High Energy Physics and Nuclear Physics

Volume 14, Number 4

A Chiral Quark-Soliton Model Theory for Nuclear Force

Xu Xiaoming and Qiu Xijun

Institute of Nuclear Research, Chinese Academy of Sciences, Shanghai, China

The chiral quark-soliton model with one gluon exchange is briefly described. The quark-gluon coupling constant and quark wave functions determined in the soliton model are used as input for the relativistic two-quark-cluster theory. In this paper the two-nucleon system is studied. Two terms of the kinetic energy and relative-motion energy in the quark-exchange kernel are taken into account. Therefore, the phase shifts for $^3S_1$ and $^1S_0$ channels are improved. The phase shifts are in agreement with Arndt's. It is shown by our calculations that one-gluon-exchange and quark-exchange provide a short-range repulsive potential and the chiral field produces the middle- and long-rang attractive potential.

1. INTRODUCTION

Since the appearance of the quark model [1], the research for nuclear force has been developed into a new stage. Using the quark-parton model, one can solve some high-energy hadron-hadron scattering processes [2]. Incorporating the quark potential model and resonating group method in the low-energy region, the baryon-baryon interaction and scattering processes have been well studied to some extent by the Yazaki and Faessler groups, etc. [3-7].

In Ref. [8], based on the chiral soliton model, which is suggested by Birse and Kahana *et al.*, the problems of the deuteron ground states etc. are studied by considering one-gluon-exchange and employing the relativistic two-quark-cluster theory [11]. Besides the one-gluon-exchange interaction, other higher-order processes are phenomenologically described by a simple chiral field (σ, $\vec{\pi}$). Although this model is an approximation for describing the nuclear structure, it is still an ingenious one, since it connects the QCD with the conventional nuclear physics. With fewer parameters, one can obtain the static properties of nucleons, Δ-resonance and nucleon-nucleon scattering phase shifts, etc. [8].

Supported by the National Natural Science Foundation of China.
Received on November 29, 1989.

Table 1
Matrices, M_{ij}

$\vec{\lambda}_i \cdot \vec{\lambda}_j$	M_{ij}
$\vec{\lambda}_1 \cdot \vec{\lambda}_4$	16/9
$\vec{\lambda}_3 \cdot \vec{\lambda}_5$	−8/9
$\vec{\lambda}_2 \cdot \vec{\lambda}_5$	4/9
1	1/3

Table 2
Matrices N_{ST}

S	T	N_{ST}
1	0	1/9
0	1	−1/3
1	1	1/9
0	0	−1/3

In our present work we follow Ref. [8] and take the quark exchanges between two nucleons more completely into account.

In Sec.2, the relativistic two-quark-cluster theory and the chiral quark-soliton model are briefly described, and the relevant calculation formulas and the detailed expression for the quark-exchange term are presented. In Sec.3, some calculated results are given. We analyse our results and draw some conclusions in Sec.4.

2. THEORY AND CALCULATION FORMULA

In Ref. [8], we employ the chiral quark-soliton model with one gluon exchange to describe baryon-baryon interactions. Suppose the baryon can be described as a Hedgehog state [14], in which the single quark wave function is

$$q(\vec{r}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} iG(r) \\ \sigma \cdot \hat{r} \dfrac{F(r)}{r} \end{pmatrix}. \tag{2.1}$$

and the constituents of the chiral fields — σ and π meson fields, can be expressed by the spherically symmetric wave functions, $\sigma(r)$ and $h(r)$, respectively. In the mean field approximation, one set of coupled equations for the quark, σ and π meson fields can be obtained:

$$\frac{d^2h(r)}{dr^2} + \frac{2}{r}\frac{dh(r)}{dr} - \frac{2}{r^2}h(r) = \frac{39}{2\pi r^2}G(r)F(r), \tag{2.2a}$$

$$\frac{d^2\sigma(r)}{dr^2} + \frac{2}{r}\frac{d\sigma(r)}{dr} = -\frac{39}{4\pi r^2}\left(G^2(r) - F^2(r)\right), \tag{2.2b}$$

$$\begin{pmatrix} -g\sigma(r) + U^E(r) & -\partial_r - \frac{1}{r} + gh(r) - U^M(r) \\ \partial_r - \frac{1}{r} + gh(r) - U^M(r) & g\sigma(r) + U^E(r) \end{pmatrix} \begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = \omega \begin{pmatrix} G(r) \\ F(r) \end{pmatrix}, \tag{2.2c}$$

$$U^E(r) = -\frac{4\alpha_s}{3}\int_0^\infty \frac{1}{r_>}\left(G^2(r') + F^2(r')\right)dr', \tag{2.2d}$$

$$U^M(r) = \frac{16}{27}\alpha_s \int_0^\infty \frac{r_<}{r_>^2} G(r')F(r')dr', \tag{2.2e}$$

where $r_> = \max(r,r')$, $r_< = \min(r,r')$. By the self-consistent-iterative numerical method, the above equations can be solved to get the quark-gluon coupling constant $\alpha_s = 0.552$, quark-chiral field coupling constant $g = 4.416$, and the numerical $G(r)$ and $F(r)$. For convenience of the RGM calculation, the numerical result of the single quark wave function may be simulated by the following analytic function:

$$q(\vec{r}) = \begin{pmatrix} i c_1 e^{-ar^2} \\ c_2(\vec{\sigma} \cdot \vec{r}) e^{-br^2} \end{pmatrix}, \tag{2.3}$$

where

$$a = 2.53,\ b = 2.575,\ c_1 = 1.21,\ c_2 = 1.324, \tag{2.4}$$

Furthermore, we use the relativistic two-quark-cluster theory in Ref. [11] to explore the baryon-baryon interaction.

The quark part of the wave function of a baryon, $|\phi>$, is a product of color-singlet $|c>$, totally symmetric spin-isospin state $|S_N T_N>$ and totally symmetric orbit wave function $|qqq>$. According to the RGM, the equation of motion for the bound state and the l-partial wave scattering state takes the following form:

$$\left(-\frac{1}{2\mu}\frac{d^2}{dR'^2} + V_D(R') - E_r\right) f_l(R') = \int_0^\infty k_l(R', R'') f_l(R'') dR'', \tag{2.5}$$

where μ is the reduced mass of nucleon A and B; E_r is the nucleon-nucleon relative energy in the center-of-mass system. $V_D(R')$ is the direct interaction; $k_l(R', R'')$ is obtained through the partial-

Table 3
Matrices K_{ijST}

S	T	K_{14ST}	K_{15ST}	K_{16ST}
1	0	19/27	−7/27	2/27
0	1	31/27	−7/27	0
1	1	59/81	−17/81	10/81
0	0	−1/9	−5/9	−1/162

Table 4
Matrices L_{ST}

S	T	L_{ST}
1	0	−25/27
0	1	−25/27
1	1	25/81
0	0	25/9

wave expansion of the exchange term $k(\vec{R}', \vec{R}'')$ [8]; the effective NN potential is defined as:

$$V_{eff,l}(R') = V_D(R') + \int_0^\infty \frac{f_l(R'')}{f_l(R')} k_l(R', R'')dR''. \tag{2.6}$$

In the present work, the one-boson-exchange potential between quarks is considered to be

$$V_{OGE}(r_{ij}) = \frac{\alpha_s}{4} \vec{\lambda}_i \cdot \vec{\lambda}_j \left(\frac{1}{r_{ij}} - \frac{\vec{\alpha}_i \cdot \vec{\alpha}_j}{r_{ij}}\right) \tag{2.7}$$

$$V_{D\sigma E}(r_{ij}) = -\frac{g^2}{4\pi} \frac{e^{-m_\sigma r_{ij}}}{r_{ij}} \gamma_0(i)\gamma_0(j), \tag{2.8}$$

$$V_{D\pi E}(r_{ij}) = \frac{g^2}{4\pi} \frac{e^{-m_\pi r_{ij}}}{r_{ij}} (\vec{\tau}_i \cdot \vec{\tau}_j)\gamma_0(i)\gamma_0(j)\gamma_5(i)\gamma_5(j), \tag{2.9}$$

here $\vec{\alpha}_i$ is the Dirac operator for quark i. In Ref. [8], the exchange term $k(\vec{R}', \vec{R}'')$ contributed from one-boson-exchange between quarks is considered; even if the theoretical results, as a whole, are satisfactory, the high-energy NN scattering phase shifts deviate greatly from the empirical ones. In order to improve the previous theory, two other energy-dependent relativistic terms are taken into account in the exchange term $k(\vec{R}', \vec{R}'')$, namely,

$$k_1(\vec{R}',\vec{R}'') = {}_{ST}\langle\phi_A\otimes\phi_B|(-E_r)\mathscr{A}''_{AB}|\phi_A\otimes\phi_B\rangle_{ST}, \tag{2.10}$$

$$k_2(\vec{R}',\vec{R}'') = {}_{ST}\left\langle\phi_A\otimes\phi_B\left|\frac{1}{3}\left(\sum_{i\in A}\vec{\alpha}_i - \sum_{j\in B}\vec{\alpha}_j\right)\cdot\vec{P}_{\vec{R}_r}\mathscr{A}''_{AB}\right|\phi_A\otimes\phi_B\right\rangle_{ST}, \tag{2.11}$$

where $\mathscr{A}''_{AB}$ is the quark-exchange operator between nucleon A and B. One can see that $k_1(\vec{R}', \vec{R}'')$ is spin-isospin-independent, and $k_2(\vec{R}', \vec{R}'')$ is spin-isospin-dependent. In the calculation of $k_1(\vec{R}', \vec{R}'')$ and $k_2(\vec{R}', \vec{R}'')$ the following integral formula is used:

$$\iiint F_2(\vec{S}_1,\vec{S}_2,\vec{S})e^{F_1(\vec{S}_1,\vec{S}_2,\vec{S})}d^3\vec{S}_1d^3\vec{S}_2d^3\vec{S}$$

$$= \frac{\pi^{9/2}}{h^{3/2}}\left(8a_{20} + \frac{24m_1}{z_3} + \frac{12m_2}{z_3h} + \frac{8m_3}{z_3^2} + \frac{4\vec{m}_4\cdot\vec{i}}{z_3^2h} + \frac{2m_2\vec{i}^2}{z_3^2h^2}\right)\times\exp\left(a_{10} + \frac{j}{z_3} + \frac{\vec{i}^2}{4hz_3}\right), \tag{2.12}$$

where

$$F_1(\vec{S}_1,\vec{S}_2,\vec{S}) = -a_1S_1^2 - a_2S_2^2 - a_3S^2 + \vec{a}_4\cdot\vec{S}_1 + \vec{a}_5\cdot\vec{S}_2 + \vec{a}_6\cdot\vec{S} + a_7\vec{S}_1\cdot\vec{S}$$
$$+ a_8\vec{S}_2\cdot\vec{S} + a_9\vec{S}_1\cdot\vec{S}_2 + a_{10},$$
$$F_2(\vec{S}_1,\vec{S}_2,\vec{S}) = a_{20} + a_{11}S_1^2 + a_{12}S_2^2 + a_{13}S^2 + \vec{A}\cdot\vec{S}_1 + \vec{B}\cdot\vec{S}_2 + \vec{C}\cdot\vec{S}$$
$$+ a_{17}\vec{S}_1\cdot\vec{S} + a_{13}\vec{S}_2\cdot\vec{S} + a_{19}\vec{S}_1\cdot\vec{S}_2,$$
$$m_1 = 2a_{11}a_2 + 2a_{12}a_1 + a_{19}a_9,$$
$$m_2 = z_3(a_{13}z_3 + a_{17}z_1 + a_{13}z_2) + a_{11}z_1^2 + a_{12}z_2^2 + a_{19}z_1z_2,$$
$$m_3 = a_{11}\vec{P}^2 + a_{12}\vec{Q}^2 + a_{19}\vec{P}\cdot\vec{Q} + z_3(\vec{A}\cdot\vec{P} + \vec{B}\cdot\vec{Q}),$$
$$\vec{m}_4 = (2a_{11}z_1 + a_{17}z_3 + a_{19}z_2)\vec{P} + (2a_{12}z_2 + a_{13}z_3 + a_{19}z_1)\vec{Q}$$
$$+ z_3(z_1\vec{A} + z_2\vec{B} + z_3\vec{C}),$$
$$h = 4a_1a_2a_3 - a_1a_8^2 - a_3a_9^2 - a_2a_7^2 - a_7a_8a_9,$$
$$\vec{i} = z_1\vec{a}_4 + z_2\vec{a}_5 + z_3\vec{a}_6, \quad j = a_2\vec{a}_4^2 + a_9\vec{a}_4\cdot\vec{a}_5 + a_1\vec{a}_5^2,$$
$$z_1 = 2a_2a_7 + a_8a_9, \quad z_2 = 2a_1a_8 + a_7a_9, \quad z_3 = 4a_1a_2 - a_9^2,$$
$$\vec{P} = 2a_2\vec{a}_4 + a_9\vec{a}_5, \quad \vec{Q} = 2a_1\vec{a}_5 + a_9\vec{a}_4,$$
$$a_1 > 0,\ a_2 > 0,\ a_3 > 0,\ 4a_1a_2 - a_9^2 > 0,\ h > 0.$$

From this formula, one realizes that the derivation of $k(\vec{R}', \vec{R}'')$ is tedious. From Eq.(2.4), we can assume $b \approx a$, in order to simplify the calculation of k_1 and k_2. Using Eq.(2.12), we eventually get the exchange terms k_1 and k_2 for the N-N spin-isospin states $|S = 1, T = 0>$ and $|S = 0, T = 1>$, namely,

$$k_1(\vec{R}',\vec{R}'') = E_rc_1^{10}t(3.524c_1^2 + 7.928c_2^2/a - 4.625c_2^2R'^2 - 6.389c_2^2R''^2$$
$$+ 10.134c_2^2\vec{R}'\cdot\vec{R}''),$$
$$k_2(\vec{R}',\vec{R}'')\Big|_{\substack{S=1\\T=0}} = t(-1.175c_1^2 - 17.442c_2^2/a + (-11.017ac_1^2 + 2.017c_2^2)R'^2 \tag{2.13}$$
$$+ (-55.083ac_1^2 + 26.301c_2^2)R''^2 + (51.411ac_1^2 - 17.314c_2^2)\vec{R}'\cdot\vec{R}''),$$

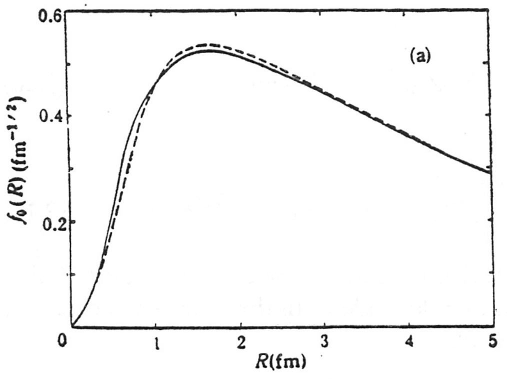

Fig. 1a
The ground state wave function of deuteron.

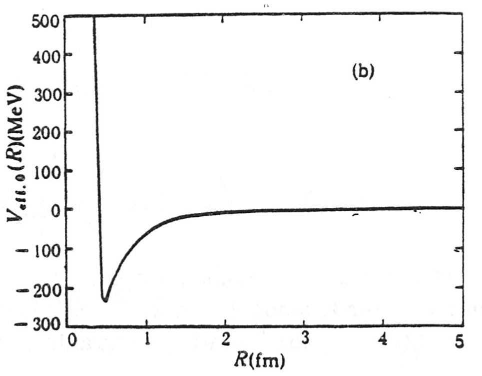

Fig. 1b
The nucleon-nucleon effective potential for deuteron.

$$k_2(\vec{R}', \vec{R}'')\Big|_{\substack{S=0\\T=1}} = t(-1.175c_1^2 - 12.084c_2^2/a + (-11.017ac_1^2 + 1.882c_2^2)R''^2 + (-55.083ac_1^2 + 23.62c_2^2)R''^2 + (51.411ac_1^2 - 16.992c_2^2)\vec{R}' \cdot \vec{R}''),$$
$$t = a^{-9/2}c_1^9c_2\exp(-2.875aR'^2 - 5.875aR''^2 + 7.25a\vec{R}' \cdot \vec{R}''). \quad (2.14)$$

3. NUMERICAL CALCULATION AND RESULTS

In accordance with the symmetry or antisymmetry of the wave functions of the N-N system the following terms need to be calculated,

$$V_{DOPE}(R') = 9\,_{ST}\langle \phi_A \otimes \phi_B | V_{OPE}(r_{14}) | \phi_A \otimes \phi_B \rangle_{ST}, \quad (3.1)$$

$$V_{DO\sigma E}(R') = 9\,_{ST}\langle \phi_A \otimes \phi_B | V_{O\sigma E}(r_{14}) | \phi_A \otimes \phi_B \rangle_{ST}, \quad (3.2)$$

$$k(\vec{R}', \vec{R}'') = -9\,_{ST}\langle \phi_A \otimes \phi_B | [(\vec{\alpha}_A - \vec{\alpha}_B) \cdot \vec{P}_{\vec{R}} - E_r + V_{OGE}(r_{14}) + 4V_{OGE}(r_{15}) + 4V_{OGE}(r_{25})](14) | \phi_A \otimes \phi_B \rangle_{ST}, \quad (3.3)$$

here (14) represents an exchange between quark 1 in nucleon A and quark 4 in nucleon B. In the calculation, these terms may be divided into the integration over the internal quark coordinates and the calculation of the following matrices:

$$M_{ij} = \langle c_A | \langle c_B | \vec{\lambda}_i \cdot \vec{\lambda}_j(14) | c_B \rangle | c_A \rangle, \quad N_{ST} = \langle ST | \vec{\sigma}_1 \cdot \vec{\sigma}_4 | ST \rangle,$$
$$k_{ijST} = \langle ST | \vec{\sigma}_i \cdot \vec{\sigma}_j(14) | ST \rangle, \quad L_{ST} = \langle ST | \vec{\tau}_1 \cdot \vec{\tau}_4 \vec{\sigma}_1 \cdot \vec{\sigma}_4 | ST \rangle.$$

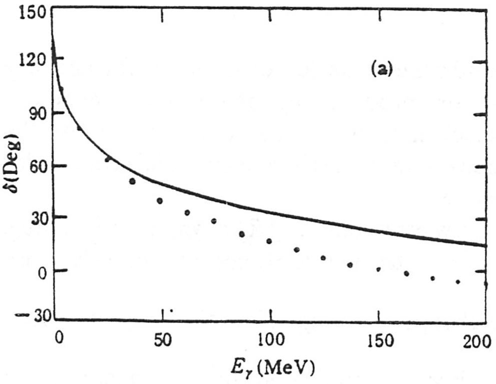

Fig.2a
The phase shifts for proton-neutron scatterings in the $^3S_1$ channel.

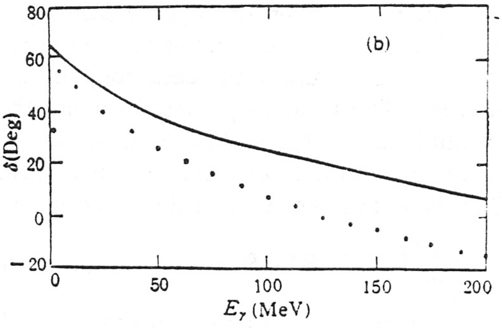

Fig.2b
The phase shifts for proton-proton scatterings in the $^1S_0$ channel.

The results of these matrices are listed in Tables 1−4, respectively.

After the evaluation of the direct term and exchange term, the $l = 0$ partial-wave Eq.(2.5) can be solved by the differential method and QR method. Readjusting the coupling constant $g = 7.27$, one can obtain the binding-energy $E_d = -2.225$ MeV for the $^3S_1(T = 0)$ ground state of the deuteron. The calculated ground-state wave function $f_0(R)$ and the effective potential $V_{eff,0}(R)$ are depicted as the solid lines in Figs.1a and 1b, respectively (the dashed lines are taken from Ref. [15]); the calculated phase shifts for the $^3S_1(t = 0)$ channel and $^1S_0(T = 1)$ channel are plotted as solid lines in Figs.2a and 2b, and the experimental phase shifts taken from Ref. [16] are represented by dots.

4. DISCUSSIONS AND CONCLUSIONS

In this work, the properties of the deuteron ground-state are determined by the one gluon exchange, one σ meson exchange and one π meson exchange between quarks. One can see from Fig. 1a that the slow decrease of the wave function represents a large size of the deuteron since the np weak combination. The shape of the deuteron wave function is in agreement with the results [15] of the conventional OPE theory. Comparing Fig. 1a with Fig. 1b, we realize that the lowest point of potential is not kinetically stable. The possibility of finding the proton and neutron in the deuteron in the middle and long range attractive region is large, the deuteron properties are, therefore, mainly determined by the σ meson and π meson. This finding verifies the description for the deuteron in the conventional one-pion-exchange theory [15].

Differing from the singular meson potential in the conventional OBE theory, our effective NN attractive potential does not have singularity at $R = 0$. At $R = 0$, two nucleons overlap completely and form a system with six quarks and meson clouds. In this case quarks feel a finite meson potential. Therefore, the corresponding effective N-N attractive potential is finite and of no singularity. As a consequence the effective NN potential from the meson exchange is exponential.

From expressions (2.8) and (2.9), one finds that the one-σ-meson-exchange provides the spin-isospin-independent and angular-momentum-independent central attractive potential between two nucleons, and the one-pion-exchange offers not only the spin-isospin-dependent effective central potential but also the tensor potential. Our calculation shows that the OPE between two quarks provides the tensor potential in the spin-triplet state but not in the spin-singlet state. This agrees with the conventional OPE theory for the deuteron.

According to the conventional OPE theory, the bound state of the deuteron may be formed only through OPE potential. But in our theory the repulsive core produced by the quark-exchange can destroy the bound state of the deuteron, hence a σ meson field should be added to increase the binding force. The attractive potential produced by the one-σ-meson-exchange potential is one order of magnitude larger than that from one-π-meson-exchange.

From the l-partial wave equation, the relative-motion wave function $f_l(R)$ changes with energy. Eq.(3.3) shows that the quark-exchange kernel is proportional to energy. Hence, the repulsive core $\propto\int_0^\infty \frac{f_l(R'')}{f_l(R')} k_l(R', R'')dR''$ changes its shape with the change of energy. The relation between the radius and the magnitude of the repulsive core and energy is very complicated, and the linear relation as in the case of the Paris potential does not show explicitly.

In the region of the repulsive core (R < 0.4 fm), the repulsive potential produced by quark-exchange dominates and increases quickly. It is two orders of magnitude larger than the attractive potential from the meson exchange. Like the results of Suzuki etc. [7] from the nonrelativistic quark potential model, our study shows that $k_2(R', R'')$ from the kinetic-energy operator and $k_1(R', R'')$ from the relative-motion energy give a remarkable contribution to the short-range repulsive core. In Ref. [8], however only the quark-exchange term with one-gluon-exchange is considered. Due to the omission of the contributions from k_1 and k_2, the NN scattering phase shifts do not agree with the empirical ones when the relative-motion energy E_r is larger. Now we correctly take into account the contributions from k_1 and k_2, and consequently the NN scattering phase shifts are improved properly.

Arndt *et al.* [16] determine the nucleon-nucleon scattering phase shifts by the one-pion-exchange potential and energy-dependent phase shift parameters. We compare their results with ours in Figs. 2a and 2b. It is clear that the phase shifts for both $^3S_1$-channel and $^1S_0$-channel are in agreement with the phenomenological results of Ref. [16].

From the above discussion, one can conclude that: (a) Our theory reasonably describes the bound state of the deuteron. (b) The quark exchange offers a spin-isospin-dependent and energy-dependent nonvocal short-range repulsive potential with radius of about 0.4 fm. (c) The complete inclusion of the k_1 and k_2 contributions and the contribution from chiral field (σ,π) coupling, the calculated phase shifts for the $^3S_1$-channel and $^1S_0$-channel are in agreement with Arndt's results [16].

ACKNOWLEDGMENTS

We are grateful to E. R. Nakamura for helpful discussions. We thank Prof. Zhou Xiaoqian for his concern and direction.

REFERENCES

[1] M. Gell-Mann, *Phys. Lett.*, **8** (1964) 214.

[2] E. R. Nakamura, Lecture in Introduction to Parton Model (Shanghai, 1989).

[3] M. Oka and K. Yazaki, *Prog. Theor. Phys.*, **66** (1981) 556, 572; Nucl. Phys. A402 (1983) 477.

[4] Y. He, F. Wang and C. W. Wong, *Nucl. Phys.*, **A448** (1986) 652.

[5] Z. Y. Zang *et al.*, *Nucl. Phys.*, **A443** (1985) 577.

[6] A. Faessler *et al.*, *Phys. Lett.*, **112B** (1982) 201.

[7] Y. Suzuki and K. T. Hecht, *Phys. Rev.*, **C27** (1983) 299.

[8] X. J. Qiu and X. M. Xu., *Nucl. Phys.*, **A** (in press).

[9] M. C. Birse and M. K. Banerjee, *Phys. Lett.*, **136B** (1984) 284.
[10] S. Kahana and G. Ripka, *Nucl. Phys.*, **A429** (1984) 462.
[11] X. J. Qiu, L. H. Wang and R. C. Hou, *Phys. Rev.* **C30** (1984) 1731.
[12] L. S. Celenza *et al.*, *Phys. Rev.*, **C31** (1985) 232.
[13] C. W. Wong, *Nucl. Phys.*, **A435** (1985) 669.
[14] A. Chodos and C. B. Thorn, *Phys. Rev.*, **D12** (1975) 2733.
[15] T. E. O. Ericson and W. Weise, Pions and Nuclei (Oxford Press, 1987) ch.3.
[16] R. A. Arndt *et al.*, *Phys. Rev.*, **C15** (1977) 1002.