal distribution of the particles in Monte Carlo events where the self correlation exists for the squares and has been removed for the black circles.

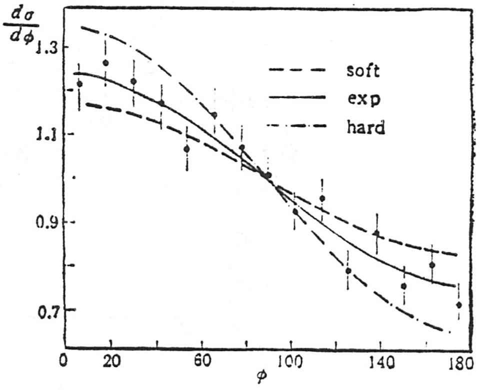

Fig. 2
The azimuthal distribution of the particles, where the self-correlation has been removed, with rapidity larger than 0.75 y_{beam} for experiment and VUU (1.2 A GeV Ar + KCl).

where

$$p_z(\nu) = p'(\nu) \cdot Q/|Q|$$

Because there is a self-correlation term in the above scalar product, the distribution of the azimuthal angles given by Eq.(5) is distorted. The distribution of Monte Carlo events showing in Fig. 1 (square) demonstrates this effect. Monte Carlo events are generated by randomly mixing particles from different events within the same multiplicity range such that the distribution of azimuthal angles of particles should be isotropic. Using the method of removing self-correlation proposed by Danielewicz [3], the azimuthal angle of the particles is calculated from the following expression

$$\phi'(\nu) = \cos^{-1}(p'_z(\nu)/|p'(\nu)|) \tag{6}$$

where

$$p'_z(\nu) = \frac{p'(\nu) \cdot Q'_\nu}{|Q'_\nu|}, \quad Q'_\nu = \sum_{\mu \neq \nu} p'(\mu)\omega(\mu).$$

The condition $\mu \neq \nu$ removes the self-correlation term. The reaction plane determined by Q'_ν and z is called the estimated reaction plane. The azimuthal distribution function, $\phi'(\nu)$, is

$$d\sigma/d\phi' = 1 + \lambda' \cos\phi' \tag{7}$$

The black circles in Fig. 1 are the azimuthal distribution of the particles in Monte Carlo events, where the self-correlation has been removed. Compared with the results where the self correlation exists, it shows that the distortion of the azimuthal distribution caused by self correlation has been eliminated.

Fig. 2 shows the azimuthal distribution of the particles with rapidity greater than 0.75 y_{beam} for

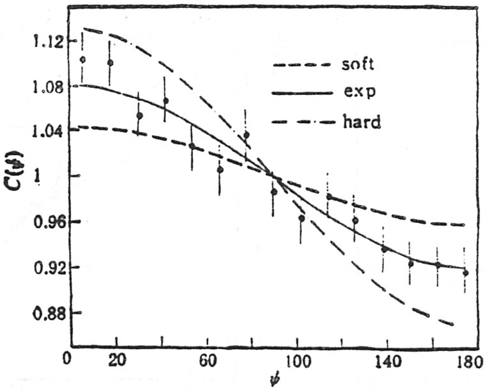

Fig. 3
The azimuthal correlation function of the particles with rapidity larger than 0.75 y_{beam} for experiment and VUU (1.2 A GeV Ar + KCl).

experiment and VUU model, where the self-correlation has been removed. The values of λ' for the three curves are 0.25 ± 0.02 (experiment), 0.17 ± 0.01 (soft), and 0.34 ± 0.01 (hard). The maximum azimuthal anisotropy R', calculated from Eq.(3), are 1.67 ± 0.07 (experiment), 1.41 ± 0.03 (soft), and 2.03 ± 0.05 (hard).

Quantities like p_x and R' are distorted to lower values because of the rms dispersion $\Delta\phi$ associated with the estimated reaction plane. This distortion can be removed if $\Delta\phi$ is known. Assuming a Gaussian distribution for ϕ_r (the angle between the estimated reaction plane and the real reaction plane) with $<\phi_r> = 0$, then

$$N(\phi_r) = \frac{1}{\Delta\phi\sqrt{2\pi}} \exp(-\phi_r^2/2\Delta\phi^2) \tag{8}$$

and

$$R = [1 + \lambda' \exp(\Delta\phi^2/2)]/[1 - \lambda' \exp(\Delta\phi^2/2)]$$

For the experimental data, $\Delta\phi$ is about 55° and R is decreased by about 30%. For VUU model predictions, the maximum azimuthal anisotropy R can be calculated by Eq.(1), Eq.(2) directly; they are 1.98 ± 0.05 (soft), 3.10 ± 0.10 (hard). Comparing with R', R is decreased by about 30% and 33% respectively because of dispersion in the estimated reaction plane.

In order to eliminate the effect of the dispersion of the reaction plane on azimuthal distribution analysis in the experiment, the azimuthal correlation function analysis is proposed to calculate the maximum azimuthal anisotropy in this paper.

From Eq.(2), the probability of observing two particles with azimuthal angle ϕ_1 and ϕ_2 is

$$d^2\sigma/d\phi_1 d\phi_2 = A^2(1 + \lambda\cos\phi_1)(1 + \lambda\cos\phi_2) \tag{9}$$

then the distribution probability of ψ, which is the angle between the transverse momenta of two correlated particles, has the form

$$P(\psi) = A^2 \cdot (1 + 0.5\lambda^2 \cos\psi)$$

From this expression, we define the azimuthal correlation function as

$$C(\psi) = P(\psi)/PM(\psi) \tag{10}$$

where $PM(\psi)$ is the distribution probability of ψ for Monte Carlo events.

The three curves in Fig.3 correspond to $C(\psi)$ for the experiment and the VUU model. Only particles with rapidity greater than 0.75 y_{beam} are used in the calculation. The experimental analysis is based on 45000 correlated particle pairs from 571 events. The uncorrelated particle pairs selected from Monte Carlo events, with ten times the statistics of the correlated pairs, make up the background. The fitted values of λ for the three curves are 0.41 ± 0.03 (experiment), 0.31 ± 0.01 (soft), 0.51 ± 0.02 (hard). The values of R are calculated using Eq.(3), and are 2.39 ± 0.08 (experiment), 1.90 ± 0.04 (soft), 3.08 ± 0.06 (hard). These results are consistent with those calculated by Eq.(1), Eq.(2) for VUU predictions. The reason for this consistency is that the angle ψ between the transverse momenta of two detected particles is independent of the azimuthal angle of the reaction plane, such that the azimuthal correlation function analysis is not affected by the dispersion of the azimuthal angle of the reaction plane.

As the azimuthal correlation function analysis is independent of the reaction plane in each event, the observational loses in measuring tracks may only reduce the statistics of the analysis and increase the statistical errors of the results. For example, if we randomly discard 40% of the particles in each event, the maximum azimuthal anisotropy R calculated by azimuthal correlation function analysis are 2.40 ± 0.15 (experiment), 1.90 ± 0.10 (soft), and 2.97 ± 0.12 (hard). In the range of statistical error, those results are consistent with the ones obtained by the same analysis where all tracks were used.

CONCLUSION

In the final state of a high energy nucleus-nucleus collision, the azimuthal angle distribution of the particles is sensitive to EOS. Using the azimuthal distribution analysis, many factors, such as the finite-multiplicity fluctuations in each event, the elimination of self-correlation, and the dispersion of the reaction plane, will make the calculated maximum azimuthal anisotropy lower than the true value. To estimate the stiffness of EOS, it is necessary to compare with a transport model prediction in which all of these effects have been correctly simulated. Being dependent on the angle between the transverse momenta of two particles, the azimuthal correlation function has no relation with the azimuthal angle of the reaction plane and eliminates the effect caused by the dispersion of the azimuthal angle of the reaction plane. The streamer chamber data for 1.2 A GeV Ar + KCl appear to favor incompressibilities intermediate between soft and hard, according to the results of the azimuthal distribution analysis and the azimuthal correlation function analysis.

REFERENCES

[1] Erez Braun and Zeev Frankel, *Phys. Rev.*, **C34** (1986) 120.
[2] H. A. Gustafsson *et al.*, *Phys. Rev. Lett.*, **52** (1984) 1590.
[3] P. Danielewicz and G. Odyniec, *Phys. Lett.*, **157B** (1985) 146.
[4] D. Beavis *et al.*, *Phys. Rev.*, **C33** (1986) 1113.
[5] K. G. R. Doss *et al.*, *Phys. Rev. Lett.*, **57** (1986) 302.

[6] D. Keane *et al.*, *Phys. Rev.*, **C37** (1988) 1447.
[7] D. Keane *et al.*, in Proceedings of Winter School on Nuclear Physics, Les Houches, France, 1989. Nuclear Matter and Heavy Ion Collisions, ed. M. Soyeur, Plenum, N. Y.
[8] D. Beavis *et al.*, *Phys. Rev. Lett.*, **54** (1985) 1652.
[9] G. M. Welke *et al.*, *Phys. Rev.*, **C38** (1988) 2101.
[10] H. Kruse *et al.*, *Phys. Rev. Lett.*, **54** (1985) 289.
[11] J. J. Molitoris *et al.*, *Phys. Lett.*, **162B** (1985) 47.
[12] J. J. Molitoris *et al.*, *Phys. Rev.*, **C37** (1988) 1014.
[13] E. A. Uehling and G. E. Uhlenbeck, *Phys. Rev.*, **43** (1933) 552.
[14] C. Gale *et al.*, *Phys. Rev.*, **C35** (1987) 1666.
[15] M. Prakash *et al.*, *Phys. Rev.*, **C37** (1988) 2253.
[16] D. Beavis *et al.*, *Phys. Rev.*, **C27** (1983) 2443.
[17] S. Y. Fung *et al.*, *Phys. Rev. Lett.*, **40** (1978) 292.
[18] M. Vient, Ph. D. Thesis, University of California, Riverside (1988).
[19] D. Keane *et al.*, in Proceedings of the 2nd Conference on the Intersection between Particle and Nuclear Physics, Lake Louise, Alberta, 1986, edited by D. F. Geesaman (Amenrican Institute of Physics, New York, 1986), p844.
[20] G. F. Bertsch and S. Das Gupta, *Phys. Rep.*, **160** (1988) 189.
[21] Y. M. Liu *et al.*, *Phys. Rev.*, **C34** (1986) 1667.