

The Polarization States of the Gluon and the Glueball Interpretation of the ξ (2230)

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The glueball interpretation of the $\xi(2230)$ is discussed when the gluon has only the transverse polarization components.

1. INTRODUCTION

The existence of the glueball, which is a color singlet composed by two or more gluons, is a direct consequence of the quantum chromodynamics. Experimentally, several glueball candidates, such as $\iota/\eta(1440)$ [1], $\theta/f_2(1720)$ [2], $G/f_0(1590)$ [3], $\xi(2230)$ [4] and three types of g_T state [5], have been discovered.

For a valence gluon bound in a glueball, we only know that its spin is 1. But it is not yet clear if it should be treated as a massive particle or a massless one. A related problem is whether the gluon possesses three polarization states or remains only two transverse polarization states as a photon. In the framework of a nonrelativistic potential model [6], the gluon is a massive particle with spin 1. The dynamic mass is generated through strong gluon-binding force. D. Robson also pointed out that as gluons are not on their mass shell when they are confined in a hadron, they should be treated as massive particles [7]. In other words, we must consider all three types of polarization state. But Barnes proposed that the gauge invariance allows only two degrees of freedom, i.e., two transverse polarization modes for each gluon color, and that other polarization modes are of gauge-dependent, non-physical degrees of freedom [8]. Therefore, there is a difference in principle between [8] and [6]. In the MIT bag model, the gluons are regarded as massless, thus there are only two transverse polarization degrees of freedom [9]. However, by using the effective Lagrangian method, Ward

discussed the possibility that the $\xi(2230)$ is considered a glueball and proposed that the transverse magnetic (TM) gluon is massive [10]. Therefore, there is no consistent answer to the question whether there are three types of polarization state or only two types of transverse polarization state for the gluon bound in a glueball. It is a very interesting question how much the glueball theory of the $\xi(2230)$ will be affected when different methods are used.

We formerly calculated the ratios of the helicity amplitudes x and y for the $\xi(2230)$ [11]. By comparing the theoretical value with experimental data, the following conclusions were reached: (1) the $\xi(2230)$ does not seem to be a 2^{++} glueball in a pure S wave (or pure D wave), but may be a 2^{++} glueball in a mixture of the S wave and two D waves, and (2) if $\xi(2230)$ is a 4^{++} glueball in a pure D wave, the theoretical values of x and y are in excellent agreement with the data. In the calculation we assumed, as did Robson [7], that the gluon possesses three types of polarization state.

In this paper, we use the MIT bag model and assume that the gluons retain only two transverse polarization modes. On this basis, the ratios x and y of the helicity amplitudes for the $\xi(2230)$ are calculated and a further test of the glueball theory of the $\xi(2230)$ is conducted. We find that the theoretical values of x and y are quite different with different treatment for the polarization state of the gluon. We find that when the gluon possesses only two transverse polarization modes, $x = 0$ for a 2^{++} gluon in the S wave with $s = 2$ and $l = 0$, $y = 0$ for a 2^{++} glueball in D wave with $s = 0$ and $l = 2$, and both $|x|$ and $|y|$ are much larger than the experimental data if the $\xi(2230)$ is a 4^{++} glueball in the D wave. Consequently, the possibility that the $\xi(2230)$ is a 4^{++} glueball in the D wave is ruled out. These results are the same as that when the gluon is assumed to have three polarization modes.

2. NUMERICAL CALCULATIONS

In 1983, the Mark III Collaboration discovered a new resonance state $\xi(2230)$ [4]. However, it could not be determined whether the spin of the $\xi(2230)$ is 2 or 4. The ratios of the helicity amplitudes for the $\xi(2230)$ have been measured [12]. They are

$$x = -0.67^{+0.14}_{-0.16}, \quad y = 0.13^{+0.21}_{-0.19}. \quad (1)$$

if J , the spin of ξ , is equal to 2 and

$$x = 1.29^{+0.62}_{-0.30}, \quad y = 0.40^{+0.76}_{-0.39}. \quad (2)$$

if $J = 4$. After discovering the ξ particle, various possible theoretical interpretations for the ξ have been proposed by physicists. One of them is the glueball interpretation [10].

Since the Mark III Collaboration discovered the ξ particle only in the K^+K^- and $K_s^0\bar{K}_s^0$ channels, if the ξ is a glueball composed by two gluons, these gluons may form a transverse magnetic (TM) glueball with spin-parity $J^{PC} = 1^{--}$. The reason, as proposed by M. Chanowitz [13], is that the TM gluon mode couples predominately to a pair of strange quarks ss , hence the glueball made of two TM gluons will decay predominately to the final state $K\bar{K}$, but not the $\pi\pi$ final state.

For a 2^{++} glueball in the S wave (or D wave) or a 4^{++} glueball in the D wave, which is composed by two TM gluons, the forms of the wave functions are completely analogous to the ones given in Ref. [11], except for a re-treatment of normalization. Now, the polarization vector e^m of the component gluon has only two transverse polarization modes, i.e.,

$$e^1 = \left(0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right), \quad e^{-1} = \left(0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right). \quad (3)$$

By using the calculation method given by Qi and Hong [11], the helicity amplitudes for various glueballs can be obtained. (we continue to use the symbols utilized by Qi and Hong [11].) For the 2^{++} glueball in the S wave with $l = 0$ and $s = 2$, we obtain the following helicity amplitudes T_λ :

$$\begin{aligned} T_2 &= -\frac{32}{3\sqrt{3}} g^2 G(0) \phi_J(0) \frac{\sqrt{m_J}}{m_c^2} \left\{ 1 - \frac{m_J^2 - m_G^2}{2m_c m_J} \right\}, \\ T_1 &= 0, \\ T_0 &= -\frac{64\sqrt{3}}{9\sqrt{2}} g^2 G(0) \phi_J(0) \frac{\sqrt{m_J}}{m_c^2} \frac{1}{m_c^2 + \frac{1}{4} m_J^2 - \frac{1}{2} m_G^2} \\ &\quad \times \left[m_c^2 + \frac{1}{8} (m_J^2 - m_G^2) - \frac{m_c}{4m_J} (m_J^2 + m_G^2) \right]. \end{aligned} \quad (4)$$

For the 2^{++} glueball in the D wave with $l = 2$ and $s = 0$, we have

$$\begin{aligned} T_2 &= 0, \\ T_1 &= -\frac{64}{3\sqrt{3}} g^2 G(0) \phi_J(0) m_G^2 \sqrt{m_J} \frac{E_J}{m_c^4 m_J}, \\ T_0 &= \frac{32}{9} g^2 G(0) \phi_J(0) m_G^2 \sqrt{m_J} \frac{p_J^4}{m_c^4} \frac{1}{m_c^2 + \frac{1}{4} m_J^2 - \frac{1}{2} m_G^2} \\ &\quad \times \left[2m_c^2 - \frac{4m_c^2}{m_J} + m_G p_J - \frac{2m_G m_c}{m_J} E_J \right]. \end{aligned} \quad (5)$$

For the 2^{++} glueball in the D wave with $l = 2$ and $s = 2$, we obtain

$$\begin{aligned} T_2 &= \frac{64}{9} g^2 G(0) \phi_J(0) \frac{m_G^2 \sqrt{m_J}}{m_c^4} \left[1 + \frac{p_J^2}{m_c^2} \left(1 - \frac{m_G p_J}{m_J m_c} \right) \right], \\ T_1 &= \frac{128}{3\sqrt{7}} g^2 G(0) \phi_J(0) \frac{m_G^2}{m_c^4 \sqrt{m_J}} \left[\frac{p_J^2 m_G}{m_c^2 + \frac{1}{4} m_J^2 - \frac{1}{2} m_G^2} + \frac{E_J}{2\sqrt{3}} \right], \\ T_0 &= \frac{128}{3\sqrt{3}} g^2 G(0) \phi_J(0) \frac{m_G^2 \sqrt{m_J}}{m_c^4} \left\{ \frac{1}{\sqrt{3}} + \frac{1}{3} \frac{p_J^2}{m_c^2 + \frac{1}{4} m_J^2 - \frac{1}{2} m_G^2} \right. \\ &\quad \left. \left[-\frac{1}{2} + \frac{m_c}{m_J} - \frac{m_G p_J}{4m_c^2} + \frac{m_G E_J}{2m_c m_J} \right] \right\}. \end{aligned} \quad (6)$$

Finally, for a 4^{++} glueball in the D wave with $l = 2$ and $s = 2$, the helicity amplitudes T_λ can be

Table 1
Change of theoretical values corresponding to glueball.

$m_c(\text{GeV})$	$l = 0, s = 2$		$l = 2, s = 0$		$l = 2, s = 2$	
	x	y	x	y	x	y
1.1	0	0.52	2.1	0	0.90	0.52
1.3	0	0.55	2.8	0	0.75	0.54
1.5	0	0.72	3.6	0	0.65	0.54
1.7	0	0.76	4.4	0	0.59	0.54

written as

$$\begin{aligned}
 T_2 &= \frac{64}{9} g^2 G(0) \psi_J(0) \frac{m_G^2 \sqrt{m_J}}{m_c^4} \left\{ 1 + \frac{p_J^2}{m_c^2} \left(1 - \frac{m_G p_J}{m_c m_J} \right) \right\}, \\
 T_1 &= \frac{128}{3\sqrt{42}} g^2 G(0) \psi_J(0) \frac{m_G^2}{m_c^4 \sqrt{m_J}} \left\{ \frac{p_J^2 m_G}{m_c^2 + \frac{1}{4} m_J^2 - \frac{1}{2} m_G^2} - \sqrt{3} E_J \right\}, \\
 T_0 &= \frac{128}{3\sqrt{114}} g^2 G(0) \psi_J(0) \frac{m_G^2 \sqrt{m_J}}{m_c^4} \left\{ 1 + \frac{\sqrt{3} p_J^2}{m_c^2 + \frac{1}{4} m_J^2 - \frac{1}{2} m_G^2} \right. \\
 &\quad \left. \times \left[1 - \frac{2m_c}{m_J} + \frac{m_G p_J}{2m_c^2} - \frac{m_G E_J}{m_c m_J} \right] \right\}.
 \end{aligned} \tag{7}$$

Therefore, the ratios of helicity amplitudes defined by

$$x = T_1/T_0, y = T_2/T_0 \tag{8}$$

include only the parameter m_c , the mass of charmed quark. In Table 1, we list the change of theoretical values of x and y with m_c corresponding to three types of 2^{++} glueball. For the D -Wave 4^{++} glueball, our numerical calculation shows that both the absolute values of x and y are larger than 3.8 in the m_c range of $1.1 \leq m_c \leq 1.7$ GeV.

3. DISCUSSION

A comparison between the theoretical values listed in Table 1 and experimental data (1) shows that when the gluon possesses only two types of transverse polarization state, the $\xi(2230)$ cannot be a pure S -wave (or pure D -wave) 2^{++} glueball. But we can use the method given by Qi and Hong [11] to mix the S wave and the D wave and to choose proper mixing parameters so that the theoretical values of x and y are in excellent agreement with the data. For example, fixing $m_c = 1.3$ GeV, we can choose $a = 0.088$ and $b = 0.097$. Therefore, particle ξ may be a 2^{++} glueball which is a mixture of the S wave and D waves. The above conclusion is the same as that deduced from the case where the gluon possesses three types of polarization state. The difference lies only in the values of mixing parameters.

If $\xi(2230)$ is a D -wave 4^{++} glueball, our calculation gives $|x|, |y| > 3.8$. This is inconsistent with the data (2), hence its possibility of being a D -wave glueball for the $\xi(2230)$ is ruled out. This conclusion contradicts the result deduced from the case where the gluon possesses three types of polarization state [11].

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