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The Staggering Phenomenon in the Nuclear Energy Spectrum and the Quadruple Interaction

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An improved approach to the treatment of the staggering phenomenon in nuclear energy spectra in the framework of the IBM2 is described in this paper. By applying this approach to the even-even isotopes, Ru and Pt, the relation between the staggering phenomenon and the quadruple-quadruple interaction is discussed.

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1. INTRODUCTION

The staggering phenomenon in the nuclear energy spectrum is such that in the quasi-γ band of the collective energy spectra of the even-even nuclei, the energy levels of the odd-spin state are close to their neighboring even-spin state and the doublets (3^+_γ, 4^+_γ), (5^+_γ, 6^+_γ), ..., are formed. The distribution of the energy spectrum in the quasi-γ band is not as even as that in the ground state band. However, what causes this phenomenon is still an open question in nuclear structure theory.

Among various nuclear structure theories, the Interacting Boson Model (IBM), which was put forward in the middle of 1970s [1-5], has been successful in the description of many properties of heavy and medium-heavy even-even nuclei. However, in the IBM, while the calculated sequences 2^+_γ, (3^+_γ, 4^+_γ), (5^+_γ, 6^+_γ), ..., for the nuclei with the $O(6)$ symmetry [6] are evident, the experimental quasi-γ band is homogeneous. This is the well-known staggering phenomenon associated with the IBM. Several solutions [7-14] have then been proposed to solve this discrepancy and to explain its physical mechanism. Casten and Van Brentano recognized this phenomenon as a consequence of the γ-

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independent interaction [6,7]. Zeng and Sun considered it a result of the progressive breaking of symmetry [12]. A recently proposed solution suggests that the staggering phenomenon is closely related with the quadruple interaction in the nucleus [13,14]. In this paper, this relationship will be analyzed after a brief description of the approach and its physical mechanism of weakening the staggering phenomenon in the IBM.

2. THE IMPROVED APPROACH TO THE STAGGERING PHENOMENON IN THE IBM2 AND ITS MECHANISM

From the shell model theory, the interaction among like nucleons is mainly the pairing force, and the interaction among unlike nucleons is mainly the quadruple force [15,16]. Therefore, the Hamiltonian in the framework of the IBM2 is usually taken as [15,16]

$$H = \varepsilon(n_{d\pi} + n_{d\nu}) + \kappa Q^{(2)}_\pi \cdot Q^{(2)}_\nu + M_{\pi\nu}. \tag{1}$$

Some take into account the residual interaction among the coherent pairs of like nucleons as well. Then the IBM2 Hamiltonian can be written as [17-20]

$$H = \varepsilon(n_{d\pi} + n_{d\nu}) + \kappa Q^{(2)}_\pi \cdot Q^{(2)}_\nu + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu} \tag{2}$$

here

$$n_{d\rho} = \sum_\mu d^\dagger_{\rho\mu} d_{\rho\mu}, \quad (\rho = \pi, \nu) \tag{3}$$

$$Q^{(2)}_\rho = (s^\dagger_\rho \tilde{d}_\rho + d^\dagger_\rho s_\rho) + \chi_\rho (d^\dagger_\rho \tilde{d}_\rho)^{(2)}, \quad (\rho = \pi, \nu) \tag{4}$$

$$V_{\rho\rho} = \frac{1}{2} \sum_{L=0,2,4} \sqrt{2L+1}\, C^{(1)}_\rho [(d^\dagger_\rho d^\dagger_\rho)^{(L)} (\tilde{d}_\rho \tilde{d}_\rho)^{(L)}]^{(0)}, (\rho = \pi, \nu) \tag{5}$$

$$M_{\pi\nu} = \xi_2 (s^\dagger_\nu d^\dagger_\pi - s^\dagger_\pi d^\dagger_\nu)^{(2)} \cdot (s_\nu \tilde{d}_\pi - s_\pi \tilde{d}_\nu)^{(2)} + \sum_{\lambda=1,3} \xi_\lambda (d^\dagger_\pi d^\dagger_\nu)^{(\lambda)} \cdot (\tilde{d}_\pi \tilde{d}_\nu)^{(\lambda)}. \tag{6}$$

Even though the calculated results with this Hamiltonian agree with experimental data globally, the staggering phenomenon in the quasi-γ bands of the energy spectra for $O(6)$ nuclei is still much more obvious than that in the experimental ones.

However, in the case of the $SU^*(3)$ limit [11,21] of the IBM2, the Hamiltonian is expressed as

$$H = \kappa(Q^{(2)}_\pi + Q^{(2)}_\nu) \cdot (Q^{(2)}_\pi + Q^{(2)}_\nu) + \kappa'(L_\pi + L_\nu) \cdot (L_\pi + L_\nu). \tag{7}$$

in which the quadruple operator $Q^{(2)}_\rho$ ($\rho = \pi, \nu$) is taken from Eq.(4) with $\chi_\pi = -\frac{\sqrt{7}}{2}$ and $\chi_\nu = \frac{\sqrt{7}}{2}$, or $\chi_\pi = \frac{\sqrt{7}}{2}$ and $x_\nu = -\frac{\sqrt{7}}{2}$. And the angular momentum operator L_ρ is written as

$$L_\rho = \sqrt{10}\,(d^\dagger_\rho \tilde{d}_\rho)^{(1)}. \quad (\rho = \pi, \nu) \tag{8}$$

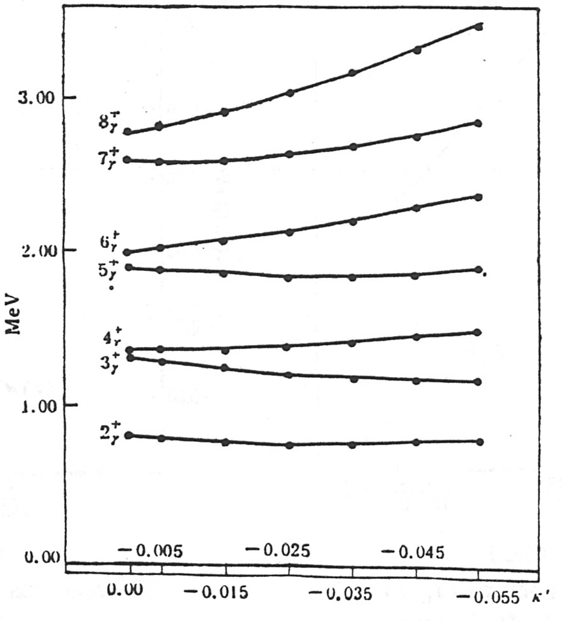

Fig. 1
The pattern in which energy levels in
the quasi-γ band change along with κ'.

Within this limit, the strength of the quadruple interaction among like coherent pairs of nucleons (approximately regarded as bosons) is half the strength of the interaction among unlike coherent pairs of nucleons, i.e., $Q_\pi^{(2)} \cdot Q_\pi^{(2)} = Q_\nu^{(2)} \cdot Q_\nu^{(2)} = Q_\pi^{(2)} \cdot Q_\nu^{(2)}/2$. The calculated energy spectrum exhibits no staggering phenomenon, but a big degeneracy for the levels appears.

Comparing these two extreme situations, we can see that with the Hamiltonian

$$H = \varepsilon(n_{d\pi} + n_{d\nu}) + \kappa Q_\pi^{(2)} \cdot Q_\nu^{(2)} + \kappa'(Q_\pi^{(2)} \cdot Q_\pi^{(2)} + Q_\nu^{(2)} \cdot Q_\nu^{(2)}) + M_{\pi\nu}, \tag{9}$$

with κ' being taken in the domain $|\kappa'| < |\kappa|/2$, the staggering phenomenon of the energy spectrum in the framework of the IBM2 can be weakened obviously.

Comparing Eqs.(2) and (9), we know that the difference between these two cases lies mainly in the choice of the interaction among like coherent pairs of nucleons. Eq.(2) regards the interaction as the one conserving the number of coherent pairs with angular momentum 2 (approximately regarded as d bosons). But Eq.(9) takes the quadruple interaction into account, which is closely related with the quadruple deformation of the nucleus. In order to show the effect of $Q_\pi^{(2)} \cdot Q_\pi^{(2)}$ and $Q_\nu^{(2)} \cdot Q_\nu^{(2)}$ interactions on weakening the staggering phenomenon, we perform the following calculation. We take the Hamiltonian as Eq.(9). Using one set of fixed parameters $\varepsilon = 0.70$ MeV, $\kappa = -0.11$ MeV, $\chi_\pi = 0.80$, $\chi_\nu = -0.90$, $\xi_1 = 0.42$ MeV, $\xi_2 = \xi_3 = 0.00$, and various values of κ', we calculate the energy spectrum of ^{104}Ru, and observe the pattern in which the energy levels change along with κ'. The result for the quasi-γ band is shown in Fig. 1. The figure shows:

(1) when $\kappa' = 0$, there is an obvious staggering phenomenon in the quasi-γ band of the

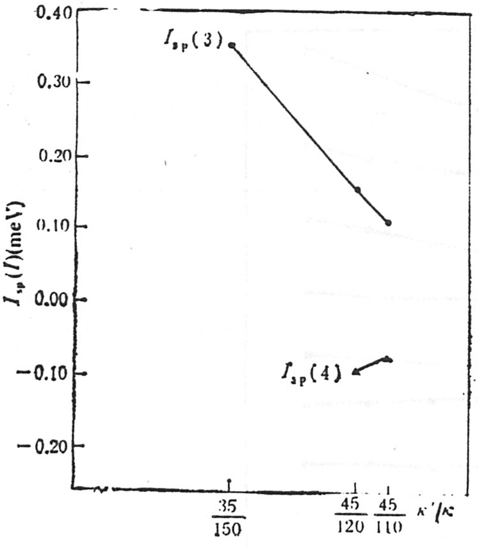

Fig. 2
The relations between $I_{sp}(I)$ and κ'/κ for the Ru isotopes.

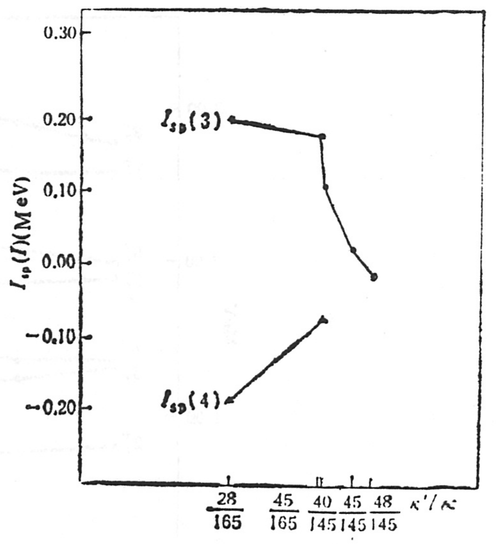

Fig. 3
The relations between $I_{sp}(I)$ and κ'/κ for the Pt isotopes.

calculated spectrum (in fact, Eq.(9) is reduced to Eq.(1) in this case);

(2) the energy level of the 2_γ^+ state does not change globally as the κ' changes;

(3) the energy levels of states 4_γ^+, 6_γ^+, . . ., rise as $|\kappa'|$ increases, and

(4) some odd-spin levels (e.g. 3_γ^+, 5_γ^+) decrease in energy as $|\kappa'|$ increases and some odd-spin levels (e.g. 7_γ^+) increase in energy as $|\kappa'|$ increases, but at a rate much lower than that of the even-spin neighbors.

This indicates that the interactions of $Q_\pi^{(2)} \cdot Q_\pi^{(2)}$ and $Q_\nu^{(2)} \cdot Q_\nu^{(2)}$ make the energy levels of even-spin state grow and those of odd-spin state decline. It reduces the depression of the levels of even-spin state relative to their odd-spin neighbors, so that the staggering phenomenon is removed. According to the results for the even-even Ru isotopes [13] and Pt isotopes [14], taking the $Q_\pi^{(2)} \cdot Q_\pi^{(2)}$ and $Q_\nu^{(2)} \cdot Q_\nu^{(2)}$ interactions into account can not only weaken the obvious staggering phenomena in previous calculations [19,20], but also describe well the electric quadruple transition rate, the electric quadruple moment of the first excited 2_1^+ state and other electromagnetic properties. It shows that the staggering phenomenon has close relationships with the quadruple interaction of the coherent pairs of nucleons in nuclei.

3. THE RELATIONS BETWEEN THE STAGGERING PHENOMENON AND QUADRUPLE INTERACTION

Since the staggering phenomenon was discovered [22], several methods [23-25] have been developed to describe it. With the recently introduced displacement $\delta E(I)$ of the odd-spin levels with respect to their even-spin neighbors [25], it is easy to depict this phenomenon. The displacement δE(I) is defined as

Table 1
Parameters used to calculate the energy spectra of Ru isotopes [13].

	ε(MeV)	κ(MeV)	κ'(MeV)	χ_ν	χ_π	ξ_1(MeV)	$\xi_2 = \xi_3$(MeV)
^{98}Ru	0.825	-0.150	-0.054	-0.100	0.800	0.300	0.000
^{100}Ru	0.800	-0.150	-0.035	-0.100	0.800	0.350	0.000
^{102}Ru	0.750	-0.120	-0.045	-0.500	0.800	0.400	0.000
^{104}Ru	0.700	-0.120	-0.045	-0.900	0.800	0.420	0.000

Table 2
Parameters used to calculate the energy spectra of Pt isotopes [14].

	ε(MeV)	κ(meV)	κ'(MeV)	χ_ν	χ_π	ξ_1(MeV)	$\xi_2 = \xi_3$(MeV)
^{188}Pt	0.480	-0.165	-0.028	0.500	-0.900	0.285	-0.085
^{190}Pt	0.460	-0.145	-0.040	0.535	-0.900	0.285	-0.085
^{192}Pt	0.460	-0.145	-0.045	0.600	-0.900	0.285	-0.085
^{194}Pt	0.460	-0.145	-0.048	0.750	-0.900	0.285	-0.085
^{196}Pt	0.460	-0.165	-0.045	0.800	-0.900	0.285	-0.085
^{198}Pt	0.460	-0.165	-0.040	0.500	-0.900	0.285	-0.085

$$\delta E(I) = E(I) - \frac{(I+1)E(I-1) - IE(I+1)}{2I+1}, \tag{10}$$

i.e.,

$$\delta E(I) = \frac{2I(I+1)}{2I+1}\left[\frac{E(I)-E(I-1)}{2I} - \frac{E(I+1)-E(I)}{2(I+1)}\right]. \tag{11}$$

It is very clear that if $\delta E(I)$ is positive, $E(I)$ is close to $E(I+1)$, but far away from $E(I-1)$. Then, if $\delta E(I_\gamma^+)$ is positive for odd-spin I_γ^+ and $E((I+1)_\gamma^+)$ is negative for even-spin $(I+1)$, even more generally, $E(I_\gamma^+) > E((I+1)_\gamma^+)$, the staggering phenomenon is caused by the depression of the even-spin levels relative to the odd-spin levels. The investigations on the nuclear energy levels show that the staggering phenomena for the nuclei in the rare-earth region are formed in this way. The interactions $Q_\pi^{(2)} \cdot Q_\pi^{(2)}$ and $Q_\nu^{(2)} \cdot Q_\nu^{(2)}$ counteract this kind depression of the levels. The staggering phenomenon in the nuclear energy spectrum is thus described quite well.

With the purpose of indicating quantitatively the obvious staggering phenomenon, we define a quantity $I_{sp}(I)$ as

$$I_{sp}(I) = \delta E(I) - \delta E(I+1). \tag{12}$$

Apparently, for odd-I, the larger the $I_{sp}(I)$ is, the more obvious the staggering phenomenon is. For even-I, the smaller the $I_{sp}(I)$ is, the more obvious the staggering phenomenon is. In order to clarify the relationships between the staggering phenomenon and the quadruple interaction of the coherent pairs of nucleons, by following the successful application of the approach [13,14] to the even-even Ru and Pt isotopes, we depict in Figs. 2 and 3 the experimental $I_{sp}(I)$ [26] with respect to the calculated values of κ'/κ [13,14] (see Tables 1 and 2).

The figures show that the smaller the κ'/κ is, the larger the $I_{sp}(I)$ for odd-I is, and vice versa, i.e., the staggering phenomenon becomes obvious as the value of κ'/κ decreases. In other words, as the interaction $Q_\pi^{(2)} \cdot Q_\pi^{(2)}$ and $Q_\nu^{(2)} \cdot Q_\nu^{(2)}$ intensifies with respect to the interaction $Q_\pi^{(2)} \cdot Q_\nu^{(2)}$, the staggering phenomenon becomes obscure. When they reach their maximum $Q_\pi^{(2)} \cdot Q_\pi^{(2)} = Q_\nu^{(2)} \cdot Q_\nu^{(2)} = Q_\pi^{(2)} \cdot Q_\nu^{(2)}/2$, i.e., $\kappa'/\kappa = 1/2$, the staggering phenomenon disappears. On the other hand, the staggering phenomenon is more obvious when the $Q_\rho^{(2)} \cdot Q_\rho^{(2)}(\rho = \pi, \nu)$ interaction is weaker with respect to the $Q_\pi^{(2)} \cdot Q_\nu^{(2)}$ interaction, and vice versa.

4. DISCUSSION

In this paper, on the basis of the successful application of the improved approach [13,14], in which the main point is to introduce the quadruple interaction among like coherent pairs of nucleons in nuclei to the traditional Hamiltonian of the IBM2, we analyze the relation between the staggering phenomenon of the nuclear energy spectrum and the quadruple interaction. The result shows that the staggering phenomenon in the nuclear energy spectrum is determined by the competition between the quadruple interactions among like coherent pairs of nucleons $Q_\pi^{(2)} \cdot Q_\pi^{(2)}$ and $Q_\nu^{(2)} \cdot Q_\nu^{(2)}$ and those among unlike coherent pairs $Q_\pi^{(2)} \cdot Q_\nu^{(2)}$. More exactly, the staggering phenomenon becomes obscure when the interaction $Q_\rho^{(2)} \cdot Q_\rho^{(2)}$ $(\rho = \pi, \nu)$ is strong with respect to the interaction $Q_\pi^{(2)} \cdot Q_\nu^{(2)}$. Meanwhile, the staggering phenomenon is the reflection of the quadruple interaction mode in nuclei.

The result also shows that with the Hamiltonian of the IBM2, into which the interactions $Q_\pi^{(2)} \cdot Q_\pi^{(2)}$ and $Q_\nu^{(2)} \cdot Q_\nu^{(2)}$ are introduced, we can describe well the energy spectrum, the electronic quadruple transition rate and the electric quadruple moment of the first excited 2_1^+ state of nuclei. However, whether this approach can be used to depict other properties of nuclei such as the magnetic dipole transition, which is an open question in the IBM, deserves further studies.

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