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The Possibility of Forming Bound State of η-Mesic Nucleus

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The formation of the bound state of the η-mesic-nucleus is investigated by the standard Green's function method used in the many-body problem. It is found that when parameters, coupling constants and harmonic oscillator potentials are chosen in the physical range, due to the attractive nature of the $\eta NN^*(1/2, 1/2^-)$ interaction the η—^{16}O bound state can possibly be formed. However, detecting such a bound state would be difficult, because its width Γ is larger than the binding energy of η in $^{16}O_\eta$.

1. INTRODUCTION

Due to the rather weak nature of the ηNN interaction, there is usually no η-meson degree of freedom in nuclear matter. However, the η-meson product has been found in the high energy π-nucleus collision [1]. Bhalerao and Liu analysed theoretically the cross-section of the reaction $\pi N \to \eta N$ [2] and pointed out that the η-meson can be produced in the πN reaction via the formation of $N^*(1535)$ where the ηNN^* coupling constant is not weak. Recently, Haider and Liu further solved the three-dimensional relativistic wave equation with the η-nucleus optical potential derived from the $t_{\eta N}$ matrix [2] and found the possibility of binding the η-meson in some nuclei. According to their analysis, the key point of the formation of such a bound state is that the strong interaction in the S_{11} channel $N^*(1535) \rightleftharpoons \eta N$ is attractive, where $N^*(1535)$ is the πN resonance with $(I, J^P) = (1/2, 1/2^-)$, its mass $M_{N^*} = 1535$ MeV and its average width $\Gamma_{N^*} = 150$ MeV. If the η-nucleus bound state predicted by Haider and Liu does exist, one can obtain not only a type of exotic state of the nuclear

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matter, but also new informations about the ηN interaction, because in contrast with binding π(K) in an atomic orbit in the π(K) mesic atom, the η-meson is bound in a nuclear orbit. Because of the importance of studying the η-mesic nucleus, and in order to understand better the formation mechanism, we re-examine the possibility of forming the bound state of the η-nucleus by using a method which entirely differs from Haider and Liu's.

As an example, we study the η-mesic nucleus $^{16}O_\eta$ by calculating its binding energy

$$E_B = m_\eta - \varepsilon_0, \tag{1}$$

where m_η is the rest mass of the η-meson (m_η = 548.8 MeV), and ε_0 is the energy difference between the ground state of $^{16}O_\eta$ and ^{16}O,

$$\varepsilon_0 = E_0(^{16}O_\eta) - E_0(^{16}O), \tag{2}$$

In this paper, by the standard Green's function method (see Section 2 for details), we solve the self-consistent equations for the self-energy of the η-meson and then obtain ε_0. In the calculation, we assume that the dominant contribution to the self-energy of the η-meson comes from the (N^*N^{-1}) intermediate state, where N* are πN resonances with $(I, J^P) = (1/2, 1/2^\pm)$, and that in $^{16}O_\eta$, each of η, N and N* moves in its own mean field respectively. For simplicity, we employ phenomenoloically the harmonic oscillator potential $U(r) = 1/2m\omega^2r^2$ for these fields. Our result shows that due to the attractive feature of the $\eta NN^*(1/2, 1/2^-)$ interaction, forming the bound state of $^{16}O_\eta$ is possible. This conclusion agrees with that drawn by Haider and Liu. Since our η — N interaction differs from theirs, our result can provide more information about the structure of the bound state of the η-mesic nucleus.

2. FORMULATION

The single particle (s.p.) Green's function for the η-meson can be written as

$$G_\eta(t' - t) = \langle\Psi_0|T\{\Phi(t')\Phi^+(t)\}|\Psi_0\rangle \tag{3}$$

where Ψ_0 denotes the ground state of ^{16}O, and where Φ^+ and Φ are the creation and annihilation operators of the η-meson in the Heisenberg representation, respectively. The Fourier transform of $G_\eta(t' - t)$ is denoted by $G_\eta(\omega)$. Then, the excitation energy ε_n, defined as the energy difference between the excited state of $^{16}O_\eta$, specified by the quantum number n, and the ground state of ^{16}O, can be obtained as a pole of $G_\eta(\omega)$. The poles of $G_\eta(\omega)$ are obtained by solving the following self-consistent eigen equations [4]:

$$\begin{cases} \varepsilon_n(\omega) = h_\eta + \Gamma(\omega), \\ \omega = \varepsilon_n(\omega), \end{cases} \tag{4}$$

where h_η is the relativistic s.p. energy of the η-meson in ^{16}O, and $\Gamma(\omega)$ is the contribution given by irreducible self-energy diagrams. These irreducible self-energy diagrams, including (N^*N^{-1}) intermediate state, in the lowest order are shown in diagrams (b) and (c) in Fig. 1.

We assume that the nuclear medium, each of the η-meson and baryons N and N* moves independently in its own mean field which is represented phenomenologically by a harmonic

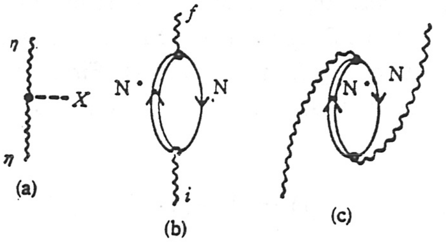

Fig. 1
Irreducible diagrams of the η-meson vertex function.

oscillator potential:

$$U_i(r) = 1/2 m_i \omega_i^2 r^2, \quad (i = \eta, \mathrm{N} \text{ or } \mathrm{N}^*) \tag{5}$$

Then, we can obtain the energy eigenvalue h_η by solving the Klein-Gordon equation

$$[-\nabla^2 + (m_\eta + U_\eta(r))^2]\phi(\vec{r}) = h_\eta^2 \phi(\vec{r}) \tag{6}$$

for the η-meson. In Eq.(5), ω_η, ω_{N} and ω_{N^*} are harmonic oscillator frequencies and will be discussed later. Similarly, we can determine the relativistic s.p. energy and wave-function for N (N*) by solving the Dirac equation

$$[\gamma_0 h_\alpha + i\vec{\gamma}\cdot\vec{\nabla} - (m_\alpha + U_\alpha(r))]\psi_\alpha(\vec{r}) = 0 \quad (\alpha = \mathrm{N} \text{ or } \mathrm{N}^*) \tag{7}$$

Now, we calculate the vertex function $\Gamma(\omega)$ shown in Fig. 1 in terms of the time-ordered formulation given in Refs. [5] and [6]. We can write this function for diagram (b) in Fig. 1 as

$$\Gamma_b(\omega) = \sum_m \frac{\langle f|H(0)|m\rangle\langle m|H(0)|i\rangle}{\omega - E_m + i0^+}, \tag{8}$$

where

$$H(t) = \int d\vec{r}\left[i\sqrt{4\pi}\, g_{\eta \mathrm{NN}^*}\bar{\Psi}_{\mathrm{N}^*}(\vec{r},t)\begin{pmatrix}\gamma_5\\ I\end{pmatrix}\Psi_N(\vec{r},t)\Phi(\vec{r},t)\right] + h.c. \tag{9}$$

is the Hamiltonian describing the strong ηNN* interaction. In our calculation, we only take those πN resonances with $(I, J^P) = (1/2, 1/2^\pm)$ and sizable (N* = Nη) branching ratios (> 25%) for N*. The Dirac matrix γ_5 and the unit matrix I in Eq.(9) are for N* with $J^P = 1/2^+$ and $1/2^-$, respectively. The intermediate state energy E_m in Eq.(8) is equal to $(h_{\mathrm{N}^*} - h_{\mathrm{N}})$, where $h_{\mathrm{N}^*(\mathrm{N})}$ determined by Eq.(7) is the relativistic s.p. energy for N*(N). Obviously, $h_{\mathrm{N}^*(\mathrm{N})}$ depends on $\omega_{\mathrm{N}^*(\mathrm{N})}$ where ω_{N} can be approximately taken to be the harmonic oscillator frequency of the nucleon in ^{16}O. In other words, $\hbar\omega_{\mathrm{N}} = 16$ MeV and ω_{N^*} can be treated as a free parameter, because we are short of the knowledge about the N*N interaction. Furthermore, since N* is a πN resonance with the average width about 100 MeV, we add an empirical width given by Particle Data Group to h_{N^*} [7]. Thus, Eqs.(4) become complex eigen equations. Of course, this is just an approximate treatment for the relativistic s.p.

motion. Similarly, we perform the calculation for diagram (c) in Fig. 1, where the intermediate energy E_m is equal to $(2h_\eta + h_{N^*} - h_N)$ and h_η is the relativistic s.p. energy for η. Since we have introduced the average field $U_\eta(r)$ for defining $\phi(\vec{r})$ and h_η, there must be some diagrams with U_η vertices for the vertex function Γ in Eq.(4). Diagram (a) in Fig. 1 is such a diagram.

We now discuss the method of solving Eqs.(6) and (7). First, we expand the scalar wave function $\phi(\vec{r})$ for the η-meson and the Dirac spinor $\psi_\alpha(\vec{r})$ for N*(N) in spherical coordinates, i.e.,

$$\phi(\vec{r}) \rightarrow \phi_{nLM}(r, \theta, \varphi) = \frac{1}{r} R_{nL}(r) Y_{LM}(\theta, \varphi), \tag{10}$$

and

$$\psi_\alpha(\vec{r}) \rightarrow \psi_{\alpha nljm\tau}(r, \theta, \varphi) = \frac{1}{r}\begin{pmatrix} i g_{\alpha nlj}(r) y_{ljm\tau}(\theta, \varphi) \\ \vec{\sigma}\cdot\hat{r} f_{\alpha nlj}(r) y_{ljm\tau}(\theta, \varphi)\end{pmatrix}, \tag{11}$$

where

$$y_{ljm\tau}(\theta, \varphi) = \sum_s C^{jm}_{lm-s,\frac{1}{2}s} Y_{lm-s}(\theta, \varphi)\chi_s\chi_c. \tag{12}$$

χ_s and χ_c are the spin and isospin wave functions for N*(N), respectively.

Then, substituting Eqs.(10) and (11) into Eqs.(6) and (7), respectively, we obtain the following set of radial equations

$$R''_{nL}(r) + \left\{h^2_{\eta nL} - [U_\eta(r) + m_\eta]^2 - \frac{L(L+1)}{r^2}\right\} R_{nL}(r) = 0, \tag{13}$$

and

$$g'_{\alpha nlj}(r) = [U_\alpha(r) + h_{\alpha nlj}] f_{\alpha nlj}(r) - \frac{K}{r} g_{\alpha nlj}(r),$$

$$f'_{\alpha nlj}(r) = [U_\alpha(r) - h_{\alpha nlj}] g_{\alpha nlj}(r) + \frac{K}{r} f_{\alpha nlj}(r), \tag{14}$$

where $K = \pm(j + 1/2)$ for $j = l \mp 1/2$, respectively, $g_{\alpha nlj}(r)$ and $f_{\alpha nlj}(r)$ satisfy the normalization condition

$$\int_0^\infty dr [g^2_{\alpha nlj}(r) + f^2_{\alpha nlj}(r)] = 1. \tag{15}$$

By using the transformation [8]

$$\tilde{g}_{\alpha nlj}(r) = [u_\alpha(r) + h_{\alpha nlj}]^{-\frac{1}{2}} g_{\alpha nlj}(r), \tag{16}$$

where $u_\alpha(r) = U_\alpha(r) + m_\alpha$, we can reduce the simultaneous equations (14) into a second order ordinary differential equation

$$\tilde{g}''_{\alpha nlj}(r) + \left\{\frac{u''_\alpha(r)}{2[h_{\alpha nlj} + u_\alpha(r)]} - \frac{3}{4}\left[\frac{u'_\alpha(r)}{h_{\alpha nlj} + u_\alpha(r)}\right]^2 - \left[\frac{u'_\alpha(r)}{h_{\alpha nlj} + u_\alpha(r)}\right]\cdot\frac{K}{r} + [E^2_{\alpha nlj} - u^2_\alpha(r)] - \frac{l(l+1)}{r^2}\right\}\tilde{g}_{\alpha nlj}(r) = 0. \tag{17}$$

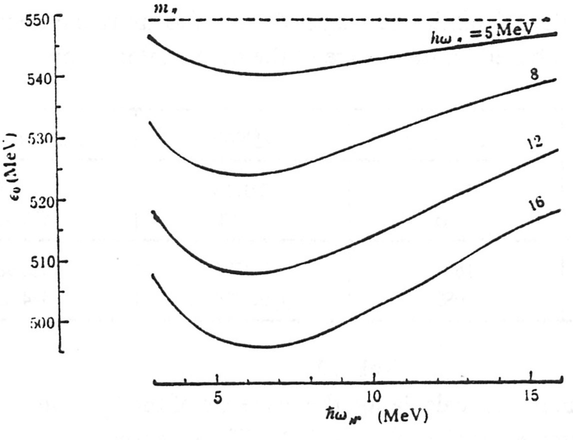

Fig. 2

Dependence of the η-meson excitation energy ε_0 on the mean-field harmonic oscillator frequencies ω_η and ω_{N^*}.

Since Eqs.(13) and (17) are both in the form of

$$U''(r) + A(r)U(r) = 0. \tag{18}$$

we can solve them numerically by using the method suggested by Blatt [9].

Finally, we present the formulas for calculating Hamiltonian matrix elements in Eq.(8). Take Fig. 1(b) as an example. The initial state $|i>$, final state $|f>$ and (N^*N^{-1}) intermediate state are, respectively,

$$\begin{aligned} |i\rangle &= |f\rangle = |\eta, nLM\rangle, \\ |m\rangle &= |[(N^*, n_1l_1j_1), (N, n_2l_2j_2)^{-1}]L, M; T = 0\rangle, \end{aligned} \tag{19}$$

where T denotes the total isospin. Using the angular-momentum coupling method given by Kuo [10], we can obtain the following matrix elements from Eq.(9):

(i) For N*(1/2, 1/2⁺)

$$\begin{aligned} \langle m|H(0)|i\rangle &= i\sqrt{4\pi}g_{NN^*\eta}[2h_{\eta nL}\cdot(2L+1)]^{-1/2} \\ &\cdot [I(n_1l_1n_2l_2nL)\langle l'_1j_1|L|l_2j_2\rangle^0 + I(n_2l_2n_1l_1nL)\langle l_1j_1|L|l'_2j_2\rangle^0], \end{aligned} \tag{20}$$

(ii) For N*(1/2, 1/2⁻)

$$\begin{aligned} \langle m|H(0)|i\rangle &= i\sqrt{4\pi}g_{NN^*\eta}[2h_{\eta nL}\cdot(2L+1)]^{-1/2} \\ &\cdot [J(n_1l_1n_2l_2nL)\langle l_1j_1|L|l_2j_2\rangle^0 - K(n_1l_1n_2l_2nL)\langle l'_1j_1|L|l'_2j_2\rangle^0], \end{aligned} \tag{21}$$

Table 1
Numerical results calculating the dependence of forming the bound state $^{16}O_\eta$, based on the nature of the ηNN^* interaction.

Resonance	m_{N^*}(MeV)	$g^2_{\eta}NN^*$	Re(ε_0)(MeV)
$N^*(1/2,1/2^+)P_{11}$	1589 1665	0.0713 0.0930	572.48 572.48
$N^*(1/2,1/2^-)S_{11}$	1608 2088	0.3795 0.5914	532.98 554.88

Table 2
Numerical results calculating the decrease of the binding energy E_B as the $g^2_{\eta NN}{}^*(1/2, 1/2^-)$ decreases.

$g^2_{\eta}(1/2,1/2^-)$	ε_0(MeV)
0.48	524.11
0.43	528.44
0.3795	532.98
0.33	537.60
0.28	542.45

with

$$I(n_1l_1n_2l_2nL) = -i\int \frac{dr}{r} f_{n_1l_1j_1}(r) g_{n_2l_2j_2}(r) R_{nL}(r), \tag{22}$$

and

$$J(n_1l_1n_2l_2nL) = \int \frac{dr}{r} g_{n_1l_1j_1}(r) g_{n_2l_2j_2}(r) R_{nL}(r), \tag{23}$$

where the radial wave functions f, g and R in Eqs.(22--24) can be determined by solving Eqs.(13) and (14), respectively. The off-diagonal matrix element is

$$K(n_1l_1n_2l_2nL) = \int \frac{dr}{r} f_{n_1l_1j_1}(r) f_{n_2l_2j_2}(r) R_{nL}(r), \tag{24}$$

where C and W are the Clebsch-Gordan coefficient and the Racah coefficient, respectively, and the definition of the quantity with the hat is $\hat{\lambda} = (2\lambda + 1)^{1/2}$.

3. RESULTS AND DISCUSSION

Using the Green's function formula mentioned above and the vertex function in Fig. 1, we calculate the binding energy E_B of the η-meson in the $(nL) = (0S)$ nuclear orbit of ^{16}O under various

Table 3

Numerical results calculating the dependence of E_B to $\hbar\omega_\eta$.

$\hbar\omega_\eta$(MeV)	E_B(MeV)	Γ(MeV)
3.0	0.77	18.80
4.0	4.20	31.41
5.0	9.13	41.90
6.0	14.65	49.28
7.0	20.04	54.26
8.0	25.04	57.75
9.0	29.63	60.34
10.0	33.83	62.35
11.0	37.70	63.97
12.0	41.28	65.32
13.0	44.59	66.47
14.0	47.66	67.48
15.0	50.53	68.37
16.0	53.21	69.17

physical conditions.

In order to discuss the dependence of forming the bound state $^{16}O_\eta$, based on the nature of the ηNN^* interaction, we first calculate ε_0 by evaluating the self energy term $\Gamma(\omega)$ in Eq.(4) with N*(1/2, 1/2$^+$) and N*(1/2, 1/2$^-$) individually. The numerical results are shown in Table 1. In order to compare our result with those given by Harider and Liu, we take two sets of values [2] for the ηNN^* coupling constant and the N* mass. We also assume that the harmonic oscillator frequencies of mean fields for N*, η and N are the same, i.e., $\hbar\omega_\eta = \hbar\omega_{N^*} = \hbar\omega_N$ = 16 MeV. Table 1 shows that forming the bound state of $\eta{-}^{16}O$ in the 0S nuclear orbit is possible (when Re(ε_0) $< m$) in the case of N*(1/2, 1/2$^-$), but not in the case of N*(1/2, 1/2$^+$). Hence, we conclude that the attractive feature in the S_{11} channel, N*(1/2, 1/2$^-$) = ηN, is very crucial for the formation of the bound state. This agrees with Haider and Liu's conclusion. Moreover, it is notable that E_B is very sensitive to the ηNN^* coupling constant ($g^2_{\eta NN^*}$). As an example, we calculate E_B with the odd parity N*(1608) by using different values of $g^2_{\eta NN^*}$(1/2, 1/2$^-$) (the other parameters keep their original values shown in Table 1). The numerical results are shown in Table 2. This table indicates that one finds that the binding energy E_B decreases rapidly with the decrease of $g^2_{\eta NN^*}$(1/2, 1/2$^-$). However, if one chooses reasonably the coupling constants in the physical region, forming the bound state of ^{-16}O in the 0S nuclear orbit is possible in the case of N*(1/2, 1/2$^-$). Thus, our theoretical calculation supports the conclusion that due to the attractive feature of the strong interaction in the S_{11} channel, N*(1/2, 1/2$^-$) = ηN, the formation of the bound state of ^{16}O is possible.

Obviously, assuming the same frequency for all harmonic oscillator potentials U_η, U_{N^*} and U_N is not realistic. We should discuss the relations among E_B, the binding energy of the η-meson in the 0S orbit of ^{16}O, and mean fields U_η and U_{N^*}. Since it was experimentally found that the ηNN interaction is not strong, it is more realistic to introduce a weaker mean field U_η for η-meson. As for N*, due to the inadequate information about the NN* interaction, we take $\hbar\omega_{N^*}$ as a free parameter. Keeping $\hbar\omega_N$ = 16 MeV for ω_N and taking different values for $\hbar\omega_\eta$, we calculate the $\hbar\omega_{N^*}$ dependence for E_B and plot our results in Fig. 2. In order to fit the actual situation better, we take the values of the masses and widths of N* from Particle Data Group [7] and the values of the coupling constant $g^2_{\eta NN^*}$ from Ref. [2]. As a result, E_B is not sensitive to the variation of the mass of N*. From Fig. 2, we find that for every given value of ω_η, E_B reaches its maximum value around $\hbar\omega_{N^*} = 6 \sim 7$ MeV. In view of the fact that $\hbar\omega \sim 1/M$ in the nuclear shell model and $M_N^*/M_N \sim 2$, the N*N interaction seems to be slightly weaker than the NN interaction. Moreover, we also see that E_B increases along with the increase of $\hbar\omega_\eta$. We tabulate the dependence of $\hbar\omega_\eta$ on E_B in Table 3, where $\hbar\omega_N = 16$ MeV and $\hbar\omega_{N^*} = 6$ MeV. This table clearly shows that the bound state of ^{16}O still exists, i.e. $E_B > 0$, even if the mean field of the η-meson in the nuclear medium of ^{16}O is much weaker ($\hbar\omega_\eta \sim 3.0$ MeV). However, the width of the ^{16}O bound state is larger than its binding energy E_B and observing such a bound state would be difficult. The conclusion agrees with that given in Ref. [11].

In summary, since the ηNN*(1/2, 1/2$^-$) interaction is attractive and quite strong, it is possible to bind the η-meson in the nucleus through forming a particle (N*)-hole (N^{-1}) state. In other words, the η-meson repeats the decay and fusion processes, $\eta \rightleftharpoons$ N*N^{-1}, in the nuclear medium and consequently forms a bound state of the η-mesic nucleus. Of course, it is necessary to test experimentally the validity of the conclusion. We believe that the bound state of ^{16}O can be obtained via the π-nucleus reaction, such as $\pi^+ + {}^{16}O \to {}^{16}O_\eta^* \to {}^{15}O + p$, at medium energies. If it is true, one can obtain more informations about the ηNN* interaction in the nuclear matter by measuring the energies and widths of the excited state of $^{16}O_\eta$.

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