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Gluon Fragmentation in e^+e^- Annihilations

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In this paper, we suggest that the gluon is hadronized by first splitting into a quark-antiquark pair. The final hadrons are produced by the color interactions among the quarks and antiquarks. Under this assumption, the calculated average charged particle multiplicities in e^+e^- three-jet events. The ratio of multiplicities between gluon and quark jets, and the rates of baryons in e^+e^- three gluons events all agree well with experiments. The angular distribution of final particles can also be explained.

1. INTRODUCTION

Detailed studies of $e^+e^- \rightarrow$ two jet events at high energies made it possible for us to accumulate a wealth of information about the features of the quark-antiquark hadronization. However, we still know very little, either experimentally or theoretically, about the gluon fragmentation, which we usually study through $e^+e^- \rightarrow$ three jet events at high energies. According to the theory of quantum chromodynamics (QCD), the gluon fragmentation is softer because its multiplicity is higher compared with the one in quark jets at the same energy. The calculated ratio of the parton multiplicities of the gluon fragmentation to that of the quark's at the same energy is [1]

$$r = \langle N \rangle_g / \langle N \rangle_q = \frac{9}{4}(1 - 0.27\sqrt{\alpha_s} - 0.07\alpha_s)$$

where α_s is the strong interaction coupling constant. Because the multiplicities of partons and hadrons can be related through a parameter which is independent of energy [2], this ratio should also be the ratio of the hadron multiplicities. The ratio r is predicted to be 9/4 at the limit of the infinite energy. However, experimental results are not completely in agreement with this theoretical prediction. The HRS collaboration [3] studied the charged particle multiplicity of the asymmetric

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three-jet events at c.m. energy $\sqrt{s}$ = 29GeV. They found that the relation between the multiplicity $<n_i>$ and the energy E_i of the jet i (i denotes quark, antiquark or gluon) is very similar to the relation between the charged particle multiplicity $<n_{2j}>/2$ and the c.m. energy $\frac{1}{2}\sqrt{s}$ of the 2-jet events. Their result shows that the multiplicity of the gluon jet, $<n_g>$, is not higher than the multiplicity of the quark jet, $<n_q>$. The difference between quark jets and gluon jets was further studied by using the three-fold symmetric three-jet events. These events provide a direct comparison of quark and gluon jets at the same energy. Although the experimental results show that there is a large difference between the quark and gluon jets, i.e., $<n_g>/<n_q> = 1.29 \pm ^{0.21}_{0.41} \pm 0.20$, it is still much smaller than the QCD theoretical expectation. The MARK II collaboration [4] measured the inclusive charged particle distribution in terms of the fractional momentum $X_p = p_j/E_i$ in each jet, where p_j is the momentum of the charged particle j in the jet i, and E_i is the energy of the jet i. Their results apparently support the prediction as to the soft gluon fragmentation.

However, the distribution of X_p given by the TASSO collaboration [5] did not show the difference between the gluon and the quark fragmentations. The JADE collaboration [6] measured the mean transverse momentum $<p_T>$ of charged particles, giving the ratio of $<p_T>$ of jet 3 (gluon jet) to that of jet 2 (quark and antiquark jet) as $<p_{T3}>/<p_{T2}> = 1.16 \pm 0.12$. The ratio of $<p_T>$ given by TPC collaboration [7] is $<p_{T3}>/<p_{T2}> = 1.08 \pm 0.02$. The CELLO collaboration [8] compared the $<p_T>$ of the jet 3 in three-jet events at c.m. energy $\sqrt{s}$ = 35 GeV with that in two-jet events at c.m. energy $\sqrt{s}$ = 14 GeV, giving $<p_{T3}(35)>/<p_T(14)> = 1.03 \pm 0.04$. None of them found notable difference in the average transverse momentum of hadrons between the gluon and the quark fragmentations. Furthermore, the ARGUS collaboration [9] found that the baryon yields are enhanced by a factor of 2.5 or more in $e^+e^- \rightarrow$ three gluons events at c.m. energy $\sqrt{s}$ = 10 GeV compared to the continuum region. This experimental result suggests that there is a difference in the baryon production between the quark and the gluon fragmentations.

By using the theory of QCD, we can quantitatively calculate the high Q^2 processes of strong interactions where perturbative calculations are valid. However, the hadronization of a system of partons (quarks and gluons) is a nonperturbative problem which remains unsolved. It can only be described by phenomenological hadronization models developed in recent years [10-13]. A good agreement with experiments can be obtained by adjusting some free parameters. However, there are some problems and difficulties. For example, the independent fragmentation model (IF) [10] cannot explain the 'string effect' which has been verified in a number of experiments [14]. As we pointed out [15], since this model cannot give the experimental average multiplicities of final particles either, it is no longer favored. The Lund string fragmentation (SF) [11] model can fit the experimental data well in many aspects. However, after the introduction of the diquark vacuum excitation, the number of adjustable parameters increases to as many as ten. The appearance of the baryonium ($qqq\bar{q}\bar{q}\bar{q}$) is also unavoidable in the model, which contradicts the experiments. Besides, none of these models can explain the baryon production in the $\gamma \rightarrow ggg$ events. In brief, the parton hadronization is still an unsolved problem. Therefore, it is important to explain and predict the experimental results based on some reasonable assumptions in order to understand the fragmentation mechanism. In this paper, we present a simple picture of gluon fragmentation and compare the calculated results with experiments.

2. GLUON FRAGMENTATION IN $e^+e^- \rightarrow$ THREE-JET EVENTS

According to the theory of quantum chromodynamics (QCD), the primary quark and antiquark produced in e^+e^- annihilation via electromagnetic excitation, i.e., $e^+e^- \rightarrow q\bar{q}$, can both emit one or several gluons with a probability proportional to the strong coupling constant α_s. To the first order, the result is $q \rightarrow q + g$ (or $\bar{q} \rightarrow \bar{q} + g$), i.e., $e^+e^- \rightarrow q\bar{q}g$. For the hard gluon emission the final state is a three-jet event. For the fragmentation of a $q\bar{q}g$ system, there are two fundamental problems: how the color-field among the gluon and other color charge (q and $\bar{q}$) is created and how the partons are produced through color interaction vacuum excitation and combining into final hadrons.

According to the theory of quantum chromodynamics (QCD), quarks and gluons have an exact $SU(3)$ color symmetry. Quarks have three different colors, e.g., red (R), yellow (Y) and blue (B). Antiquarks have the corresponding anticolors ($\bar{R}$, $\bar{Y}$ and $\bar{B}$). Gluons form the $SU(3)$ color octet which can be characterized by a color and an anticolor index (e.g. $B\bar{Y}$, $B\bar{R}$, etc.). Considering that the initial state of $e^+e^- \rightarrow$ three-jet events is colorless, the $q\bar{q}g$ also must be in a color singlet. If the quark is red (R), the antiquark is antiblue ($\bar{B}$). According to the requirement of color confinement, the gluon g must be blue-antired ($B\bar{R}$). The gluon g interacts with the quark (antiquark) through a red (blue) color force line which runs from the gluon to the quark (antiquark). In this sense, we can replace the color and anticolor index of the gluon by an effective quark and an antiquark, i.e., we assume that the gluon splits into a pair of effective quark and antiquark:

$$g(p_3) \rightarrow q'(zp_3) + \bar{q}'((1-z)p_3), \tag{1}$$

where p_3 is the momentum of the gluon, z is the momentum fraction of the quark q'. In this way, the color field between the gluon-quark and the gluon-antiquark can be made to be equivalent to that between q' and $\bar{q}$, and that between $\bar{q}'$ and q, respectively. In fact, this simple assumption about splitting is also used in the Lund string [11] model (SF) and QCD-cluster [12] model, etc. to study the gluon fragmentation.

The equivalent interactions in the $q\bar{q}'$ and $\bar{q}q'$ systems formed by the effective $q'\bar{q}'$ and the initial q, $\bar{q}$ are completely the same as that between the qq in the $e^+e^- \rightarrow$ two-jet events. The hadronization mechanism should also be the same. In other words, we can treat the hadronization of the two systems ($q\bar{q}'$, $\bar{q}q'$) in the same way as in $e^+e^- \rightarrow$ two-jet events in their own center-of-mass systems, which are different from the laboratory system or the e^+e^- center-of-mass system. The hadronization results of $e^+e^- \rightarrow$ three-jet events can be obtained by Lorentz transformation of the hadronization results of these two systems.

From the above simple analysis we can treat the $e^+e^- \rightarrow$ three-jet events as two equivalent two jet events. This simple picture can easily explain the 'string effect' of the final hadron distribution. When the hydrons are transformed into a laboratory system, the directions of their momenta fall on a hyperbola with asymptotes being along the directions from the gluon to the quark and to the antiquark, respectively. This makes the particle distribution between the gluon jet and quark jet, or gluon jet and antiquark jet much higher than that between the quark jet and antiquark jet. This is the 'string effect' which has been seen in many experiments. Some other fragmentation pictures, such as the independent fragmentation model, cannot explain it.

Further calculations on the c.m. energy of each two-jet system can provide the quantitative results to be compared with experiments. Let us discuss the $q\bar{q}'$ system first. The system can transform from the e^+e^- c.m. to the $q\bar{q}'$ c.m. through two successive Lorentz transformations. The first one is along the direction of the bisector of the angle θ_1 in Fig. 1 (as the X direction). Assuming that the momenta of q and $\bar{q}'$ along X direction are zero, we obtain

$$\beta_x = \cos(\theta_1/2), \tag{2}$$

Thus, the momenta of q and $\bar{q}'$ become equal through the transformation along the direction vertical to the angle θ_1 bisector (i.e. the Y direction in Fig. 1). After that, one reaches the $q\bar{q}'$ c.m. system. The relative velocity for the second transformation is

$$\beta_y = \frac{p_2 - p_3/2}{p_2 + p_3/2}, \tag{3}$$

where p_2 is the momentum of the quark q. In the calculation, Z in formula (1) is taken as $z = 1/2$, which means that quark q′ and antiquark $\bar{q}'$ each takes a half of the total momentum of the gluon. In the same way, we discuss the $q'\bar{q}$ system and obtain similar results.

We use X_i to denote the energy fraction of the parton i, i.e.,

$$X_i = \frac{2E_i}{\sqrt{S}}, \quad (i = 1, 2, 3), \tag{4}$$

where E_i is the energy of the parton i. According to the law of conservation of energy, we have

$$X_1 + X_2 + X_3 = 2 \tag{5}$$

By using momentum conservation, we have the relation of X_i and θ_i as follows:

$$X_i = \frac{2\sin\theta_i}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}, \tag{6}$$

where θ_i is the angle between the momenta of the partons j and k in Fig. 1.

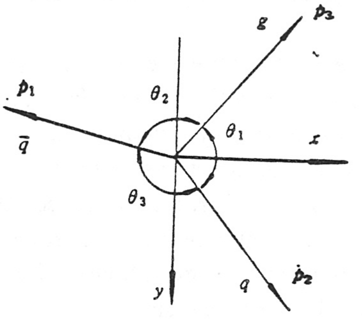

Fig. 1
Illustrated calculations of the c.m. energy of two-jet systems.

From the above formula, we obtain the c.m. energies W_1 and W_2 of $q\bar{q}'$ and $\bar{q}q'$ systems,

$$W_i = \sqrt{S}\,\frac{(2\sin\theta_3\sin\theta_i)^{1/2}\sin(\theta_i/2)}{\sin\theta_1+\sin\theta_2+\sin\theta_3},\quad (i=1,2). \tag{7}$$

Now the fragmentation result of each system can be obtained. Making two times of Lorentz transformations from the $q\bar{q}'$ and $\bar{q}q'$ system to the c.m. system of e^+e^-, we get the final fragmentation results which can be directly compared with the experiments of three-jet events $e^+e^-\rightarrow q\bar{q}g$. Being a Lorentz scalar, the average charged particle multiplicities can be obtained directly by summing up the results of the two systems

$$<n_{3j}> = <n_1> + <n_2>, \tag{8}$$

With the quark production rule for $e^+e^-\rightarrow$ two-jet events [15], we can calculate the multiplicities $<n_1>$ and $<n_2>$ of the two systems. For the three-fold symmetric three-jet events at c.m. energy $\sqrt{S}$ = 29 GeV, the calculated average charged particle multiplicity is $<n_{ch}> = 16.86$. The result from the Lund string model is $<n_{ch}> = 16.7$. Dzhaparidze [2] gave the result of $<n_{ch}> = 15.85$ by calculating the parton multiplicity of the quark and the gluon fragmentation. All of them are in agreement with the result $<n_{ch}> = 16.3 \pm 0.3 \pm 0.7$ from the HRS collaboration [3]. We can also calculate the final particle multiplicity of asymmetric three-jet events by using the above picture.

According to the simple picture in this paper, it is easy to obtain the ratio $<n_g>/<n_q>$ of the charged particle multiplicity $<n_g>$ of gluon jet to that of the quark jet $<n_q>$ from the particle multiplicity in $e^+e^-\rightarrow$ three-jet events. The n_{2j} charged particles produced from the fragmentation in their own c.m. system can be divided into two back-to-back quark jets according to the momentum direction of the particles. When they are transformed to the laboratory system, the magnitudes of their momenta will be changed. For example, for the particles originally belonging to $\bar{q}'$ jet in the c.m. system of $q\bar{q}'$, the sign of its momentum along y direction p_y will be changed after the Lorentz transformation if its longitudinal momentum $p_y < p_y^{cut}$. p_y^{cut} can be obtained by the Lorentz transformation as

$$p_y^{cut} = \left(\frac{m^2+p_x^2}{1-\beta_y^2}\right)^{1/2}\beta_y. \tag{9}$$

where m and p_x^2 are the mass and the squared transverse momentum of the particle. β_y is given in Eq.(3). This part of the particles with $p_y < p_y^{cut}$ and all the particles originally belonging to the q jet form the quark jet in the laboratory system. Using the longitudinal momentum distribution of the final particles in two-jet events, we can get the ratio R_1 of the number of these particles to n_{2j}. By the same method, we can obtain the corresponding R_2 in the $\bar{q}q'$ system. The particles with $p_y < p_y^{cut}$ originally belonging to $\bar{q}'$ jet and q' jet form the gluon jet after being transformed to the laboratory system. In order to compare with experiments directly, we calculate the ratio of the charged particle multiplicity of the gluon jet to the one of the quark jet at c.m. energy $\sqrt{S}$ = 29 GeV in three-fold symmetric three-jet events, i.e., with $R_1 = R_2$ and obtain

$$\frac{\langle n_g\rangle}{\langle n_q\rangle} = \frac{2\langle n_{2j}\rangle(1-R_1)}{\langle n_{2j}\rangle(1+R_1)} = 1.27, \tag{10}$$

This fits the result $1.29 \pm ^{0.21}_{0.41} \pm 0.20$ from the HRS collaboration [3] very well. The detailed calculation process will be discussed in another paper. Along with the increase of the energy $\sqrt{s}$, the longitudinal momentum distribution of final charged particle for $e^+e^- \rightarrow$ two-jet events [18] becomes broader. So the probability of the charged particles with large longitudinal momentum in the final state increases with c.m. energy. The calculated ratio R_1 or R_2 will decrease along with the increase of the c.m. energy. As a result, the ratio $<n_g>/<n_q>$ increases with c.m. energy and nears 2 at $\sqrt{s} \rightarrow \infty$. This prediction is in agreement with the result given by QCD.

3. THE GLUON FRAGMENTATION IN $e^+e^- \rightarrow \gamma$ EVENTS

In the c.m. energy range from 9.46 GeV to 10.35 GeV, e^+e^- annihilates into three bound state of the bb systems, i.e., γ (1s), γ (2s) and γ (3s). In the lowest order of QCD [19], they decay through an intermediate state of three gluons and the gluons fragmentat into final hadrons. Consequencely, we can treat the decay with the above picture of gluon fragmentation, i.e., each gluon splits into an effective quark and antiquark pair, such as $g_i \rightarrow q_i\bar{q}_i$. In this way, the fragmentations of the three gluons are changed into the fragmentations of three qq systems. Using the above Lorentz transformation method in e^+e^- three-jet events, we can obtain the c.m. energy of each system. Furthermore, we can calculate the average numbers of quark and antiquark pairs and then the average numbers of final hadrons as described in Ref. [17] and compare them with experiments.

Recently, ARGUS collaboration [9] measured various baryon yields in e^+e^- annihilation events at $\sqrt{s}$ = 10 GeV and found that the production rates are enhanced by a factor of 2.5 to 3 in upsilon resonance region compared to the continuum region. According to the above picture, the ratios of various baryons are from 2.81 to 2.88, which agrees with the experiment very well. The detailed results are given in Table 2 of Ref. [20].

4. DISCUSSION

Under the hypothesis of gluon splitting into an effective quark and antiquark pair, we change the fragmentation of three-jet events and three-gluon events into two and three two-jet events, respectively. The calculated various average charged particle multiplicities, the rates of the production, the ratio of baryon yields, etc. are in good agreement with experiments. These are essential tests to our simple picture. In this picture, we can also calculate the multiplicity distribution, the angular distribution (i.e., 'string effect'), etc. quantitatively. The characteristics at very high energy can also be predicted. These can be tested by further experiments. As we point out in the introduction, the mean transverse momenta of charged particles in a jet given by different experimental collaborations are different. It is difficult to test the theoretical predictions. Nevertheless, in the fragmentation picture of this paper, the gluon jet is formed by those particles from two independent fragmentation systems after being transformed into the laboratory system via Lorentz transformation. The transverse momenta of the particles become larger after the transformation. This leads to a large mean transverse momentum of the charged particles in the gluon jet.

At first sight, the above fragmentation picture is very similar to the Lund fragmentation model: both of them treat the three-jet as two independent quark-antiquark fragmentation systems; both can explain the 'string effect' of the final particle distribution in three-jet events, etc. However, there is a significant difference between them. In the Lund model, the fragmentation of the $q\bar{q}$ system is calculated by using the Monte-Carlo method. To explain the baryon production, the vacuum excitation of the diquark is involved. The ratio of the diquark to quark, which is an adjustable parameter, should be the same in the γ events and the continuum two-jet events in Lund model. So it cannot explain the experimental results that the baryon yields in γ events are 2.5 times higher than those in the continuum region. In this paper, we use our quark production rule and quark combination rule to deal with the $q\bar{q}$ system, and provide a further test of the rules. Our calculation shows that both the mean numbers of mesons and baryons increase linearly with the average numbers of quark-antiquark pairs. So it can explain the increase of the baryon yields in the γ event. As to the final meson yields, by including the mesons from various resonance decays, what we obtained is that the ratio of the yields in the γ events to the one in the continuum is approximately one which agrees with experiments within the error bars. In addition, in the Lund model the gluon in the qqg event is regarded as a kink in the color string of q and $\bar{q}$ and the color string breaks into a leading meson at the breaking point, with the rest two parts forming two 1+1-dimensional strings. In this paper, however, we assume that the color field between the gluon (g) and quark (q) can be replaced by the color field between the antiquark ($\bar{q}$) and quark (q). Thus, the three-jet events were treated as two independent two-jet fragmentation systems. The influence to the prediction originating from this detailed difference is expected to be analysed further and compared with the experiments.

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