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Bjorken-Johnson-Low Technique and a Perturbative Study on Chiral Anomaly in Pure Abelian Coset Gauge Field Theory

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The perturbative theory in the pure coset gauge field theory is studied in this paper. By using the Bjorken-Johnson-Low technique and calculating the Schwinger term in related commutators, the anomalous Ward identity in the pure Abelian coset gauge field theory is derived and found consistent with the non-perturbative calculation.

1. INTRODUCTION

In 1986, Faddeev *et al.* pointed out that during the quantization of the gauge theory with anomaly, the first class of the original classical constraint changes into the second class and brings difficulties to the normal quantization of path integration of the gauge field [1]. They suggested a method of introducing a chiral auxiliary field and a counteractive term of action to solve these difficulties. This reminds us of the pure coset gauge field theory [2]. The theory extended the original symmetry by introducing the pure gauge filed in the coset space, and for the usual chiral group, the commutative relations of the pure coset gauge field is very similar to the chiral auxiliary field suggested by Faddeev. In addition, after introducing the pure coset gauge field into the path integration form of generating functional, the chiral anomalous term, which is very similar to the additional counteractive term suggested by Faddeev [3], can be obtained automatically for the Abelian chiral group. So we shall study the chiral anomaly of the pure coset gauge field theory carefully in order to search a specific method for solving the aforementioned difficulties.

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The non-perturbative calculation of the chiral anomaly of the pure coset gauge field theory was discussed carefully in Ref. [4]. Since the Lagrangian of the usual pure coset gauge field theory does not contain an explicit kinetic energy term, the perturbative calculation of this theory has not been studied. Under the physical gauge, we introduce a kinetic energy term to the Lagrangian of the pure coset gauge field by using frozen approximation, and make the perturbative calculation of this theory.

Let us consider the following Lagrangian

$$\mathscr{L}^{(A)} = -\frac{1}{4}F_{\mu\nu}^2 - \bar{\psi}\gamma_\mu\left(\partial_\mu - ieA_\mu\frac{1+\gamma_5}{2}\right)\psi - g\bar{\psi}\left(\frac{1+\gamma_5}{2}\phi^+ + \frac{1-\gamma_5}{2}\phi\right)\psi$$
$$-(\partial_\mu + ieA_\mu)\phi^+(\partial_\mu - ieA_\mu)\phi - V(\phi^+\phi), \tag{1.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and ϕ is the Higgs field. In the canonical transformation

$$\psi' = e^{i\theta\frac{1-\gamma_5}{2}}\psi, \qquad \bar{\psi}' = \bar{\psi}e^{-i\theta\frac{1+\gamma_5}{2}},$$
$$\phi' = e^{-i\theta}\phi, \qquad \phi^{+\prime} = e^{i\theta}\phi, \tag{1.2}$$

we have

$$\mathscr{L}^{(A)\prime} = -\frac{1}{4}F_{\mu\nu}^2 - \bar{\psi}'\gamma_\mu\left(\partial_\mu - ieA_\mu\frac{1+\gamma_5}{2} - i\partial_\mu\theta\frac{1-\gamma_5}{2}\right)\psi'$$
$$- g\bar{\psi}'\left(\frac{1+\gamma_5}{2}\phi^{+\prime} + \frac{1-\gamma_5}{2}\phi'\right)\psi' - (\partial_\mu + ieA_\mu$$
$$- i\partial_\mu\theta)\phi^{+\prime}(\partial_\mu - ieA_\mu + i\partial_\mu\theta)\phi' - V(\phi^{+\prime}\phi'). \tag{1.3}$$

and obtain the pure coset gauge field $\theta(x)$. For the convenience in calculation, we ignore the prime symbol in Eq.(1.3), and obtain

$$\mathscr{L}^{(B)} = -\frac{1}{4}F_{\mu\nu}^2 - \bar{\psi}\gamma_\mu\left(\partial_\mu - ieA_\mu\frac{1+\gamma_5}{2} - i\partial_\mu\theta\frac{1-\gamma_5}{2}\right)\psi$$
$$- g\bar{\psi}\left(\frac{1+\gamma_5}{2}\phi^+ + \frac{1-\gamma_5}{2}\phi\right)\psi - (\partial_\mu + ieA_\mu$$
$$- i\partial_\mu\theta)\phi^+(\partial_\mu - ieA_\mu + i\partial_\mu\theta)\phi - v(\phi^+\phi), \tag{1.4}$$

It is easy to see that $\mathscr{L}^{(B)}$ is invariant under the gauge transformation of $U(1)_L \times U(1)_R$

$$\psi' = e^{i\alpha\frac{1+\gamma_5}{2}}e^{i\beta\frac{1-\gamma_5}{2}}\psi, \quad \bar{\psi}' = \bar{\psi}e^{-i\beta\frac{1+\gamma_5}{2}}e^{-i\alpha\frac{1-\gamma_5}{2}},$$
$$\phi' = e^{i\alpha}e^{-i\beta}\phi, \qquad \phi^{+\prime} = e^{-i\alpha}e^{i\beta}\phi^+,$$
$$A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha, \quad \theta' = \theta + \beta. \tag{1.5}$$

Under the physical gauge, by using the frozen approximation, we first introduce a kinetic energy

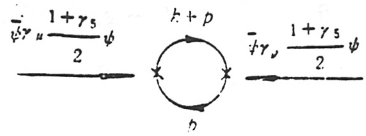


Fig. 1

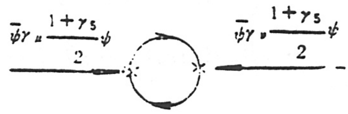


Fig. 2

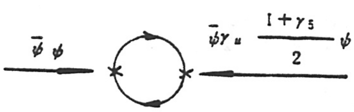


Fig. 3

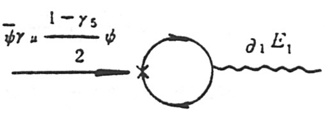


Fig. 4

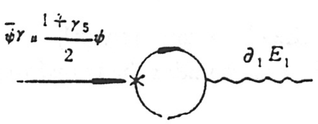


Fig. 5

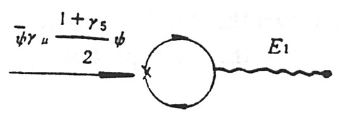


Fig. 6

Figs. 1-6
Processes of perturbative calculation.

term to the Lagrangion of the pure coset gauge field in this paper, and at the same time the gauge field acquires a mass. Then we calculate all the related Green functions by using the BJL technique [5] in order to obtain the Schwinger terms that we need [6], and get the anomalous Ward identity of the divergence of the left-hand current. Finally, we obtain the Ward identity of the left-hand current, which is consistent with the perturbative theory in terms of the non-perturbative method. For simplicity, we make the calculation in two-dimensional space-time, but it is not difficult to extend it to the four-dimensional space-time. In this paper, we define the γ matrix as

$$\gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \gamma_5 = -i\gamma_1\gamma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1.6}$$

In Section 2, we will analyze the constraint problems in the pure coset gauge field theory according to Dirac's theory on the generalized Hamiltonian system [7], and obtain the useful Feynman rules. In Section 3, we will perform the perturbative calculation of the chiral anomaly in the theory by the BJL technique. In Section 4, we will make the corresponding non-perturbative calculation of the chiral anomaly, and obtain the anomalous Ward identity which is consistent with the perturbative calculation.

2. CONSTRAINT CONDITIONS AND FEYNMAN RULES

We parametrize the Higgs field as follows

$$\phi = \rho^{i\xi}, \phi^+ = \rho e^{-i\xi} \tag{2.1}$$

so that the Lagrangian density Eq.(1.4) can be written as

$$\mathscr{L}^{(B)} = -\frac{1}{4}F_{\mu\nu}^2 - \bar{\psi}\gamma_\mu\left(\partial_\mu - ieA_\mu\frac{1+\gamma_5}{2} - i\partial_\mu\theta\frac{1-\gamma_5}{2}\right)\psi - g\rho\bar{\psi}e^{-i\xi\gamma_5}\psi$$
$$-(\partial_\mu + ieA_\mu - i\partial_\mu\theta - i\partial_\mu\xi)\rho(\partial_\mu - ieA_\mu + i\partial_\mu\theta + i\partial_\mu\xi)\rho - V(\rho^2). \qquad (2.2)$$

It is not difficult to show that Eq.(2.2) is invariant under the gauge transformation

$$\psi' = e^{i\alpha\frac{1+\gamma_5}{2}}e^{i\beta\frac{1-\gamma_5}{2}}\psi, \quad \bar{\psi}' = \bar{\psi}e^{-i\beta\frac{1+\gamma_5}{2}}e^{-i\alpha\frac{1-\gamma_5}{2}},$$
$$A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha, \quad \theta' = \theta + \beta,$$
$$\xi' = \xi + \alpha - \beta, \qquad \rho' = \rho. \qquad (2.3)$$

It was shown that the Higgs field does not affect the chiral anomaly [8], and the last equation in (2.3) shows that the radial part of the Higgs field is invariant under the transformation. So we can choose the frozen approximation, i.e., $\rho = \frac{\rho_0}{\sqrt{2}}$. Introducing

$$m = \frac{\rho_0 g}{\sqrt{2}}, \quad B = \rho_0\theta, \quad C = \rho_0\xi, \qquad (2.4)$$

we can rewrite Eq.(2.2) as

$$\mathscr{L}^{(B)} = -\frac{1}{4}F_{\mu\nu}^2 - \bar{\psi}\gamma_\mu\left(\partial_\mu - ieA_\mu\frac{1+\gamma_5}{2} - \frac{i}{\rho_0}\partial_\mu B\frac{1-\gamma_5}{2}\right)\psi - m\bar{\psi}e^{-i\frac{1}{\rho_0}C\gamma_5}\psi$$
$$-\frac{1}{2}(\rho_0 eA_\mu - \partial_\mu(B+C))^2 - V(\rho_0^2), \qquad (2.5)$$

Then we have the following canonical momenta

$$\pi_\mu = \frac{\partial\mathscr{L}^{(B)}}{\partial\dot{A}_\mu} = iF_{2\mu} \equiv -E_\mu, \quad \pi_\psi = \frac{\partial\mathscr{L}^{(B)}}{\partial\dot{\psi}} = i\psi^+,$$
$$\pi_B = \frac{\partial\mathscr{L}^{(B)}}{\partial\dot{B}} = \frac{1}{\rho_0}\psi^+\frac{1-\gamma_5}{2}\psi + \rho_0 eA_0 + \dot{B} + \dot{C},$$
$$\pi_C = \frac{\partial\mathscr{L}^{(B)}}{\partial\dot{C}} = \rho_0 eA_0 + \dot{B} + \dot{C}. \qquad (2.6)$$

It is easy to see that there are two original constraints in the theory:

$$\pi_2 = 0, \quad G_1 \equiv \pi_B - \pi_C - \frac{1}{\rho_0}\psi^+\frac{1-\gamma_5}{2}\psi = 0 \qquad (2.7)$$

and the Hamiltonian density can be written as

$$
\begin{aligned}
\mathscr{H} &= \pi_\psi \dot{\psi} + \pi_\mu \dot{A}_\mu + \pi_B \dot{B} + \pi_C \dot{C} - \mathscr{L}^{(B)} \\
&= \frac{1}{2} E_1^2 + \bar{\psi}\gamma_1 \left(\partial_1 - ieA_1 \frac{1+\gamma_5}{2} - \frac{i}{\rho_0} \partial_1 B \frac{1-\gamma_5}{2} \right) \psi + m\bar{\psi} e^{-\frac{i}{\rho_0} C\gamma_5} \psi \\
&\quad + \frac{1}{2} \pi_c^2 + \frac{1}{2} (e\rho_0 A_1 - \partial_1 B - \partial_1 C)^2 \\
&\quad + A_0 \left(-\partial_1 E_1 + e\psi^+ \frac{1+\gamma_5}{2} \psi - e\rho_0 \pi_c \right).
\end{aligned}
\tag{2.8}
$$

where we ignore the constant $V(\rho_0^2)$. According to Dirac's theory on a generalized Hamiltonian system, the original constraints should satisfy the self-consistent condition as follows

$$
\dot{\pi}_2 = [\pi_2, H]_{\text{P.B.}} = \left[\pi_2, \int dx \mathscr{H} \right]_{\text{P.B.}} \approx 0, \tag{2.9}
$$

where "≈" means a loose zero. The basic Poisson brackets can be defined as

$$
\begin{aligned}
&[\pi_\mu(x,t), A_\nu(x',t)]_{\text{P.B.}} = [A_\nu(x',t), E_\mu(x,t)]_{\text{P.B.}} = -\delta_{\mu\nu}\delta(x-x'), \\
&[\psi(x,t), \psi^+(x',t)]_{\text{P.B.}} = -i\delta(x-x'), \\
&[B(x,t), \pi_B(x',t)]_{\text{P.B.}} = [C(x,t), \pi_c(x',t)]_{\text{P.B.}} = \delta(x-x').
\end{aligned}
\tag{2.10}
$$

A secondary constraint is obtained from Eq.(2.6):

$$
G = -\partial_1 E_1 + e\psi^+ \frac{1+\gamma_5}{2} \psi - e\rho_0 \pi_c = 0. \tag{2.11}
$$

Furthermore, we are unable to acquire new constraints from the self-consistent conditions

$$
\dot{G}_1 = [G_1, H]_{\text{P.B.}} = 0, \quad \dot{G} = [G, H]_{\text{P.B.}} = 0
$$

but we have

$$
[\pi_2, G]_{\text{P.B.}} = [\pi_2, G_1]_{\text{P.B.}} = [G, G_1]_{\text{P.B.}} = 0. \tag{2.12}
$$

Therefore, there are two original constraints and one secondary constraint in this theory. They are all first-class constraints.

Combining these constraints, we get the Hamiltonian as follows:

$$
H_T = \int dx (\mathscr{H} + \lambda G + \lambda_1 G_1 + \lambda_2 \pi_2), \tag{2.13}
$$

where λ, λ_1 and λ_2 are arbitrary functions of the field quantities and conjugate momenta. Because our discussion is under the temporal gauge and the physical gauge

$$
A_2 = iA_0 = 0, C = 0, \tag{2.14}
$$

we can ignore $\lambda_2\pi_2$ in Eq.(2.13), and the secondary constraint Eq.(2.11) can be written as

$$G = -\partial_1 E_1 + e\psi^+ \frac{1+\gamma_5}{2}\psi - e\rho_0 \dot{B} = 0. \tag{2.15}$$

This also can be seen from the equation of motion of the gauge field. In fact, the equation of motion of the gauge field can be obtained from Eq.(2.5) as follows

$$\partial_\mu F_{\mu\nu} = -ie\bar{\psi}\gamma_\nu \frac{1+\gamma_5}{2}\psi + e\rho_0(e\rho_0 A_\nu - \partial_\nu(B+C)), \tag{2.16}$$

and the gauge condition (2.14) with $\nu = 2$ leads to Eq.(2.15).

In order to obtain the Feynman rules in the perturbative calculation, the Hamiltonian Eq.(2.8) is divided into two parts

$$H = \int dx \mathscr{H} = \int dx(\mathscr{H}_0 + \mathscr{H}_i), \tag{2.17}$$

where

$$\mathscr{H}_0 = \frac{1}{2}E_1^2 + \frac{1}{2}\mu^2 A_1^2 - A_0\partial_1 E_1 + \bar{\psi}(\gamma_1\partial_1 + m)\psi + \frac{1}{2}\pi_B^2 + \frac{1}{2}(\partial_1 B)^2,$$

$$\mathscr{H}_i = -ieA_\mu\bar{\psi}\gamma_\mu \frac{1+\gamma_5}{2}\psi - \frac{i}{\rho_0}\partial_\mu B\bar{\psi}\gamma_\mu\frac{1-\gamma_5}{2}\psi - e\rho_0 A_\mu\partial_\mu B. \tag{2.18}$$

where $\mu = e\rho_0$, and the normal term is ignored [9].

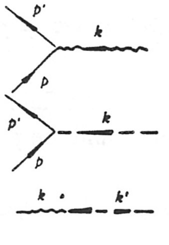
$$-eA_0\psi^+\frac{1-\gamma_5}{2}\psi - e^2\rho_0^2 A_0^2 - \frac{1}{2\rho_0^2}\left(\psi^+\frac{1-\gamma_5}{2}\psi\right)^2.$$

We obtain the Feynman rules as follows. For the vertex, we have

$$-e\gamma_\mu\frac{1+\gamma_5}{2}(2\pi)^2\delta(p'-p-k), \quad -\frac{i}{\rho_0}k_\mu\gamma_\mu\frac{1-\gamma_5}{2}(2\pi)^2\delta(p'-p-k), \tag{2.19}$$

$$e\rho_0 k_\mu(2\pi)^2\delta(k'-k),$$

where the solid lines stand for the Fermi field, the waved lines the gauge field, and the broken lines the pseudoscalar field (the pure coset gauge field).



In addition, the propagator of the spinor field is

$$S_F(p) = \frac{-1}{\hat{p} - im - i\varepsilon}, \tag{2.20}$$

The propagator of the pseudoscalar field (the pure coset gauge field) is

$$\Delta_F(k) = \frac{-i}{k^2 - i\varepsilon}, \tag{2.21}$$

It is not difficult to obtain the propagator of the gauge field A_μ under the temporal gauge:

$$D_{\mu\nu}(k) = \frac{-i}{k^2 + \mu^2 - i\varepsilon}\left\{\delta_{\mu\nu} - \frac{(k\cdot n)(k_\mu n_\nu + k_\nu n_\mu) + \mu^2 n_\mu n_\nu}{(k\cdot n)^2 + \mu^2} + \frac{k_\mu k_\nu}{(k\cdot n)^2 + \mu^2}\right\}. \tag{2.22}$$

where $n_\mu = (n_1, n_2) = (0, 1)$, $k \cdot n = k_\mu n_\mu$.

3. PERTURBATIVE CALCULATION OF THE CHIRAL ANOMALY

Under the temporal gauge $A_0 = 0$,

$$E_1 = A_1, \tag{3.1}$$

the Hamiltonian can be written as

$$H_T = H_1 + H_2 + H_3 + H_4, \tag{3.2}$$

where

$$\begin{aligned} H_1 &= \int dx\left(-\frac{1}{2}E_1^2 + \frac{1}{2}\mu^2 A_1^2 - ieA_1\bar{\psi}\gamma_1\frac{1+\gamma_5}{2}\psi - \frac{i}{\rho_0}\partial_\mu B\bar{\psi}\gamma_\mu\frac{1-\gamma_5}{2}\psi + m\bar{\psi}\psi\right), \\ H_2 &= \int dx\left(\frac{1}{2}\pi_B^2 + \frac{1}{2}(\partial_1 B)^2 - e\rho_0 A_1\partial_1 B\right), \\ H_3 &= \int dx\bar{\psi}\gamma_1\partial_1\psi, \quad H_4 = \int dx\lambda G, \end{aligned} \tag{3.3}$$

After quantization, the following terms become operators:

$$j_\mu^L = i\bar{\psi}\gamma_\mu\frac{1+\gamma_5}{2}\psi, \quad j_\mu^R = i\bar{\psi}\gamma_\mu\frac{1-\gamma_5}{2}\psi, \quad (j_2^{L,R} = ij_0^{L,R}) \tag{3.4}$$

In order to obtain the Ward identity of the divergence of the left-hand current in the theory, we consider

$$-i\dot{G} = [H_T, G], \tag{3.5}$$

where H_T is given by Eq.(3.2). In the following, we are going to calculate the equal-time commutators of the operators in Eq.(3.5) by the BJL technique. In general, we define the matrix elements of two operators A and B in the time-ordered product as

$$T(p) = \int d^2x e^{ipx}\langle\alpha|TA(x)B(0)|\beta\rangle, \tag{3.6}$$

where $p = (p_1, ip_0)$, $x = (x, it)$ are the momentum and coordinate in two dimensions, respectively, and $d^2x = dxdt$. $|\alpha\rangle$ and $|\beta\rangle$ are the orthonormal bases. Consider the following limit

$$\lim_{p_0\to\infty} p_0 T(p) = -i\int dx e^{ip_1 x}\langle\alpha|[A(x,0)B(0,0)]|\beta\rangle,$$

According to the BJL method, we have

$$[A(x,0),B(0,0)] = \begin{cases} i\delta(x) & \text{if } \lim_{p_0\to\infty} p_0 T(p) = 1 \\ \frac{\partial}{\partial x}\delta(x), & \text{if } \lim_{p_0\to\infty} p_0 T(p) = p_1. \end{cases} \tag{3.7}$$

and

$$[\dot{A}(x,0),B(0,0)] = \begin{cases} \delta(x) & \text{if } \lim_{p_0\to\infty} p_0^2 T(p) = 1 \\ -i\frac{\partial}{\partial x}\delta(x) & \text{if } \lim_{p_0\to\infty} p_0^2 T(p) = p_1. \end{cases} \tag{3.8}$$

Now we calculate some commutators in $[H_T, G]$. We first consider $[j_\mu^L(x), j_\mu^L(y)]$. In the Heisenberg picture we take

$$T_{\mu\nu}(p) = \int d^2x e^{ipx}\langle 0\|Tj_\mu^L(x)j_\nu^L(0)\|0\rangle, \tag{3.9}$$

and in the interaction picture, we have

$$T_{\mu\nu}(p) = \int d^2x e^{ipx}\langle 0|Tj_\mu^L(x)j_\nu^L(0)S|0\rangle. \tag{3.10}$$

For the processes shown in Figs. 1-6, we only need to regard matrix S as 1, and obtain

$$T_{\mu\nu}(p) = \int d^2x e^{ipx}\mathrm{Tr}S_F(-x)\gamma_\mu\frac{1+\gamma_5}{2}S_F(x)\gamma_\nu\frac{1+\gamma_5}{2}, \tag{3.11}$$

where

$$S_F(x) = \frac{1}{(2\pi)^2}\int d^2k e^{ikx}\frac{-1}{\not{k} - im - i\varepsilon}. \tag{3.12}$$

It is easy to obtain

$$T_{\mu\nu}(p) = \frac{i}{4\pi}\frac{\delta_{\mu\nu}p^2 - 2p_\mu p_\nu - i\varepsilon_{\mu\nu}p^2 - 2i\varepsilon_{\lambda\nu}p_\mu p_\lambda}{p^2}\int_0^1 dx\frac{x(1-x)}{\frac{m^2}{p^2} + x(1-x)}. \tag{3.13}$$

In the limit $p_0 \to \infty$, we have

$$\int_0^1 dx\frac{x(1-x)}{\frac{m^2}{p_1^2 - p_0^2} + x(1-x)}\Bigg|_{p_0\to\infty} = 1,$$

By using Eq.(3.7), we obtain

$$[j_1^L(x), j_1^L(y)] = [j_1^L(x), j_0^L(y)] = [j_0^L(x), j_0^L(y)] = -2\delta'(x-y)k, \tag{3.14}$$

where $k = -i/4\pi$, $\delta'(x-y) \equiv \partial/\partial y\ \delta(x-y)$. The right-hand side of (3.14) is the so-called Schwinger term.

Similarly, we have the following equal-time commutation relations:

$$[j_\mu^L(x), j_\nu^R(y)] = 0, \tag{3.15}$$

$$[\bar{\psi}(x)\psi(x), j_\mu^{L,R}(y)] = 0, \tag{3.16}$$

$$[j_1^L(x), \partial_1 E_1(y)] = [j_0^L(x), \partial_1 E_1(y)] = -e\delta'(x-y)k, \tag{3.17}$$

$$[j_\mu^R(x), \partial_1 E_1(y)] = 0, \tag{3.18}$$

$$[j_1^L(x), E_1(y)] = [j_0^L(x), E_1(y)] = -e\delta(x-y)k. \tag{3.19}$$

and

$$[\partial_1 E_1(x), \bar{\psi}(y)\psi(y)] = 0, \quad [j_\mu^L(x), \partial_\nu B(y)] = 0, \quad [j_\mu^L(x), \pi_B(y)] = 0. \tag{3.20}$$

Since $[j_\mu^R(x), \dot{B}(y)] \neq 0$, the commutators of this form contribute to $[H_T, G]$. By using Eq.(2.15), the left-hand side of (3.5) can be rewritten as follows

$$\begin{aligned} -i\partial_0 G &= -i\partial_0(-\partial_1 E_1 + ej_0^L - e\rho_0 \dot{B}) \\ &= -ie\partial_0 j_0^L - \partial_1[H_T, E_1] - e\rho_0[H_T, \dot{B}], \end{aligned} \tag{3.21}$$

It is easy to see that there is also a contribution of $[j_\mu^R(x), \dot{B}(y)]$ in the right-hand side of (3.21). Since the term $-i\partial_0 G$ calculated from Eq.(3.5) and from Eq.(3.21) are equal to each other, when we collect these two results, the contributions from $[j_\mu^R(x), \dot{B}(y)]$ offset each other. From Eqs.(3.14-3.20) and Eq.(3.5) we have

$$-i\dot{G} = [H_T, G(x)] = -\frac{ie^2}{4\pi}(E_1(x) + \partial_1 A_1(x)), \tag{3.22}$$

where $\partial_1 = \partial/\partial x$, and we have ignored the contribution of $[H_T, -e\rho_0\dot{B}(x)]$. On the other hand, from Eq.(3.21) we also have

$$-i\dot{G} = -ie\partial_\mu j_\mu^L + \frac{ie^2}{4\pi}\partial_1(A_1 - \lambda), \tag{3.23}$$

where we have also ignored the contribution of $[H_T, -e\rho_0\dot{B}(y)]$. Therefore we have

$$\partial_\mu j_\mu^L(x) = \frac{-e}{4\pi}(\partial_0 A_1(x) + C\partial_1 A_1(x)), \tag{3.24}$$

where C is an arbitrary function. Suppose $C = 0$, we obtain the usual Ward identity of the divergence of the left-hand current

$$\partial_\mu j_\mu^L(x) = \frac{-e}{4\pi}\partial_0 A_1(x). \tag{3.25}$$

4. NON-PERTURBATIVE CALCULATION OF THE CHIRAL ANOMALY

In the previous section, we chose the frozen approximation under the physical gauge, and obtained the anomalous Ward identity of the left-hand current divergence through the perturbative calculation. The same result can be obtained by non-perturbative calculation. To do so, we write the generating functional of the theory as follows:

$$Z[J] = \int [d\psi d\bar{\psi} dA_\mu d\theta] \exp\left\{i\int d^2x(\mathscr{L} + j_\mu A_\mu + j\theta + \bar{\eta}\psi + \bar{\psi}\eta)\right\}, \tag{4.1}$$

where j_μ, j, $\bar{\eta}$ and η are the external sources, which correspond to the fields A_μ, θ, ψ and $\bar{\psi}$, respectively. Because the Higgs field is no longer a varying quantity when we choose the frozen approximation under the physical gauge, it can be ignored in the integral measure. In addition, since the Higgs field does not enter the chiral anomaly, for simplicity, in Eq.(4.1) we take

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}^2 - \bar{\psi}\gamma_\mu\left(\partial_\mu - ieA_\mu\frac{1+\gamma_5}{2} - i\partial_\mu\theta\frac{1-\gamma_5}{2}\right)\psi. \tag{4.2}$$

Now we perform the left-hand chiral transformation in Eq.(4.1).

$$\psi' = e^{i\alpha\frac{1+\gamma_5}{2}}\psi, \quad \bar{\psi}' = \bar{\psi}e^{-i\alpha\frac{1-\gamma_5}{2}}, \tag{4.3}$$

where α is a parameter of the local transformation, and the corresponding change in the path integral measure of the Fermi field is [4]

$$[d\psi d\bar{\psi}] = [d\psi' d\bar{\psi}']e^{i\mathrm{Tr}\alpha\gamma_5}, \tag{4.4}$$

where $\mathrm{Tr}\alpha\gamma_5$ is the so-called chiral phase factor. To calculate $\mathrm{Tr}\alpha\gamma_5$, we write the covariant derivative in the theory as

$$\hat{D} = \hat{\partial} - ie\hat{A}\frac{1+\gamma_5}{2} - i\hat{\partial}\theta\frac{1-\gamma_5}{2} = \hat{D}_L\frac{1+\gamma_5}{2} + \hat{D}_R\frac{1-\gamma_5}{2}, \tag{4.5}$$

where

$$\hat{D}_L = \hat{\partial} - ie\hat{A} = \gamma_\mu(\partial_\mu - ieA_\mu), \quad \hat{D}_R = \hat{\partial} - i\hat{\partial}\theta = \gamma_\mu(\partial_\mu - i\partial_\mu\theta). \tag{4.6}$$

and the chiral phase factor $\mathrm{Tr}\alpha\gamma_5$ has the following form

$$\mathrm{Tr}\alpha\gamma_5 = \lim_{M\to\infty}\frac{1}{2}\int d^2x\alpha(x)\mathrm{tr}\gamma_5\left[\left(e^{\frac{i}{M^2}\hat{D}_L\hat{D}_R} + e^{\frac{i}{M^2}\hat{D}_R\hat{D}_L}\right)\delta(x-x')\right]\Big|_{x'=x}.$$

and we obtain

$$\mathrm{Tr}\alpha\gamma_5 = \frac{ie}{8\pi}\int d^2x\alpha\varepsilon_{\mu\nu}F_{\mu\nu}. \tag{4.7}$$

Under the transformation (4.3), Eq.(4.1) becomes

$$Z[J] = \int[d\psi d\bar{\psi}dA_\mu d\theta]\exp\Big\{i(\mathscr{L} + j_\mu A_\mu + j\theta + \bar{\eta}\psi + \bar{\psi}\eta - i\partial_\mu\alpha \cdot \bar{\psi}\gamma_\mu\frac{1+\gamma_5}{2}\psi + i\bar{\eta}\alpha\frac{1+\gamma_5}{2}\psi - i\alpha\bar{\psi}\frac{1-\gamma_5}{2}\eta - \frac{ie}{8\pi}\alpha\varepsilon_{\mu\nu}F_{\mu\nu})\Big\}. \tag{4.8}$$

Because

$$\int d^2x i\partial_\mu\alpha\cdot\bar{\psi}\gamma_\mu\frac{1+\gamma_5}{2}\psi = -\int d^2x\alpha\partial_\mu\left(i\bar{\psi}\gamma_\mu\frac{1+\gamma_5}{2}\psi\right),$$

Eq.(4.8) becomes

$$Z[J] = \int[d\psi d\bar{\psi}dA_\mu d\theta]\exp\Big\{i(\mathscr{L} + j_\mu A_\mu + j\theta + \bar{\eta}\psi + \bar{\psi}\eta + \alpha\partial_\mu\left(i\bar{\psi}\gamma_\mu\frac{1+\gamma_5}{2}\psi\right) + i\bar{\eta}\alpha\frac{1+\gamma_5}{2}\psi - i\alpha\bar{\psi}\frac{1-\gamma_5}{2}\eta - \frac{ie}{8\pi}\alpha\varepsilon_{\mu\nu}F_{\mu\nu})\Big\}. \tag{4.9}$$

We take the functional derivative with respect to α in both sides of Eq.(4.9), and then by letting $\alpha \to 0$, we obtain

$$\int[d\psi d\bar{\psi}dA_\mu d\theta]\exp\Big\{i\int d^2x(\mathscr{L} + j_\mu A_\mu + j\theta + \bar{\eta}\psi + \bar{\psi}\eta)\Big\} \cdot\left[\partial_\mu\left(i\bar{\psi}\gamma_\mu\frac{1+\gamma_5}{2}\psi\right) - \frac{ie}{8\pi}\varepsilon_{\mu\nu}F_{\mu\nu} + i\bar{\eta}\frac{1+\gamma_5}{2}\psi - i\bar{\psi}\frac{1-\gamma_5}{2}\eta\right] = 0. \tag{4.10}$$

By taking the functional derivatives $\delta/i\delta\eta(x)$, $\delta/i\delta\bar{\eta}(x)$ with respect to the external sources η and $\bar{\eta}$ in Eq.(4.10), and setting all the external sources to zero, we obtain the Ward identity of the left-hand current.

$$\partial_\mu\langle T\psi(z)j_\mu^L(x)\bar{\psi}(z')\rangle_0 = \frac{-e}{4\pi}\langle T\psi(z)\partial_0A_1(x)\bar{\psi}(z')\rangle_0 + \delta(x-z')\left\langle T\psi(z)\bar{\psi}(x)\frac{1-\gamma_5}{2}\right\rangle_0 - \delta(x-z)\left\langle T\frac{1+\gamma_5}{2}\psi(x)\bar{\psi}(z')\right\rangle_0. \tag{4.11}$$

It is easy to see that this result coincides with Eq.(3.25), which is obtained in the coordinate representation when $C = 0$.

In sum, in this paper we choose the frozen approximation under the physical gauge, and study the perturbative approach of the pure coset gauge field theory. We work out the explicit expressions for the propagator of the pure coset gauge field and the propagator of the massive gauge field in the temporal axial gauge. By using the BJL technique, we obtain the anomalous Ward identity of the left-hand current, which is consistent with that obtained in the non-perturbative calculation. The chiral group under investigation in this paper is abelian. The generalization of the results obtained to the non-abelian case is very interesting. Meanwhile, it is necessary to solve the quantization problem of the gauge theory with anomaly. Such work is in progress.

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