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Relationships Between the Gap at Z = 64 Subshell and the Abrupt Transition in the Region of N = 88 ~ 90

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The onset of deformation in the $N = 88 \sim 90$ region and the effects of the gap at the $Z = 64$ subshell on nuclear structure are investigated in a microscopic approach of the interacting boson model. Besides the n-p interaction, the distribution of single particle levels in the shell model space also is found to play a direct and important role in the rapid transition of the spectrum structure. For Nd ~ Dy nuclei, the calculations show excellent agreement with the experimental energy systematics.

1. INTRODUCTION

In the region of nuclei with neutron number $N = 88 \sim 90$, there has been extensive discussion on the mechanism of the abrupt transition of the spectrum from the vibrational to the rotational type. This problem remains of wide interest [1-5]. Based on the fact that $^{146}_{64}Gd_{82}$ has properties of the double-magic nucleus [6], R. F. Casten *et al.* studied the phase transition of nuclei with $A \sim 150$ in the interacting boson model. In their opinion, the major shell $Z = 50 \sim 82$ is divided into two subshells, $Z = 50 \sim 64$ and $Z = 64 \sim 82$, when $N \leq 88$; but because of the enhancement of the proton-neutron quadrupole interaction, this effect of subshell closure disappears when $N \geq 90$, and $Z = 50 \sim 82$ major shell is formed again [3,4]. In this analysis, two methods are adopted to count the proton boson number in the cases of $N \leq 88$ and $N \geq 90$. Thus the proton boson number N_p changes greatly from $N = 88$ to $N = 90$ for the isotopes with the same Z. For example, $N_p = 1$ for $^{150}_{62}Sm_{88}$

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and $N_p = 6$ for $^{150}_{62}Sm_{90}$. Their work shows that the abrupt transition can indeed be described by this approach.

Considering that the $Z = 64$ subshell effects indicate the existence of an energy gap between the two groups of proton single particle levels ($1g_{7/2}$, $2d_{5/2}$) and ($1h_{11/2}$, $2d_{3/2}$, $3s_{1/2}$), we believe that the energy gap should decrease as the number of neutrons increases [6]. Therefore, the work of Gill and Casten *et al.* [3] mainly shows whether the effects caused by the energy gap can be simulated by the change of boson numbers. As we know, under certain conditions, i.e., when the parameters in the IBM Hamiltonian are selected properly, the characteristics in the spectrum, which reflects a transition from the vibrational to the rotational modes, can be reproduced provided that the boson number increases [7]. It implies that under certain conditions the abrupt transition of the spectrum structure may be achieved by the sudden change of boson numbers. Of course, this is an oversimplified approach [3], and even its proponents pointed out that the approach cannot be used in describing Gd isotopes [3].

In the interacting boson modes, one starts with a number of valence nucleons outside closed major shells and the structure of low-lying levels is assumed to be dominated by excitations among these particles. This is one of the basic assumptions of the model. Instead of considering $Z = 64$ subshell as a major shell, to investigate the effects of the subshell energy gap on the spectrum structure seems much more direct because the subshell cannot be treated as a major shell in all cases. In this work, the subshell gap is taken into account explicitly so that one can ascertain how the abrupt transition depends on the change of the subshell gap. We will study the low-lying collective properties for the $A \sim 150$ nuclei from the shell model description. The shell model calculations are of course very difficult. Although some calculations have been made for studying the onset of deformations in this region, attention was focused on the influence of the n-p interaction on the nuclear deformation rather than the energy gap. One the other hand, if one works in one of the collective models, as did Casten *et al.*, the energy gap is still a parameter of the model. Therefore, we think that one of the microscopic approaches would be a good starting point to acquire the knowledge about the relationships between the abrupt transition and the subshell gap (or the distribution of single particle levels). Therefore, from among the proposed approaches to study the microscopic foundation of the IBM [8,9,10], we choose one which is based on the boson expansion theory and the so-called Modified Jancovici-Schiff (MJS) substitution [8] because it has been tested in some aspects. In order to investigate the above-mentioned relationships, besides theoretical qualitative analysis, the spectra for several sets of isotopes, including Gd nuclei, are calculated through this approach.

2. OUTLINE OF THE METHOD

The approach based on the boson expansion theory and the MJS substitution has been expounded previously [8,11,12]. In formulating the outline of the approach, we will emphasize some details related to the study in this paper.

The starting point of the microscopic investigation is the shell model configurations of the valence nucleons and the nucleon-nucleon interaction. For the nuclei with $A \sim 150$, the configuration can be written as

$$(1g_{7/2}, 2d_{5/2}, 1h_{11/2}, 2d_{3/2}, 3s_{1/2})^{n'}, \tag{2.1}$$

$$(2f_{7/2}, 1h_{9/2}, 3p_{3/2}, 1i_{13/2}, 2f_{5/2}, 3p_{1/2})^{N'}, \tag{2.2}$$

where $Z' = Z - 50$ is the number of valence protons and $N' = N - 82$ is the number of valence neutrons. If the valence nucleon number is more than half of the upper limit of the major shell, we will deal with the hole configurations instead of particles. In general, the Hamiltonian of the valence nucleons has the following form:

$$H_f = H_f^{(n)} + H_f^{(p)} + V_f^{(np)}, \tag{2.3}$$

$$H_f^{(\sigma)} = \sum_{\alpha} E_{\alpha}^{(\sigma)} a_{\alpha}^{(\sigma)+} a_{\alpha}^{(\sigma)} + \sum_{\alpha\beta\gamma\delta} P_{\alpha\beta\gamma\delta}^{(\sigma)} a_{\alpha}^{(\sigma)+} a_{\beta}^{(\sigma)+} a_{\gamma}^{(\sigma)} a_{\delta}^{(\sigma)}, \tag{2.4}$$

in which index σ = n and p is used to distinguish the degrees of freedom of valence neutrons and protons, respectively, and $V_f^{(np)}$ is the neutron-proton interaction.

First, let us consider the proton configurations (2.1). If the $Z = 64$ subshell is regarded as a major shell and the proton boson number is counted from it, the configurations studied become $(1g_{7/2}, 2d_{5/2})^{(64-Z)}$ or $(1h_{11/2}, 2d_{3/2}, 3s_{1/2})^{(Z-64)}$. The number of the proton bosons would be much less than that counted according to (2.1) for those nuclei near $Z = 64$. This corresponds to a truncation scheme of configuration space, and a part of the interaction related to the valence protons in Eq.(2.3) is ignored. Meanwhile, the subshell gap $\Delta = E_{1h_{11/2}} - E_{2d_{5/2}}$ affects at most the binding energy, but does not contribute to the excitation spectrum. Theoretically, these factors are all favorable to forming a vibrational spectrum. On the other hand, if the configurations (2.1) are studied, the subshell gap Δ appears as a characteristic quantity in the distribution of the five levels in 50 ~ 82 major shell. The effects of the gap will manifest itself in the single particle term of Eq.(2.4) and the dependence of the interaction on the energy levels. Therefore, as mentioned in the introduction, it is possible for us to investigate the interrelation between the gap Δ and the spectrum structure.

In the microscopic approach, the valence nucleon description of the system is first transformed into the ideal boson description. The boson image of the nucleon Hamiltonian is written as

$$H_B = H_B^{(n)} + H_B^{(p)} + V_B^{(np)}, \tag{2.5}$$

$$H_B^{(\sigma)} = H_{B1}^{(\sigma)} + H_{B2}^{(\sigma)}, \tag{2.6}$$

where $H_{B1}^{(\sigma)}$ and $H_{B2}^{(\sigma)}$ stand respectively for the one-body and two-body terms of the ideal bosons $A_{\alpha\beta}$. Then we introduce a set of Q-bosons whose creation operators are linear combinations of the ideal boson creation operators:

$$Q_{r\pi JM}^{(\sigma)+} = \sum_{\alpha \leqslant \beta} x_{\alpha\beta}^{(r\pi JM)} A_{\alpha\beta}^{(\sigma)+}. \tag{2.7}$$

In terms of the eigenvalue equation

$$H_{B1}^{(\sigma)} Q_{r\pi JM}^{(\sigma)+} |0) = \varepsilon_{r\pi J}^{(\sigma)} Q_{r\pi JM}^{(\sigma)+} |0) \tag{2.8}$$

the structural constant $x_{\alpha\beta}^{(r\pi JM)}$ in Eq.(2.7) can be found. Thus we can transform the Hamiltonian H_B

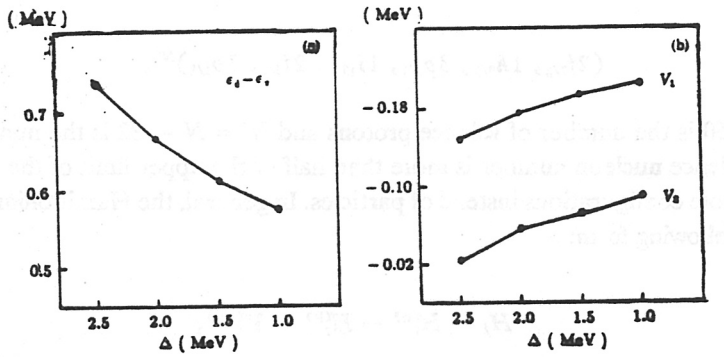

Fig. 1
$\varepsilon_d - \varepsilon_s$ role for the vibrational spectrum.

from the $\{A^+, A\}$ representation into the $\{Q^+, Q\}$ representation.

For the one-body term $H_{B1}^{(\sigma)}$ obtained by means of boson expansion, part of it comes from the single particle term of the valence nucleon Hamiltonian (2.4) directly, the rest comes from the interaction. This implies that the contribution of the nucleon-nucleon interaction has been taken into account in determining the Q-bosons according to Eqs.(2.7) and (2.8). So the lowest Q-bosons with $r = 0$ may have remarkable collectivity and can be used in describing the low-lying collective excitations. Therefore, by considering the angular momentum, parity and valence nucleon number carried by the bosons, in the lowest order approximation, the IBM s- and d-boson operators are defined as follows:

$$s^{(\sigma)+} = Q_{0+L0}^{(\sigma)+}, \tag{2.9}$$

$$d_m^{(\sigma)+} = Q_{0+2m}^{(\sigma)+}, \tag{2.10}$$

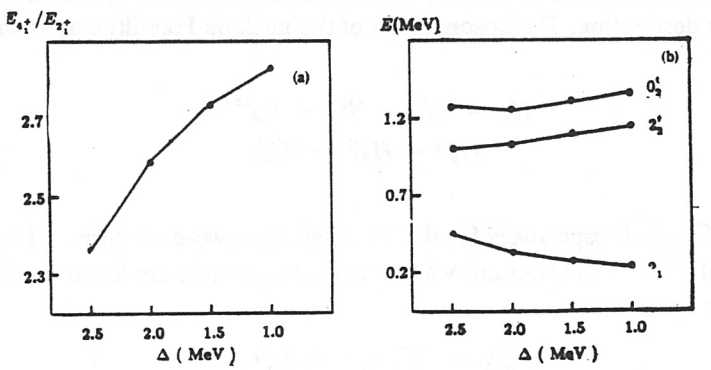

Fig. 2
Calculations resulting from the eigenvalue equation of the s-d Hamiltonian.

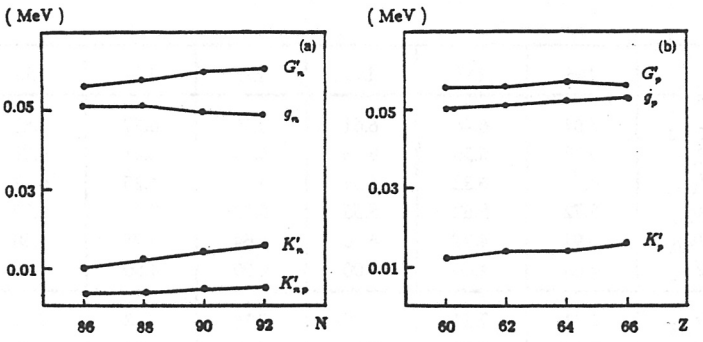

Fig. 3
Parameters of 16 nuclei.

Consequently, we can make the s-d truncation of the state space, i.e., a truncation to the collective subspace, and derive the effective operator of H_B in the s-d subspace:

$$h_{sd} = h^{(n)} + h^{(P)} + h^{(nP)} \tag{2.11}$$

which is just a general form of the microscopic IBM-2 Hamiltonian.

Eqs.(2.7)-(2.10) show that since both the single particle term and the interaction in H_f contribute to $H_{B1}^{(\sigma)}$, the distribution of the valence proton levels and the dependence of the interaction on the level distribution are important in determining the microscopic structure of the s, d bosons, and that their effects are manifested in the derived coefficients of h_{sd} in Eq.(2.11). In other words, the properties of the Hamiltonian h_{sd} depend in some ways on the value of the subshell gap Δ. This makes it possible to study the relationships between Δ and the structure of the spectrum by solving the eigenvalue equation of h_{sd}. In fact, the above qualitative conclusion does not depend on which microscopic approach is used. For example, if the study is based on the OAI model [9] or the approach proposed by Yang *et al.* [10], the definition of the s, d bosons are different, but the gap Δ still has its influence on the microscopic structure of the s, d bosons and the properties of the

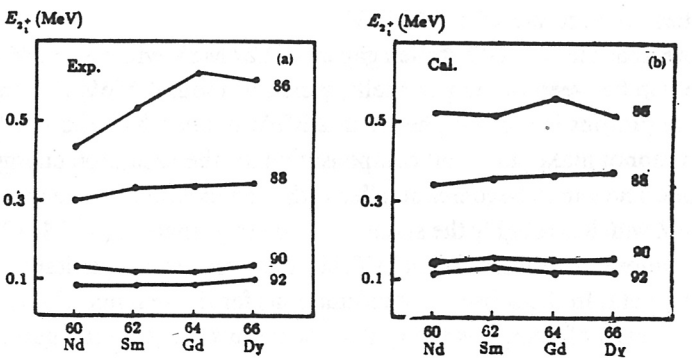

Fig. 4
Calculated results for 16 nuclei.

Table 1
The levels of valence nucleons.

A		146	148	150	152	154	156	158
Neutron	$3p_{1/2}$	6.64	6.46	6.61	6.58	6.57	6.68	6.79
	$2f_{5/2}$	6.59	6.39	6.50	6.44	6.41	6.39	6.37
	$1i_{13/2}$	6.52	6.32	6.39	6.30	6.25	6.02	6.17
	$3p_{3/2}$	5.72	5.62	5.65	5.73	5.75	5.77	5.79
	$1h_{9/2}$	4.91	4.77	4.70	4.64	4.75	4.50	4.43
	$2f_{7/2}$	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Proton	$3s_{1/2}$	7.41	7.34	7.35	7.34	7.28	7.31	7.36
	$2d_{3/2}$	7.18	7.00	7.01	7.00	6.91	6.90	6.92
	$1h_{11/2}$	6.95	6.89	6.87	6.84	6.73	6.70	6.68
	$2d_{5/2}$	4.68	4.73	4.80	4.89	4.91	4.95	5.10
	$1g_{7/2}$	4.00	4.00	4.00	4.00	4.00	4.00	4.00

Hamiltonian h_{sd}, provided that the valence proton configurations (2.1) are considered. Therefore, the effects of Δ may still be studied by investigating the properties of h_{sd} and the characteristics of the spectrum.

3. CALCULATED RESULTS AND DISCUSSIONS

It is certainly necessary to choose the single particle energy of the levels in the major shell and determine the form and strength of the interaction during the calculations of a real nucleus or the theoretical research through numerical calculations.

As for the choice of single particle energy of the valence shells, the energy of the lowest level for both neutrons and protons is unimportant since it contributes only to the binding energy rather than the execution spectrum. What is significant is the distribution of levels in the major shell. In this paper, since we concentrate on the effects of the proton subshell gap, not the single particle energy, we choose the level distribution obtained from the extrapolation of the Woods-Saxon potential [6]. The relationships between the valence nucleon number A and the level distribution are given in Table 1. The values are accurate within 10 keV.

Table 1 shows that the $Z = 64$ subshell gap $\Delta = 2.27$ MeV when $A = 146$. Although Δ is much smaller than the gap between the major shells, which is about 4 MeV, the unperturbed excitation energy of a pair of protons is much higher than 2 MeV, under which the low-lying state is located. If the interaction cannot make sufficient compensation for the excitation energy, the subshell effects will be remarkable. The gap Δ becomes smaller with the increase of nucleon number A. When $A = 158$, $\Delta \approx 1.58$ MeV, which is roughly the same as the average spacing (~1 MeV) between the valence proton levels. Therefore, the values shown in Table 1 are consistent qualitatively with those inferred from experimental data. In the subsequent calculations for the real nuclei, the values in Table 1 will be adopted as the levels of energy, while in theoretical investigation the gap Δ will vary within an appropriate range in order to study its effects.

For the nucleon-nucleon effective interaction, see previous work [8,11,12]. The interaction between identical nucleons consists of a pairing force, a quadrupole pairing force and a quadrupole-

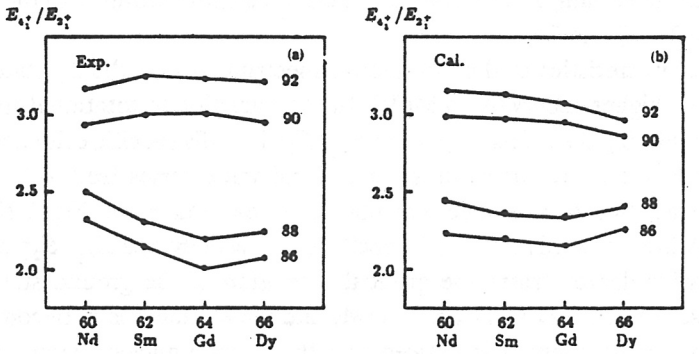

Fig. 5
Calculated results for 16 nuclei.

quadrupole force. The corresponding strength parameters are denoted respectively by g_σ, G'_σ and K'_σ. The neutron-proton interaction is considered quadrupole-quadrupole force whose strength parameter is K'_{np}. These parameters are fixed also according to the previous calculations [8,11,12]. However, the calculations by Liu *et al.* [12] show that the variation of the interaction strength within a relatively larger range may simulate those effects caused by several factors, such as the increase of the valence nucleon number, and consequently lead to a transition of the spectrum structure from the vibrational type to the rotational type. In order to reduce the influence of the variation of the interaction strength, the strength parameters in this paper are only allowed to vary smoothly within a small range with the change of nucleon numbers. Moreover, the parameters related to valence neutrons do not vary with the proton number Z and the parameters related to valence protons do not vary with the neutron number N. It will be convenient to analyze the calculated results.

It is our intention to make a theoretical investigation on the basis of some typical configurations. It is well known that the gadolinium isotope $^{146}_{64}Gd_{82}$ behaves as a doubly magic nucleus [13]. As the neutron number N increases from 82, the equilibrium shape of the gadolinium isotopes deviates rapidly from the spherical one. When $N = 90$, it becomes a well deformed nucleus [2]. On the other hand, in the work where the $Z = 64$ subshell is regarded as equivalent to a major shell, the Gd isotopes with $N \geq 82$ are magic nuclei. Consequently, they are excluded from the study [3]. Therefore, we believe that it is of great significance to study the typical configurations such as those of $^{152}_{64}Gd_{88}$. The parameters of the interaction are fixed in the unit of MeV as $g_n = 0.051$, $G'_n = 0.045$, $K'_n = 0.010$, $g_p = 0.042$, $G'_p = 0.060$, $K'_p = 0.014$, $K'_{np} = 0.010$. The subshell gap Δ decreases from 2.5 MeV, which may reflect the effects of the subshell closure, to 1 MeV, which is about the average spacing of the valence proton levels. The calculated results for the coefficients of the pure proton terms in the s-d Hamiltonian are given in Fig. 1, in which $\varepsilon_d - \varepsilon_s$ plays an important role for a vibrational spectrum, and V_1 and V_2 are the coefficients of the U(5)-symmetry-breaking interaction terms.

Fig. 1 shows that $\varepsilon_d - \varepsilon_s$ decreases gradually but V_1 and V_2 increase rapidly as Δ decreases. It indicates that the variance of the structure of the s, d bosons with the decreasing Δ results in a change of the properties of the s-d Hamiltonian. As might have been expected, this change describes a transition of the spectrum structure from vibrational to rotational types. The above conclusion is more straight-forward in Fig. 2, which gives a part of the calculated results by solving the eigenvalue equation of the s-d Hamiltonian in terms of the program NPBOS [14]. As we are interested mainly

in the features of the spectrum, a basic conclusion can be drawn according to the ratio $E_{4_1^+}/E_{2_1^+}$ and the energy of state 2_1^+, 2_2^+ and 0_2^+.

One of the characteristics in the vibrational spectrum is that the 2_1^+ state is located at the position of relatively higher energy (~0.5 MeV). In the transition to rotational spectrum, $E_{2_1^+}$ drops remarkably. In considering these facts, the ratio $E_{4_1^+}/E_{2_1^+}$ is really specific evidence to gain an insight into the structure of a nucleus, because its theoretical value varies from ~2 up to ~3.33, which corresponds to a transition from a spherical vibrator to an ideal rotor. Fig. 2 shows that when Δ decreases from 2.5 MeV towards 1 MeV, E_2 declines appreciably and $E_{4_1^+}/E_{2_1^+}$ increases from 2.36 to 2.83. The behavior demonstrates the gradual formation of the ground state rotational band. Meanwhile, the excited states 2_2^+ and 0_2^+ rise slowly and the tendency is to become the heads of the β-band and γ-band, respectively. It is noticed that the valence nucleon number and the strength parameters of the interaction remain unchanged in the above calculations. Therefore, the results seem to indicate that the reduction of the subshell gap is one of the most important factors which play a direct role in the abrupt transition in the $A \sim 150$ region.

Using the distribution of levels in Table 1, we made calculations for 16 nuclei with $N = 86$ to 92, including the $_{60}$Nb, $_{62}$Sm, $_{64}$Gd and $_{66}$Dy isotopes. These nuclei are in the region of the abrupt transition. The parameters are given in Fig. 3 and the calculated results are given in Figs. 4 and 5.

The results are in good agreement with the experiment. The agreement is mainly reflected by the fact that it is possible to describe the abrupt transition when N varies from 88 to 90. In the calculation, the transudation is caused by the following three factors: The first is the increase of valence neutron number; the second is the variance of the parameters, which are not the basis for the change since they vary smoothly; the third is the change of the level distribution with nucleon number A. It implies the reduction of Δ. The above results are compared with those obtained from similar calculations in the IBM phenomenological work [9]. Our results are much closer to the experimental data. Since in the phenomenological work, the former two factors are already taken into account, the difference may be attributed to the fact that the influence of reducing Δ on the structure of the s, d bosons is not considered, i.e., it is replaced simply by a rapid change of the valence proton numbers. As we know, even for the $Z \sim 50$ nuclei, such as Sn, Cd isotopes, the intruder state formed by the proton 2p-2h excitation across the major shell has been observed in the low-lying nuclear spectrum as long as the nucleus has certain number of valence neutrons [15]. Therefore, considering the $Z = 64$ subshell an inert core seems to be an over-simplified assumption. From this, we can infer that it is possible to better understand the abrupt transition in the $A \sim 150$ region by considering the Δ effects.

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