

The $\pi N \rightarrow \eta N$ Reactions near the Threshold and η -N Scatterings

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The $N^*(1535)$ model is proposed for pionic η -production reactions. The $\pi N \rightarrow \eta N$ reaction cross-sections near the threshold and the S -wave η -N scattering length are calculated and compared with the experimental data and other theoretical calculations.

1. INTRODUCTION

It is well-known that the η , K and π mesons are the members of an $SU(3)$ octet. The studies of the pion-nucleus and kaon-nucleus interactions have made great progress and provided important information about the excitation modes of nuclei and new forms of nuclear matter [1,2]. On the other hand, very little is known about η -nucleon interactions, and almost nothing is known about η -nucleus interactions. One of the major difficulties lies in the fact that the mean lifetime of the η particle is only about 0.75×10^{-18} seconds, which makes it impossible to produce η beam in the laboratory. All the information concerning η -N interactions is indirect. Recent experiments on the production of pionic η [1,2] at the Clinton P. Anderson Meson Physics Facility (LAMPF) are providing valuable information on η -nucleus physics. R. S. Rhalerao and L. C. Liu [3,4] have proposed a theoretical model for the η -nucleon interaction by using the coupled channel method. They included the $N^*(1440)$, $N^*(1520)$ and $N^*(1535)$ resonances. By fitting the S -wave π -N phase shifts, the coupling constants of πNN^* , ηNN^* and $\pi\pi NN^*$ are determined and the features of the η -N interactions as well as the possibilities of the existence of η -nuclear-bound state are predicted. In the framework of the distorted wave impulse approximation, Y. G. Li and H. C. Chiang [5] have predicted and discussed the characteristics of the η -production reaction by means of the pion "charge exchange" reactions

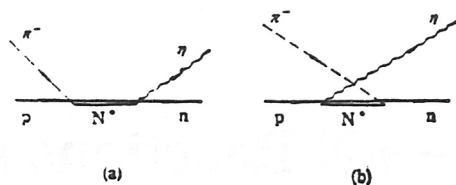


Fig. 1

Feynman Diagram for $\pi^- p \rightarrow \eta n$ reactions.

(a) Direct term; (b) exchange term.

on nuclei. The input elementary process is believed to be a parametrized form.

The particle data table [6] indicates that below 1700 MeV C-M energy, among a series of resonances, only $N^*(1535)$ has a large branching ratio to decay into the η and nucleon, the η -production probability via other resonances is very small and can be ignored. In other words, the coupling of the ηN to the $N^*(1535)$ is very strong while the coupling of others is very weak. Therefore, in studying the η -N interaction, the $N^*(1535)$ resonance model may be assumed. On the other hand, the important information on the ηNN^* coupling is the branching ratios of the ηN , πN and $\pi\pi N$ decay modes, and the $\pi^- p \rightarrow \eta N$ reaction cross sections. As for the πN channel, many more mechanisms besides the $N^*(1535)$ can contribute. Hence, in studying the ηN interaction, the S -wave πN phase shifts are not relevant, while the branching ratios and the production cross sections are. Based on the above considerations, H. C. Chiang, E. Oset and L. C. Liu [7] calculated the coupling constants of πNN^* , ηNN and $\pi\pi NN^*$ on the basis of the branching ratios of the $N^*(1535)$ decays. Then an η -nucleus optical potential was derived and the possibility of the η -nucleus-bound state was investigated by solving the Klein-Gordon equation.

However, in the energy region of 0.6-1 GeV, the pionic η -production reaction, namely $\pi^- p \rightarrow \eta n$, is an important reaction channel. The ηNN^* coupling constant determined above should reproduce the $\pi^- p \rightarrow \eta n$ reaction cross sections near the threshold. In this paper we use the interaction Hamiltonian given by Chiang *et al.* [7] to calculate the $\pi^- p \rightarrow \eta n$ reaction cross section, investigate the influences of the mass and mean total width of the $N^*(1535)$ on the production cross sections as well as the S -wave ηN scattering length. The characteristics of the ηN interaction at low energy are analyzed.

In section 2 we briefly summarize our theoretical formula. Numerical results and discussions are presented in section 3.

2. THEORETICAL FORMULA

It is known from the compilation of particle properties that the mass of the $N^*(1535)$ is between 1520 and 1560 MeV, and its width ranges from 110 MeV to 250 MeV. Their mean values are 1535 MeV and 150 MeV, respectively. The branching ratios of the πN , ηN and $\pi\pi N$ decay channels are 35-45%, 45-55% and 10%, respectively. Considering the three decay modes of the $N^*(1535)$, it is necessary to know the πNN^* , ηNN^* and $\pi\pi NN^*$ couplings. By taking into account the quantum numbers of the particles involved, we take the following interaction Hamiltonians for the πNN^* ,

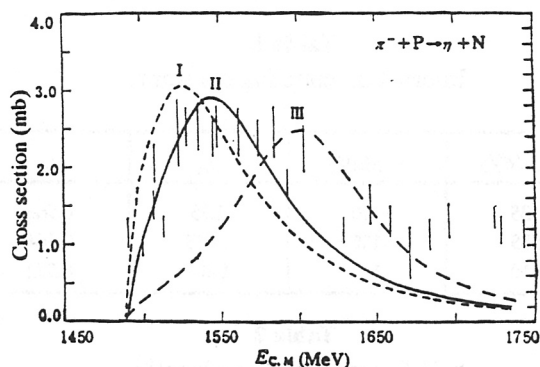


Fig. 2

The total cross sections of the $\pi^- p \rightarrow \eta n$ reaction.

ηNN^* and $\pi\pi NN^*$ couplings:

$$\begin{aligned}\delta H_{\pi NN^*} &= -ig_{\pi}\bar{\psi}_{N^*}(x)\phi(x)\psi_N(x) + \text{h.c.}, \\ \delta H_{\eta NN^*} &= -ig_{\eta}\bar{\psi}_{N^*}(x)\psi_N(x)\phi_{\eta}(x) + \text{h.c.}, \\ \delta H_{\pi\pi NN^*} &= -iC\bar{\psi}_{N^*}(x)\gamma_5\psi_N(x)\phi(x)\phi(x) + \text{h.c.}\end{aligned}\quad (1)$$

where g_{π} , g_{η} and C refer to the corresponding coupling constants. Using standard Feynman rules, we evaluate the self energy of the $N^*(1535)$ due to the πN , ηN and $\pi\pi N$ decay channels. The partial decay width $\Sigma^{(i)}$ and the corresponding self energy associated with the decay channel have the following relation,

$$\frac{\Gamma^{(i)}(q)}{2} = -\text{Im}\Sigma^{(i)}(q). \quad (2)$$

Here, q is the four-momentum of the $N^*(1535)$. With the experimental values for the total width and branching ratios of the $N^*(1535)$, it is easy to determine the coupling constants g_{π} , g_{η} and C in Eq.(1).

For the $\pi N \rightarrow \eta N$ reaction, we need to evaluate the diagram of Fig. 1. The T matrix is

$$\begin{aligned}-iT &= (-i)g_{\pi} \frac{1}{\sqrt{s} - M_{N^*} + i\Gamma(\sqrt{s})/2} (-i)g_{\eta} \\ &+ (-i)g_{\pi} \frac{1}{\sqrt{s} - M_{N^*} + i\Gamma(\sqrt{s})/2 - \omega_{\pi}(K_{\pi}) - \omega_{\eta}(K_{\eta})} (-i)g_{\eta}\end{aligned}\quad (3)$$

here $\Gamma(\sqrt{s}) = \Gamma(\pi N) + \Gamma(\eta N) + \Gamma(\pi\pi N)$, \sqrt{s} being the invariant mass of the system, M_{N^*} the mean mass of $N^*(1535)$, and k_{π} and k_{η} the momenta of π and η . The first of the terms in Eq.(3) corresponds to the direct term of Fig. 1(a) and the second to the exchange terms in Fig. 1(b). By using the Bjorken-Drell convention, the cross section of the $\pi N \rightarrow \eta N$ reaction can be written as

Table 1
Interaction coupling constants.

	$M_{N^*}(\text{MeV})$	$\Gamma(\text{MeV})$	g_η	g_π	$C(m_\pi^{-1})$
I	1535	110	1.75	0.568	11.18
II	1555	110	1.613	0.564	10.09
III	1610	110	1.408	0.552	7.99

Table 2
 η -N *S*-wave scattering lengths.

Theory	$a_0(\text{fm})$
This paper I	$0.56 + i0.256$
This paper II	$0.340 + i0.117$
This paper III	$0.155 + i0.027$
Ref. [3]	$0.28 + i0.19$
Ref. [9]	$0.83 + i0.05$

$$\frac{d\sigma}{d\Omega} = \frac{M_p^2}{(4\pi)^2} \cdot \frac{1}{s} \cdot \frac{\lambda^{1/2}(s, m_\pi^2, M_p^2)}{\lambda^{1/2}(s, m_\eta^2, M_n^2)} \cdot \bar{\Sigma} |T|^2, \quad (4)$$

Here $\bar{\Sigma}$ refers to the sum and average over final and initial polarizations of the nucleon, m_π , m_η and $M_{n(p)}$ the masses of π , η and neutron (or proton), respectively, and $\lambda(x, y, z)$ the Källén function.

With Eq.(4), the pionic η -production cross sections at different energy can be calculated.

3. NUMERICAL RESULTS AND DISCUSSIONS

Using the formula given in Section 2 we first discuss the dependence of the coupling constants g_π , g_η and C on the values of the mass and width of the N^* . The data table of particle properties [6] suggest that the mean values of the mass and width of N^* are 1535 MeV and 150 MeV, respectively. However, these values have large uncertainty. With these values of the mass and width and by taking the branching ratios of the $N^* \rightarrow \pi N$, $N^* \rightarrow \eta N$ and $N^* \rightarrow \pi\pi N$ decay modes as 40%, 50% and 10%, respectively, we obtain the coupling constants, $g_\pi = 0.664$, $g_\eta = 2.06$ and $C = 9.23 (m_\pi^{-1})$. In order to study the influence of the mass and width on N^* , we list in Table 1 the results for the coupling constants in other three cases. Apparently, different choices of the values affect the coupling constants. We will further determine these values by $\pi N \rightarrow \eta N$ reactions.

By using Eq.(4) and the coupling constants in Table 1, we calculate the $\pi N \rightarrow \eta N$ reaction cross sections near the threshold. Fig. 2 shows the energy dependences of the reaction cross sections for different choices of the mass and width of N^* . Three curves in Fig. 2 correspond to three sets of values in Table 1, respectively. Experimental data are taken from Flaminio *et al.* [8]. From a comparison between the curves and experiment, we find that with the choice of $M_{N^*} = 1555$ MeV and $\Gamma = 110$ MeV, our theory reproduces the data reasonably well near the threshold. When the C. M. energy is larger than 1600 MeV, the theoretical cross sections are smaller than the data systematically. This is because at higher energy other resonances may contribute to the η production.

For example, the branching ratio of the $N^*(1710)$ ηN decay is 25%, which may make an important contribution to the $\pi N \rightarrow \eta N$ reaction at energy greater than 1600 MeV. However, near the threshold it has little contribution. Therefore we conclude that the coupling constants obtained with $M_{N^*} = 1555$ MeV and $\Gamma = 110$ MeV are reliable at low energy.

Using the above results, we further calculate the S -wave ηN scattering length, and look into the characteristics of the η - N interaction. The calculated S -wave η - N scattering lengths as well as the results from other calculations are presented in Table 2. The table shows one common feature of the S -wave ηN scattering lengths in different models, namely, the scattering length is positive. This means that the ηN interaction is attractive at low energy. However, the absolute values of the scattering length in different models are quite different. Our results, from the second set of coupling constants in Table 1, are close to those of Bhalerao and Liu [3]. More experiments on the η - N interaction strength are expected.

In summary, the $\pi N \rightarrow \eta N$ reactions are investigated and compared with the data by using the $N^*(1535)$ resonance model. Our model reasonably reproduces the η production cross section near the threshold. The property of the ηN interaction at low energy is predicted. This results may be used for a further study of the pionic η production reactions on nuclei.

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