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# A Microscopic IBM-2 Description of the Phase Transitions of Nuclear Spectrum Structure Near A = 130

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**The phase transitions of the low-lying energy spectrum structure of the Ba isotopes and Sm isotopes are studied in the framework of the microscopic IBM-2 based on the boson expansion. It seems possible to describe the phase transitions in a uniform way by this approach.**

## 1. INTRODUCTION

The microscopic theory of the Interacting Boson Model (IBM) [1-4], based on the boson expansion and the modified Jancovici-Schiff (MJS) substitution, has been successful in describing the low-lying nuclear collective motion. Earlier work [5] usually adopted the maximum-$F$-spin truncation and the symmetric approximation of the neutron and proton degrees of freedom. In this paper, we propose a new approach, which solves the problem directly from the IBM-2 Hamiltonian through expanding NPBOS [6]. We perform calculations for a series of the Ba and Sm isotopes, and analyze the ability of our approach in describing various phase transitions.

Experimental and phenomenological work [7,8] showed that the phase transitions are manifest in the low-lying spectrum of the Ba, Sm and Xe isotopes. We find through our calculations that the nuclear spectrum structure of the Ba isotopes presents a phase transition from rotational to $\gamma$-unstable motions, while the nuclear spectrum structure of the Sm isotopes has a phase transition from vibrational to rotational motions. We also find from the latest results of the calculation on the Xe isotopes [9] that the spectra present a phase transition from vibrational to $\gamma$-unstable motions.

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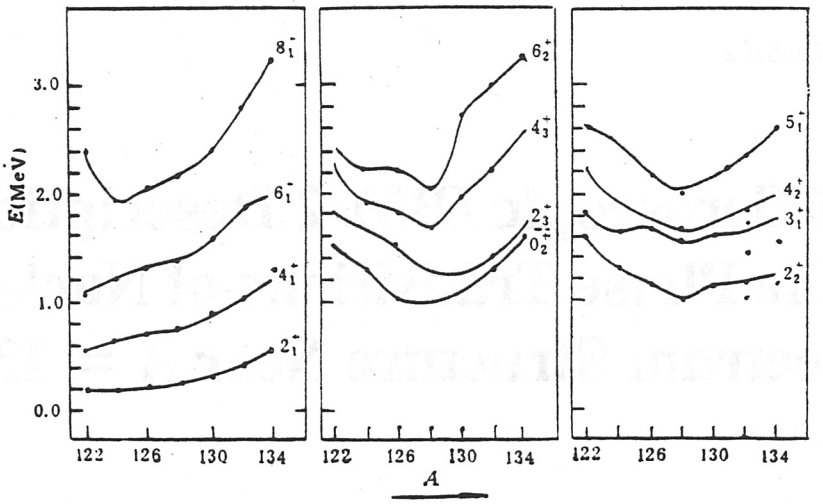


**Fig. 1**

A comparison between calculated and experimental energy levels for Ba. The experimental levels are taken from Ref.[11]. The solid lines denote theoretical values.

Therefore, it is possible to describe various phase transitions in a uniform way by extending the method of Yang *et al.* [2].

## 2. OUTLINE OF OUR THEORETICAL APPROACH

The nucleon-nucleon effective interaction and the shell model configurations are the starting point of our microscopic approach. We assume that there are $x$ valence neutrons and $x'$ valence protons moving in the $s$ and $s'$ single-particle levels, respectively. The shell model configuration is

$$(i_1, i_2 \cdots, i_s)^x (i_{1'}, i_{2'}, \cdots, i_{s'})^{x'},$$

The nucleon-nucleon effective interaction consists of a pairing force, a quadrupole pairing force and a quadrupole force [5], i.e., pairing force

$$-g_\sigma P^{(\sigma)} P^{(\sigma)+},$$

quadrupole pairing force

$$-\frac{1}{2} G_\sigma \sum P^{(\sigma)} P_\mu^{(\sigma)+}, \quad (\mu = 0, \pm 1, \pm 2) \tag{2.1}$$

and quadrupole force

$$-\frac{1}{2} K_\sigma \sum_{\mu ij} q_\mu^{(\sigma)}(i) q_\mu^{(\sigma)+}(j),$$

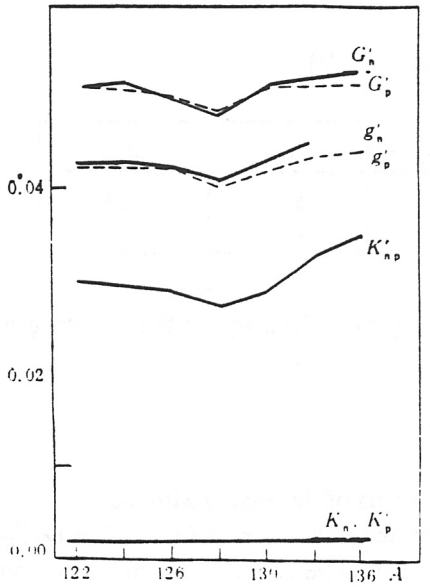


**Fig. 2**
Relationships between $A$ and the interaction parameters.

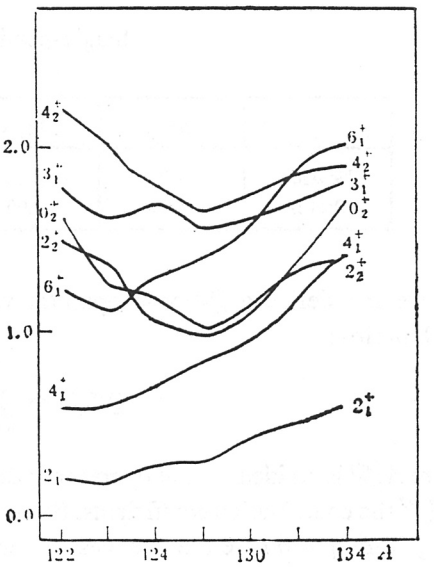


**Fig. 3**
Phase transition in Ba.

where $\sigma$ = n, p stand for the valence neutron and valence proton, respectively, $g_\sigma$, $G_\sigma$, $K_\sigma$ are the strength parameters.

The proton-neutron interaction is a quadrupole-quadrupole force:

$$-\frac{1}{2}K_{np}\sum_{\mu}\left(\sum_{i}q_{\mu}^{(n)}(i)\right)\left(\sum_{j}q_{\mu}^{(p)}(j)\right)^{+}, \tag{2.2}$$

The effective interaction Hamiltonian of the system is taken to be the following form

$$H_f = H_f^{(\sigma)} + H_f^{(np)} \tag{2.3}$$

where

$$H_f^{(\sigma)} = \sum_{\alpha}{}' E_\alpha^{(\sigma)} a_\alpha^{(\sigma)+} a_\alpha^{(\sigma)} + \sum_{\alpha\beta\gamma\delta} P_{\alpha\beta\gamma\delta}^{(\sigma)} a_\alpha^{(\sigma)+} a_\beta^{(\sigma)+} a_\gamma^{(\sigma)} a_\delta^{(\sigma)}, \tag{2.4}$$

$$H_f^{(np)} = \sum_{\alpha\beta\gamma\delta} P_{\alpha\beta\gamma\delta}^{(np)} a_\alpha^{(n)+} a_\beta^{(p)+} a_\gamma^{(p)} a_\delta^{(n)}, \tag{2.5}$$

Here $P^{(\sigma)}_{\alpha\beta\gamma\delta}$ and $P^{(np)}_{\alpha\beta\gamma\delta}$ are the matrix elements of the interaction.

According to the Dyson expansion, if the shell model space of the valence nucleon is mapped into the ideal boson space, the boson image of $H_f$ is [5]

$$H_B = H_B^{(\sigma)} + H_B^{(np)}. \tag{2.6}$$

**Table 1**
Single-particle energy levels (MeV).

| | $2d_{5/2}$ | $1g_{7/2}$ | $1h_{11/2}$ | $2d_{3/2}$ | $3s_{1/2}$ |
|---|---|---|---|---|---|
| Neutron | 4.00 | 4.88 | 5.70 | 6.24 | 7.24 |
| Proton | 4.54 | 4.00 | 5.80 | 7.47 | 6.54 |

and we can define a $Q$-boson operator with the property of collectivity by the following linear combination:

$$Q^{(\sigma)+}_{\gamma\pi JM} = \sum_{\alpha<\beta} \chi^{\sigma\gamma\pi J}_{\alpha\beta}(M) A^{(\sigma)+}_{\alpha\beta}, \tag{2.7}$$

where $A^{(\sigma)}_{\alpha\beta}$ is an ideal boson operator, $\gamma$ the ordering numbers of the energy with the same $\pi J$, and $\chi^{\sigma\ \gamma\ \pi J}_{\alpha\ \beta}$ the combination coefficients. For an appropriate choice of the effective interaction, $Q^{(\sigma)}_{0+00}$ and $Q^{(\sigma)}_{0+2m}$ with $\gamma = 0$ have the strongest collectivity. So they should be taken as the main components of the creation operators of the s and d bosons:

$$\begin{aligned} s^+_\sigma &\sim Q^{(\sigma)+}_{0+00}, \\ d^+_\sigma &\sim Q^{(\sigma)+}_{0+2m}. \end{aligned} \tag{2.8}$$

After the s-d truncation, we can obtain the Hamiltonian of the microscopic IBM-2 from (2.6):

$$h = h^{(\sigma)} + h^{(\mathrm{np})}, \tag{2.9}$$

This form can be used in describing the low-lying collective motion.

By choosing the shell model configurations, the single-particle levels and interaction parameters, we can diagonalization $h$ and calculate its eigenvalues and corresponding eigenvectors.

## 3. RESULTS AND DISCUSSION

The valence nucleons for the Ba isotopes are in major shell 50-82. The energy of the single-particle levels needed in our calculation are shown in Table 1.

Fig. 1 shows the comparison between the calculated and experimental energy levels in even Ba. For the sake of clarity, the spectrum is divided into three groups, i.e., quasi-ground, quasi-bata and quasi-gamma bands. We find that the calculated results of the spectrum structure are in qualitative agreement with the experimental ones. In particular, they are quantitatively consistent in both the quasi-ground bans, where much experimental information exists. Even in the quasi-bata and quasi-gamma bands, where there is less experimental information, the results are consistent with the phenomenological work [8,10,12]. The IBM-2 Hamiltonian we selected successfully describes the decline of the $2^+_1$ state as a function of the neutron boson number (similarly in other types of states). We also find the staggering effect [13] in the $\gamma$ and $\beta$ bands. The effective interaction parameters in our calculation are shown in Fig. 2.

In order to investigate the phase transition in the Ba isotopes, we show in Fig. 4 the calculated values of seven levels ($2^+_1$, $4^+_1$. $2^+_2$, $0^+_2$, $4^+_2$, $6^+_1$, and $3^+_1$) appearing in Fig. 1. We find that for $^{122}$Ba $E_{4^+_1}$

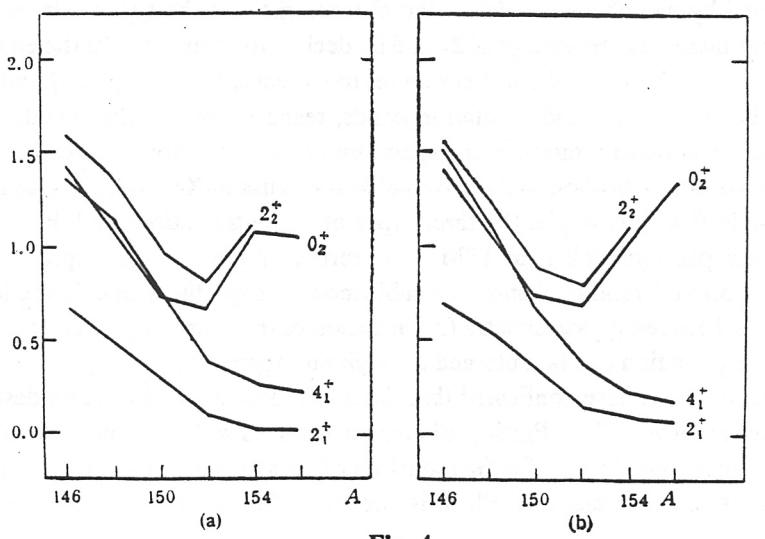


**Fig. 4**
Phase transitions in Sm.
(a) Experimental results [11].
(b) Our calculations.

$/E_{2_1^+} \approx 3.3$, $E_{6_1^+}/E_{4_1^+} \approx 2.09$ ···, i.e., the spacings of the energy levels in energy band obey the $l(l+1)$ law. It is a typical rotational spectrum structure. In $^{134}$Ba, $E_{4_1^+}/E_{2_1^+} \approx 2.31$, $E_{2_1^+}/E_{2_1^+} \approx 1.91$ [7]. Moreover, $4_1^+$ and $2_2^+$ constitute a doublet, while $0_2^+$, $4_2^+$, $3_1^+$, and $6_1^+$ form a quartet. The $0_2^+$ state is lower as expected than the $3_1^+$ state, which presents the characteristic of the $\gamma$-unstable spectrum. However, during the experiments, they are close or the former is slightly higher than the latter [14]. We also show how the phase transition from the rotational to $\gamma$-unstable motions in the Ba isotopes is reproduced in our scheme.

The spectrum of $^{146-156}$Sm is shown in Fig. 4. From the calculated and experimental results of the four levels ($2_1^+$, $4_1^+$, $2_2^+$ and $0_2^+$), we find that the energy of $4_1^+$, $2_2^+$ and $0_2^+$ are about twice as much

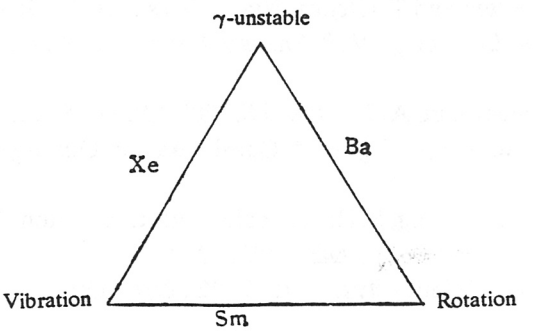


**Fig. 5**
The symmetrical triangle of the IBM.

as that of $2_1^+$, where there are fewer nucleons (i.e., $^{146,148}$Sm). The triplet, as illustrated in the Bohr model, is formed by two-photon, which is the characteristic of the typical vibrational spectra. As nucleon number increases, the energy of $2_2^+$ and $0_2^+$ decline to $^{152}$Sm just like the energy of $2_1^+$ and $4_1^+$, which is a transitional nucleus. When $A$ continues to increase, the energy of $2_2^+$ and $0_2^+$ grow sharply, and form the heads of the $\gamma$ and $\beta$ rotation bands, respectively. In other words, there is a phase transition from vibration to rotation in the spectrum of the Sm isotopes. Recent work [9] indicated a phase transition from vibration to the $\gamma$-unstable spectrum in Xe nuclei. We can use a symmetry triangle (shown in Fig. 5) to depict the three types of phase transitions and the three typical limits presented in the phenomenological IBM. The vertex of the triangle represents the spectrum structures of vibrational, rotational and $\gamma$-unstable modes, respectively, and their edges represent the phase transitions between typical spectra (no inclusion of the third one). It is clear that the uniform and consistent explanation can be obtained through our approach.

As stated above, we have confirmed that the nuclei near $A = 130$ can be described according to the Hamiltonian form of (2.9). Particularly, for the isotopes with fewer neutron bosons, the IBM-2 is a satisfactory approximation. As for the reduction of the spacing between $0_1^+$ and $2_1^+$ for nuclei lying in the middle of the two major shell, it is probably due to the strong attractive quadrupole interactions.

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