High Energy Physics and Nuclear Physics

Volume 15, Number 4

# Sum Rules in Quantum Mechanics and the Electric Dipole Transitions of the ψ (3770)

Ding Yibing[1,3,4], Zhou Lei[1], Zhao Guangda[2,3,4] and Qin Danhua[2,4]

[1]Department of Physics, Graduate School, Chinese Academy of Sciences, Beijing, China
[2]Department of Physics, Beijing University, Beijing, China
[3]CCAST (World Laboratory)
[4]Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, China

**By using sum rules in quantum mechanics, the upper and lower bounds on the electric dipole transition widths of the $\psi$ (3770) are estimated. It indicates that the experimental values given by the Mark III Collaboration are larger than the theoretical values evidently.**

## 1. INTRODUCTION

Recently, in the framework of potential model, we have made some theoretical calculations [1] on the electric dipole transitions ($E1$ transition) for the $\psi$ (3770) ($1^3D_1$ state of $c\bar{c}$ family). We found that whether we use the purely non-relativistic potential model or take into consideration the first order relativistic correction, the resulting widths of the $\psi$ (3770) $\rightarrow \gamma\chi_J(J = 0, 1, 2)$ decay are much smaller than the values [2] reported by the Mark III Collaboration. In Ref.[3], we studied the possibility that the $\psi$ (3770) is a mixing state between the $^3D_1$-state and $^3S_1$-state. It turned out that the inconsistency between the theory and the experiment cannot be resolved whatever mixing prescription is used.

In this paper, we try to further study the problem from another point of view, i.e., from the sum rule in quantum mechanics, and try to give some theoretical results which are model-independent or at least less model-dependent.

The sum rule in quantum mechanics has many applications in atomic physics [4]. It was employed to study the $E1$ transition of the heavy quarkonium and provided much valuable theoretical reference [5]. For instance, it has been used to study the possible form of the confinement potential [6].

This paper is organized as follows. In Section 2 we generalize the Thomas-Reiche-Khun (TRK) sum rule used by Novikov *et al.* [5]. In Section 3 we discuss the general relation between these sum

---

Supported by the National Natural Science Foundation of China.
Received on February 13, 1991.

rules and the $E1$ transition widths. With the requirement to suppress the model-dependence being taken into consideration, the upper bounds on the widths for the $E1$ transition $\psi(3770) \rightarrow \gamma\chi_J (J = 0, 1, 2)$ from both the first and second order sum rule are obtained in Section 4. In Section 5 the Wigner sum rules [5] are used to obtain not only the upper bounds but also the lower bounds on the same processes. We find that all these theoretical results are consistent with each other, that they require rigid restrictions on the corresponding experiments, and that they are much smaller than the data reported by the Mark III Collaboration but are basically in agreement with the theoretical values calculated from our potential model. We discuss these results briefly in the last section.

## 2. THE GENERAL FORM OF THE DIPOLE SUM RULE

Assuming the Hamiltonian of a quantum system is

$$H = \frac{p^2}{2\mu} + U(r) \tag{1}$$

where $\mu$ is the reduced mass and $U(\vec{r})$ is the interaction potential. Let the eigenvalue of $\bar{H}$ be $E_n$ and the corresponding eigenvector be $|n\rangle$, where $n$ represents all quantum numbers. Assuming $A$ is a hermitian operator and defining

$$A(0) \equiv A, \tag{2}$$

$$A(k) \equiv [H, A(k-1)] \quad (k = 1, 2, \cdots). \tag{3}$$

it can be easily proved

$$\sum_m |\langle m|A(k)|n\rangle|^2 = \langle n|A^+(k)A(k)|n\rangle. \tag{4}$$

From the above equations and considering both $|n\rangle$ and $|m\rangle$ are the eigenvectors of $\bar{H}$, we can derive a general form of even-order sum rule:

$$\sum_m (E_m - E_n)^{2k}|\langle m|A|n\rangle|^2 = \langle n|A^+(k)A(k)|n\rangle. \tag{5}$$

Similarly, using the formula

$$2\sum_m (E_m - E_n)|\langle m|A(k)|n\rangle|^2 = \langle n|[[A^+(k), H], A(k)]|n\rangle, \tag{6}$$

we can obtain a general form of odd-order sum rule:

$$\sum_m (E_m - E_n)^{2k+1}|\langle m|A|n\rangle|^2 = \frac{1}{2}\langle n|[[A^+(k), H], A(k)]|n\rangle. \tag{7}$$

If we take $A = \vec{r}$, then

$$A(0) = r, \tag{8}$$

$$A(1) = [H, r] = -\frac{ip}{\mu}. \tag{9}$$

$$A(2) = \frac{1}{\mu}\nabla U. \tag{10}$$

Substituting these results into (5) and (7), the dipole sum rules can be found one by one. Defining $\vec{r}_{mn} = <m|\vec{r}|n>$, the first five dipole sum rules are:

$$\sum_m (E_m - E_n)|\boldsymbol{r}_{mn}|^2 = \frac{3}{2\mu}, \tag{11}$$

$$\sum_m (E_m - E_n)^2|\boldsymbol{r}_{mn}|^2 = \frac{1}{\mu^2}\langle n|\boldsymbol{p}^2|n\rangle, \tag{12}$$

$$\sum_m (E_m - E_n)^3|\boldsymbol{r}_{mn}|^2 = \frac{1}{2\mu^2}\langle n|\nabla^2 U|n\rangle, \tag{13}$$

$$\sum_m (E_m - E_n)^4|\boldsymbol{r}_{mn}|^2 = \frac{1}{\mu^2}\langle n|(\nabla U)^2|n\rangle, \tag{14}$$

$$\sum_m (E_m - E_n)^5|\boldsymbol{r}_{mn}|^2 = \frac{1}{2\mu^3}\langle n|(\nabla\nabla U):(\nabla\nabla U)|n\rangle. \tag{15}$$

Eq.(11) is the so-called TRK sum rule, which has many applications [4] in atomic physics together with formula (12). Novikov *et al.* [5] used (11) to discuss the $E1$ transition of charmonium, while Song [6] used (13) to discuss the possible form of the confinement potential.

## 3. THE SUM RULE REPRESENTED BY THE E1 TRANSITION WIDTHS

The matrix element $\vec{r}_{mn}$ and the width of the $E1$ transition are directly related. This relation can be used to express the above sum rules in terms of the width of the $E1$ transition.

For charmonium, the eigenstate of $H$ is usually denoted as $|nlsjm>$, where $n$ is the main quantum number, $l$ is the orbit angular momentum number, $s$ is the total spin, $j$ and $m$ are the total angular momentum number and its projection, respectively. Since the energy levels are degenerated for quantum number $m$ and the left hand sides of the above sum rules are independent of quantum number $m$, if defining

$$\bar{r}(n'l's'j', nlsj) \equiv \frac{1}{2j+1}\sum_m \sum_{m'} |\langle n'l's'j'm'|\boldsymbol{r}|nlsjm\rangle|^2, \tag{16}$$

then Eq.(11) can be written as

$$\sum_{n'l's'j'} (E_{n'l's'j'} - E_{nlsj})\bar{r}(n'l's'j', nlsj) = \frac{3}{2\mu}. \tag{17}$$

and Eqs.(12)-(15) also can be converted into the similar forms.

The measured width of the $E1$ transition from an initial state $|n_il_is_ij_i>$ into a final state $|n_fl_fs_fj_f>$ with a photon of energy $\omega_{if}$ being emitted is

$$\Gamma(n_il_is_ij_i \to n_fl_fs_fj_f) = \frac{1}{2j_i+1}\sum_{m_i}\sum_{m_f}\frac{4}{3}\alpha e_Q^2\omega_{if}^3|\langle n_il_is_ij_im_i|\boldsymbol{r}|n_fl_fs_fj_fm_f\rangle|^2, \tag{18}$$

where $\alpha$ is the fine-structure constant, $e_Q$ is the electric charge of the quark and

$$\omega_{if} = E_{n_il_is_ij_i} - E_{n_fl_fs_fj_f}. \tag{19}$$

After summing over the final state polarizations and averaging over the initial states, we can have [7]

$$\Gamma(n_i l_i s_i j_i \rightarrow n_f l_f s_f j_f) = \frac{4}{3} e_Q^2 \alpha \omega_{if}^3 C(j_i l_i j_f l_f s) \delta_{s_i s_f} \langle r \rangle^2, \tag{20}$$

where the statistical factor $C(j_i l_i j_f l_f s)$ is

$$C(j_i l_i j_f l_f s) = \max(l_i, l_f)(2j_f + 1)\begin{Bmatrix} l_f & j_f & s \\ j_i & l_i & 1 \end{Bmatrix}^2, \tag{21}$$

and $<\vec{r}>$ is the radical integral

$$\langle r \rangle = \int_0^\infty r R_{n_f l_f}(r) R_{n_i l_i}(r) r^2 \mathrm{d}r. \tag{22}$$

According to Eqs. (16), (18) and (20), if $j = j_i$ and $j' = j_f$, then

$$\bar{r}(n'l's'j', nlsj) = \frac{\Gamma(nlsj \rightarrow n'l's'j')}{\frac{4}{3} \alpha e_Q^2 \omega_{if}^3}, \tag{23}$$

and

$$\bar{r}(n'l's'j', nlsj) = C(jlj'l's)\langle r \rangle^2. \tag{24}$$

While if $j' = j_i$ and $j = j_f$, then

$$\bar{r}(n'l's'j', nlsj) = \frac{2j' + 1}{2j + 1} \cdot \frac{\Gamma(n'l's'j' \rightarrow nlsj)}{\frac{4}{3} \alpha e_Q^2 \omega_{if}^3} \tag{25}$$

and

$$\bar{r}(n'l's'j', nlsj) = \frac{2j' + 1}{2j + 1} \cdot C(j'l'jls)\langle r \rangle^2. \tag{26}$$

Using these relations, we can express the first five order sum rules mentioned above as

$$\sum_{n'l's'j'} \frac{1}{(E_{n'l's'j'} - E_{nlsj})^2} \left(\frac{2j' + 1}{2j + 1}\right) \Gamma(n'l's'j' \rightarrow nlsj)$$

$$- \sum_{n'l's'j'} \frac{1}{(E_{nlsj} - E_{n'l's'j'})^2} \Gamma(nlsj \rightarrow n'l's'j') = \frac{2\alpha e_Q^2}{\mu}, \tag{27}$$

$$\sum_{n'l's'j'} \frac{1}{E_{n'l's'j'} - E_{nlsj}} \left(\frac{2j' + 1}{2j + 1}\right) \Gamma(n'l's'j' \rightarrow nlsj)$$

$$+ \sum_{n'l's'j'} \frac{1}{E_{nlsj} - E_{n'l's'j'}} \Gamma(nlsj \rightarrow n'l's'j') \tag{28}$$

$$= \frac{4\alpha e_Q^2}{3\mu^2} \langle nlsj | \mathbf{p}^2 | nlsj \rangle,$$

$$\sum_{n'l's'j'}\left(\frac{2j'+1}{2j+1}\right)\Gamma(n'l's'j'\rightarrow nlsj)-\sum_{n'l's'j'}\Gamma(nlsj\rightarrow n'l's'j')$$
$$=\frac{2\alpha e_Q^2}{3\mu^2}\langle nlsj|\nabla^2U|nlsj\rangle, \tag{29}$$

$$\sum_{n'l's'j'}(E_{n'l's'j'}-E_{nlsj})\left(\frac{2j'+1}{2j+1}\right)\Gamma(n'l's'j'\rightarrow nlsj)$$
$$+\sum_{n'l's'j'}(E_{nlsj}-E_{n'l's'j'})\Gamma(nlsj\rightarrow n'l's'j')$$
$$=\frac{4\alpha e_Q^2}{3\mu^2}\langle nlsj|(\nabla U)^2|nlsj\rangle, \tag{30}$$

$$\sum_{n'l's'j'}(E_{n'l's'j'}-E_{nlsj})^2\left(\frac{2j'+1}{2j+1}\right)\Gamma(n'l's'j'\rightarrow nlsj)$$
$$-\sum_{n'l's'j'}(E_{nlsj}-E_{n'l's'j'})^2\Gamma(nlsj\rightarrow n'l's'j)$$
$$=\frac{2\alpha e_Q^2}{3\mu^3}\langle nlsj|(\nabla\nabla U):(\nabla\nabla U)|nlsj\rangle. \tag{31}$$

With the properly simplified assumptions, in principle, each order sum rule can give a theoretical restriction on the width of the $E1$ transition for a given $|nlsj>$ state. However, because the right hand side of the fifth order sum rule Eq.(31) is quite complicated and the integration needs be performed numerically an explicit result cannot be obtained from it. The third order and fourth order sum rules will be discussed in another article. Here we are going to do some quantitative study on the first order and second order sum rules.

## 4. THE RESTRICTIONS ON THE WIDTHS OF THE $E1$ TRANSITION FOR THE $\psi$ (3770) BY SUM RULES

Let us take $|nlsj> = |1^3P_J>$, i.e., $n = 1$, $l = 1$, $s = 1$ and $j = J$. Taking this state (i.e., $\chi_{cJ}(J = 0, 1, 2)$) as an initial state, the possible final state is only $|1^3S_1>$ (i.e., $J/\psi$). Taking $|1^3P_J>$ as a final state, the possible initial state can be $|2^3S_1>$, $|3^3S_1>$, . . .; $|1^3D_J'>$, $|2^3D_J'>$, . . .; . . . For the cc family, the observed $E1$ transitions in experiments are $\chi_{cJ}\rightarrow\gamma J/\psi$, $\psi$ (3686) $\rightarrow\gamma\chi_{cJ}$, and the processed of $\psi$ (3770) $\rightarrow\gamma\chi_{cJ}(J = 0, 1, 2)$ which are reported by the Mark III Collaboration recently. Some theories [8] estimate that the widths of the $E1$ transition from higher quantum number levels into the $\chi_{cJ}$ are very narrow, and can be ignored in the sum rule. Now using the first order and the second order sum rule separately, we will derive the restrictions on the decay width of $\psi$ (3770) $\rightarrow\gamma\chi_{cJ}$ from the experimental results of $\chi_{cJ}\,\gamma J/\psi$ and $\psi$ (3686) $\rightarrow J\chi_{cJ}$ and then compare them with the data reported by the Mark III Collaboration.

(1) The First Order Sum Rule

Considering the statistical factors listed in Table 1, we know that

$$\Gamma(1^3D_2\rightarrow 1^3P_0)=0, \tag{32}$$

$$\Gamma(1^3D_3\rightarrow 1^3P_0)=0. \tag{33}$$

Hence, when $J = 0$, Eq.(27) leads to

$$\frac{1}{(E_{3770} - E_{\chi_{c0}})^2} \cdot 3\Gamma(1^3D_1 \to 1^3P_0) < \frac{2\alpha e_Q^2}{\mu} - \frac{1}{(E_{3636} - E_{\chi_{c0}})^2} \cdot 3\Gamma(2^3S_1 \to 1^3P_0) + \frac{1}{(E_{\chi_{c0}} - E_{J/\psi})^2} \Gamma(1^3P_0 \to 1^3S_1). \tag{34}$$

Taking $e_Q = 2/3$, choosing $m_c = 1.84$ GeV we used before [1] and inputting $\mu = m_c/2$ and the experimental values of $\Gamma(2^3S_1 \to 1^3P_0)$ and $\Gamma(1^3P_0 \to 1^3S_1)$ [9],

$$\Gamma(2^3S_1 \to 1^3P_0) \sim 23\text{keV}, \tag{35}$$

$$\Gamma(1^3P_0 \to 1^3S_1) \sim 95\text{keV}, \tag{36}$$

we can yield

$$\Gamma(1^3D_1 \to 1^3P_0) < 0.29\text{MeV}. \tag{37}$$

For $J = 1$ and $J = 2$, the contributions from the $1^3D_2$ and $1^3D_3$ states are no longer zero. However, these two states have not been observed experimentally. We assume that under the non-relativistic approximation, the three states of $1^3D_{J'}$ ($J' = 1, 2, 3$) are degenerated, their masses are
all taken to be 3770 MeV and the radial integral $<1D|r|1P>$ is considered identical for all $1^3D_{J'} \to 1^3P_J$ transitions. Therefore, the differences between the widths are completely determined by the statistical factor. Thus from Table 1 we can obtain

$$\frac{\Gamma(1^3D_2 \to 1^3P_1)}{\Gamma(1^3D_1 \to 1^3P_1)} = \frac{9}{5}, \tag{38}$$

$$\frac{\Gamma(1^3D_2 \to 1^3P_2)}{\Gamma(1^3D_1 \to 1^3P_2)} = 9, \tag{39}$$

$$\frac{\Gamma(1^3D_3 \to 1^3P_2)}{\Gamma(1^3D_1 \to 1^3P_2)} = 36. \tag{40}$$

Putting the above results into Eq.(27), similar to the case of $J = 0$, we can have

$$\Gamma(1^3D_1 \to 1^3P_1) < 0.14\text{MeV}. \tag{41}$$

and

$$\Gamma(1^3D_1 \to 1^3P_2) < 0.006\text{MeV}. \tag{42}$$

(2) The Second Order Sum Rule

The right hand side of the second order sum rule (28) is proportional to the average kinematic energy. According to the Virial theorem, when $U(\vec{r}) \equiv U(r)$, one finds the following relation,

$$\left\langle \frac{p^2}{2\mu} \right\rangle = \frac{1}{2} \left\langle r \frac{dU}{dr} \right\rangle. \tag{43}$$

If a logarithm potential [10] is chosen,

$$U(r) = c \ln\left(\frac{r}{r_0}\right), \tag{44}$$

Table 1
The statistical factor $C(J2J11)$ for transition ${}^3D_{J'} \to {}^3P_J$.

| $J'$ \ $J$ | 0 | 1 | 2 |
|---|---|---|---|
| 1 | $\frac{2}{9}$ | $\frac{1}{6}$ | $\frac{1}{90}$ |
| 2 | 0 | $\frac{3}{10}$ | $\frac{1}{10}$ |
| 3 | 0 | 0 | $\frac{2}{5}$ |

where

$$c = 0.733\text{GeV}, \quad r_0 = 0.89\text{GeV}^{-1}, \tag{45}$$

from Eq.(28) we have

$$\sum_{n'l's'j} \frac{1}{E_{n'l's'j'} - E_{nlsj}} \left(\frac{2j'+1}{2j+1}\right) \Gamma(n'l's'j' \to nlsj) + \sum_{n'l's'j'} \frac{1}{E_{nlsj} - E_{n'l's'j'}} \Gamma(nlsj \to n'l's'j') = \frac{4\alpha e_Q^2 c}{3\mu}. \tag{46}$$

Taking $m_c = 1.84$ GeV as before and treating the equation the same way as the first order sum rule (27), we can obtain

$$\Gamma(1^3D_1 \to 1^3P_0) < 0.34\text{MeV}, \tag{47}$$

$$\Gamma(1^3D_1 \to 1^3P_1) < 0.16\text{MeV}, \tag{48}$$

$$\Gamma(1^3D_1 \to 1^3P_2) < 0.006\text{MeV}. \tag{49}$$

## 5. THE WIGNER SUM RULE AND ITS RESTRICTION ON THE WIDTH OF THE $E1$ TRANSITION FOR THE $\psi$ (3770)

The TRK-type sum rule can only give the upper bounds on the widths of the $E1$ transitions for the $\psi$ (3770). Wigner introduced another kind of sum rule which gives not only the upper bound on the width of the $E1$ transition but also the lower one on it. In the following, using the commutation relations for the operator of the angular momentum and the ones of coordinator and momentum, we shall first derive the concrete form of this kind of sum rule according to the treatment in Ref.[5], and then give some quantitative discussion.

Defining $c = l(l+1)$, $c_+ = (l+1)(l+2)$, $c_- = (l-1)l$, and introducing two operators,

$$\pi^{\pm} = \frac{L^2 - c_{\mp}}{c_{\pm} - c_{\mp}}. \tag{50}$$

we can easily prove

$$\pi^{+}|l+1,m\rangle = |l+1,m\rangle, \tag{51}$$

$$\pi^{+}|l-1,m\rangle = 0, \tag{52}$$

$$\pi^{-}|l+1,m\rangle = 0, \tag{53}$$

$$\pi^{-}|l-1,m\rangle = |l-1,m\rangle, \tag{54}$$

i.e., $\pi^{\pm}$ are two projection operators.

Using the basic commutation relations for $\vec{r}$, $\vec{p}$ and $\vec{L}$ and

$$[\boldsymbol{p},\boldsymbol{L}^2] = i(\boldsymbol{L}\times\boldsymbol{p}-\boldsymbol{p}\times\boldsymbol{L}), \tag{55}$$

$$[\boldsymbol{r},\boldsymbol{L}] = i(\boldsymbol{L}\times\boldsymbol{r}-\boldsymbol{r}\times\boldsymbol{L}), \tag{56}$$

$$\boldsymbol{p}\times\boldsymbol{L}+\boldsymbol{L}\times\boldsymbol{p} = 2i\boldsymbol{p}, \tag{57}$$

$$\boldsymbol{r}\times\boldsymbol{L}+\boldsymbol{L}\times\boldsymbol{p} = 2i\boldsymbol{r}. \tag{58}$$

it can be proved that

$$\boldsymbol{r}\cdot\pi^{+}\boldsymbol{p}-\boldsymbol{p}\cdot\pi^{+}\boldsymbol{r} = i\,\frac{7\boldsymbol{L}^2+6-3c_{-}}{2(2L+1)}, \tag{59}$$

$$\boldsymbol{r}\cdot\pi^{-}\boldsymbol{p}-\boldsymbol{p}\cdot\pi^{-}\boldsymbol{r} = -i\,\frac{7\boldsymbol{L}^2+6-3c_{+}}{2(2L+1)}. \tag{60}$$

Using Eqs.(59) and (60) and the properties of projection operator $\pi^{\pm}$ and the selective rule of $\Delta l = \pm 1$ subject to the matrix elements of $\vec{r}$ and $\vec{p}$ between the eigenstates of the orbit angular momentum, we can derive the following two Wigner sum rules

$$\sum_{n}\sum_{m'=-(l+1)}^{l+1}|\langle n,l+1,m'|\boldsymbol{r}|klm\rangle|^2 m_c(E_{n,l+1}-E_{k,l}) = \frac{(2l+3)(l+1)}{2l+1}, \tag{61}$$

and

$$\sum_{n}\sum_{m'=-(l-1)}^{l-1}|\langle n',l-1,m'|\boldsymbol{r}|klm\rangle|^2 m_c(E_{n,l-1}-E_{k,l}) = -\frac{l(2l-1)}{2l+1}, \tag{62}$$

where $m_c$ is the mass of the c-quark.

Since the right hand sides of Eqs.(61) and (62) are independent of quantum number $m$, they can further evolve as

$$\sum_{n} m_c(E_{n,l+1}-E_{k,l})\cdot\frac{1}{2l+1}\sum_{m}\sum_{m'}|\langle n,l+1,m'|\boldsymbol{r}|klm\rangle|^2 = \frac{(2l+3)\,(l+1)}{2l+1}. \tag{63}$$

and
$$\sum_n m_c(E_{n,l-1} - E_{k,l}) \cdot \frac{1}{2l+1} \sum_m \sum_{m'} |\langle n, l-1, m' | r | klm \rangle|^2 = -\frac{l(2l-1)}{2l+1}. \tag{64}$$

In the light of the properties of the irreducible tensor and Wigner-Eckart Theorem, it can be easily proved that

$$\frac{1}{2l+1} \sum_m \sum_{m'} |\langle n, l+1, m' | r | klm \rangle|^2 = \frac{l+1}{2l+1} |\langle n, l+1 | r | kl \rangle|^2, \tag{65}$$

$$\frac{1}{2l+1} \sum_m \sum_{m'} |\langle n, l-1, m' | r | klm \rangle|^2 = \frac{l}{2l+1} |\langle n, l-1 | r | kl \rangle|^2. \tag{66}$$

Putting (65) and (66) into (61) and (62) respectively, we obtain the resulting form of the Wigner sum rules:

$$\sum_n (E_{n,l+1} - E_{k,l}) |\langle n, l+1 | r | kl \rangle|^2 = \frac{2l+3}{m_c}. \tag{67}$$

and

$$\sum_n (E_{n,l-1} - E_{k,l}) |\langle n, l-1 | r | kl \rangle|^2 = -\frac{2l-1}{m_c}. \tag{68}$$

Letting $l = 1$ and ignoring all the contributions from $n > 1$ states in Eq.(67), we can obtain

$$\langle 1D | r | 1P \rangle^2 < \frac{5}{m_c(E_{1D} - E_{1P})}. \tag{69}$$

Taking still $m_c = 1840$ MeV and considering

$$E_D - E_P = 3770 - M_{c.o.g}(P_J) = 245\text{MeV}. \tag{70}$$

we have

$$\langle 1D | r | 1P \rangle^2 < 11.1 \times 10^{-6} (\text{MeV})^{-2}. \tag{71}$$

Putting it into Eq.(20), we can calculate the upper bounds on the decay widths for the $\psi$ (3770) $\rightarrow \gamma\chi_{cJ}$, ($J$ = 0, 1, 2) decay,

$$\Gamma(1^3D_1 \rightarrow 1^3P_0) < 0.48\text{MeV}, \tag{72}$$

$$\Gamma(1^3D_1 \rightarrow 1^3P_1) < 0.14\text{MeV}, \tag{73}$$

$$\Gamma(1^3D_1 \rightarrow 1^3P_2) < 0.0052\text{MeV}. \tag{74}$$

From Eq.(68), letting $l = 2$ and ignoring all the contributions from $n > 1$ states, we can obtain

$$\langle 1D | r | 1P \rangle^2 > \frac{3}{m_c(E_{1D} - E_{1P})}. \tag{75}$$

With the same input, we can have

$$\langle 1D | r | 1P \rangle^2 > 6.7 \times 10^{-6} (\text{MeV})^{-2}. \tag{76}$$

**Table 2**
Electric dipole transition widths of the $\psi$ (3770).

| Process | TRK sum rules upper values (keV) | | Wigner sum rules results (keV) | | Calculation of potential model (keV) | Mark III experimental values (keV) |
|---|---|---|---|---|---|---|
| | First | Second | Upper | Lower | | |
| $1^3D_1 \to 1^3P_0$ | 290 | 340 | 480 | 290 | 312 (183) | 500 ± 200 |
| $1^3D_1 \to 1^3P_1$ | 140 | 160 | 140 | 85 | 95 (70) | 430 ± 180 |
| $1^3D_1 \to 1^3P_2$ | 6 | 6 | 5.2 | 3.1 | 3.6 (3.0) | ≤ 500 |

Note: The physical states are the following: $1^3D_1$: $\psi$ (3770); $1^3P_0$:$\chi_{c0}$(3415); $1^3P_1$: $\chi_{c1}$(3510); $1^3P_2$:$\chi_{c2}$(3555). In the column of "potential model", the values outside the parenthesis are non-relativistic values and those in the parenthesis are the results with the first order relativistic corrections. In addition, the "upper bound" is abbreviated to "U. B." and the "lower bound" to "L. B.".

Putting it into Eq.(20), we can calculate the lower bounds on the decay width for $\psi$ (3770) → $\gamma\chi_J$ ($J$ = 0, 1, 2),

$$\Gamma(1^3D_1 \to 1^3P_0) > 0.29\text{MeV}, \tag{77}$$

$$\Gamma(1^3D_1 \to 1^3P_1) > 0.085\text{MeV}, \tag{78}$$

$$\Gamma(1^3D_1 \to 1^3P_2) > 0.0031\text{MeV}. \tag{79}$$

## 6. DISCUSSIONS AND CONCLUSIONS

(1) In order to compare the upper and lower bounds on the widths of the transition $\psi$ (3770) → $\gamma\chi_J$ obtained above with the theoretical values calculated from our potential model and the experimental values reported by the Mark III Collaboration, we list them in Table 2 altogether. From this table, it is clear that the widths predicted by our potential model are in agreement with the upper and lower bounds from the sum rules, but they have obvious discrepancies with the experimental results reported by the Mark III Collaboration, which further confirms our previous conclusion [1,2]. In addition, the events collected by Mark III are not yet sufficient and the experimental error is rather large, therefore it is premature to make a judgment. If BEPC (Beijing Electron Position Collider) can accumulate more $\psi$ (3770) events and give more precise experimental results, it will undoubtedly be significant to test the potential model.

(2) The sum rules are derived from the non-relativistic Hamiltonian. In the $c\bar{c}$ system, because of the relativistic effects, there should be significant corrections to the upper and lower bounds given by the sum rules. But the experiences demonstrate that the corrections may further suppress the decay width. So it would not mount any influence on our general conclusion that the experimental results turn out to be larger.

(3) Not only are the second order sum rules of stronger model dependence because of our assumption about the logarithm potential, the first order sum rule and the Wigner sum rule rely upon the model only through the c-quark mass and have no elicit relationship with the concrete form of the potential. With the decrease of the c-quark mass, both the upper bound and lower bound will rise. For example, if we select $m_c$ = 1.6 GeV instead of 1.84 GeV, then the bounds will increase by

about 10%, which causes no substantial change to our conclusion.

## REFERENCES


[1] D. H. Qin, Y. B. Ding and G. D. Zhao, *High Energy Phys. and Nucl. Phys.* (in Chinese), **14** (1990) 380.

[2] R. H. Schindler, SLAC-Pub-4694 (1988).

[3] D. H. Qin, Y. B. Ding and G. D. Zhao, *High Energy Phys. and Nucl. Phys.* (in Chinese), **15** (1991) 284.

[4] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One and Two-Electron Atoms*, (Springer, 1957).

[5] V. A. Novikov *et al.*, *Phys. Reports*, **41** (1978) 1.

[6] X. T. Song, *Phys. Lett.*, **246B** (1990) 195.

[7] W. Kwong and J. L. Rosner, *Phys. Rev.*, **38D** (1988) 279.

[8] W. Buchmüller and S. H. H. Tye., *Phys. Rev.*, **24D** (1981) 32.

[9] G. P. Yost *et al.*, Particle Data Group, *Phys. Lett.*, **204** (1988) 1.

[10] C. Quigg and Rosner, *Phys. Lett.*, **71B** (1977) 153.

[11] E. Wigner, *Phys. Z.*, **32** (1931) 450.