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Chiral Symmetry, Ward-Takahashi Identities and Mass spectra in (2 + 1) Dimensional Chiral Gross-Neveu Model

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Chiral Ward-Takahashi identities with composite fields are applied to investigate mass spectra in (2 + 1) dimensional chiral gross-Neveu model. The fermion mass and bound state spectra are obtained, which are in agreement with large-N expansion in the lowest approximation. When the chiral symmetry is an approximate one, we obtain the PCAC.

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1. INTRODUCTION

QCD is currently accepted as the most possible candidate of the fundamental theory of strong interactions [1]. Since it has asymptotic freedom at high energies, we can apply the perturbative theory to study hard processes. However, in the low-energy case the interaction becomes so strong that the perturbative expansion fails and a non-perturbative scheme should be applied. Due to the inherent difficulties in the study of nonperturbative phenomena, the bound states, chiral symmetry and the mass spectra in QCD are still unsolved problems [2,3].

As Gross-Neveu model possesses some features of QCD, asymptotic freedom, chiral symmetry dynamical breaking and fermion mass generation etc. [4], it is expected that the study of chiral Gross-Neveu model will help us understand the low energy properties of QCD. Recently, in addition to large N expansion, such as the methods of variation, background field and the Gauss effective potential [5-7], many methods have been proposed and used to investigate the properties of mass spectra in Gross-Neveu model. In this paper, we develop Ward-Takahashi identities to include composite fields and use them to study the mass spectra of fermion and the bound states in chiral Gross-Neveu model.

Given the facts that the spontaneous breaking of a continuous symmetry does not occur in (1 + 1) dimensions [8] and that (2 + 1) dimensional chiral Gross-Neveu model is renormalizable in the

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$1/N$ expansion [9], we take (2 + 1) dimensional chiral Gross-Neveu model as an example to investigate the properties of mass spectra.

In order to describe dynamical breakings, composite external sources are introduced in the generating functional in Section 2. According to chiral symmetry, chiral Ward-Takahashi identities with composite fields are obtained. In Section 3, with the aid of Ward-Takahashi identities we obtain the mass spectra of fermion and bound states. In the lowest order of approximation, the fermion mass spectrum is identical to that in the $1/N$ expansion. If $<\bar{\psi}i\gamma_5\psi> = 0$, composite field $\bar{\psi}(x)i\gamma_5\psi(x)$ corresponds to a Goldstone boson after symmetry breaking. Since quarks are massive in QCD and chiral symmetry is an approximative symmetry, we study this case in Section 4. As a fermion mass term can be regarded as a composite external source, the generating functional with massive fermion can be expressed by the chiral symmetric generating functional. So chiral ward-Takahashi identities and the mass spectra are obtained, and the properties of partially conserved axial current are discussed.

2. EFFECTIVE ACTION AND CHIRAL WARD-TAKAHASHI IDENTITIES

The Gross-Neveu model is a low-dimension field model with asymptotic freedom [4]. It is renormalizable and possesses dynamical breaking.

The (2 + 1) dimensional chiral Gross-Neveu model is described by the following Lagrangian [5]:

$$\mathscr{L} = -\bar{\psi}\gamma \cdot \partial\psi - \frac{g}{2N}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2]. \tag{2.1}$$

where ψ^a $(a = 1, 2, \ldots, N)$ is a massless fermion field.

When the field transforms is

$$\delta\psi(x) = \frac{i}{2}(\alpha + \gamma_5\beta)\psi(x), \tag{2.2a}$$

$$\delta\bar{\psi}(x) = \bar{\psi}(x)\frac{i}{2}(-\alpha + \gamma_5\beta), \tag{2.2b}$$

the Lagrangian remains invariant. Correspondingly, the Noether currents are

$$J_\mu(x) = \bar{\psi}(x)\frac{1}{2}\gamma_\mu\psi(x), \tag{2.3a}$$

$$A_\mu(x) = \bar{\psi}(x)\frac{1}{2}\gamma_\mu\gamma_5\psi(x). \tag{2.3b}$$

The chiral transformative generators are

$$Q = \int d^2x J_0(x), \tag{2.4a}$$

$$Q_5 = \int d^2x A_0(x). \tag{2.4b}$$

According to Eq.(2.1), if $g < 0$, the attractive interaction between fermions makes the perturbative vacuum unstable and there are fermion pair condensates. Then, chiral symmetry will break down spontaneously, which is usually called dynamical spontaneous breaking [4].

In order to describe dynamical breakings, composite external sources $K(x)$, $K_5(x)$, corresponding to the composite fields $\bar{\psi}(x)\psi(x)$, $\bar{\psi}(x)i\gamma_5\psi(x)$ are introduced in the generating functional [4,10]. Thus, the generating functional is

$$Z[\bar{\eta},\eta;K,K_5] = e^{iW[\bar{\eta},\eta;K,K_5]} = \int D[\bar{\psi},\psi]\exp$$

$$\left(i\int d^3x[\mathscr{L} + \bar{\eta}(x)\psi(x) + \bar{\psi}(x)\eta(x) + \bar{\psi}(x)\psi(x)K(x) + \bar{\psi}(x)i\gamma_5\psi(x)K_5(x)]\right). \quad (2.5)$$

where $W[\bar{\eta},\eta;K,K_5]$ is called the generating functional of connected Green's functions. Define

$$\frac{\delta W}{\delta\eta(x)} = -\bar{\psi}_c(x), \quad (2.6a)$$

$$\frac{\delta W}{\delta\bar{\eta}(x)} = \psi_c(x), \quad (2.6b)$$

$$-\frac{1}{i}\frac{\delta}{\delta\eta(x)}\frac{\delta}{\delta\bar{\eta}(x)}W = G(x), \quad (2.6c)$$

$$-\frac{1}{i}\frac{\delta}{\delta\eta(x)}i\gamma_5\frac{\delta}{\delta\bar{\eta}(x)}W = G_5(x). \quad (2.6d)$$

According to Eq.(2.6),

$$\frac{\delta W}{\delta K(x)} = G(x) + \bar{\psi}_c(x)\psi_c(x), \quad (2.7a)$$

$$\frac{\delta W}{\delta K_5(x)} = G_5(x) + \bar{\psi}_c(x)i\gamma_5\psi_c(x). \quad (2.7b)$$

Taking Legendre transformation, we obtain the effective action

$$\Gamma[\bar{\psi}_c,\psi_c;G,G_5] = W[\bar{\eta},\eta;K,K_5] - \int d^3x[\bar{\psi}_c(x)\eta(x) + \bar{\eta}(x)\psi_c(x)]$$

$$-\int d^3x(K(x)[G(x) + \bar{\psi}_c(x)\psi_c(x)] + K_5(x)[G_5(x) + \bar{\psi}_c(x)i\gamma_5\psi_c(x)]). \quad (2.8a)$$

Correspondingly,

$$\frac{\delta\Gamma}{\delta\psi_c(x)} = \bar{\eta}(x) + \bar{\psi}_c(x)[K(x) + i\gamma_5K_5(x)], \quad (2.9a)$$

$$\frac{\delta\Gamma}{\delta\bar{\psi}_c(x)} = -\eta(x) - [K(x) + i\gamma_5K_5(x)]\psi_c(x), \quad (2.9b)$$

$$\frac{\delta\Gamma}{\delta G(x)} = -K(x), \quad (2.9c)$$

$$\frac{\delta\Gamma}{\delta G_5(x)} = -K_5(x). \quad (2.9d)$$

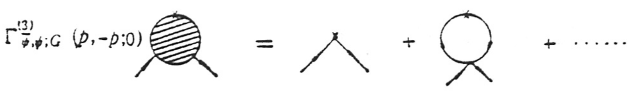

Fig. 1
The exact proper vertex.

Under the chiral transformation, the external sources transform as

$$\delta\bar{\eta}(x) = \bar{\eta}(x)\frac{-i}{2}(\alpha + i\gamma_5\beta), \tag{2.10a}$$

$$\delta\eta(x) = \frac{-i}{2}(\gamma_5\beta - \alpha)\eta(x), \tag{2.10b}$$

$$\delta K(x) = \beta K_5(x), \tag{2.10c}$$

$$\delta K_5(x) = -\beta K(x). \tag{2.10d}$$

Since $W[\bar{\eta},\eta;K,K_5]$ remains invariant, we have

$$\int d^3x\left[\bar{\eta}(x)\frac{\delta W}{\delta\bar{\eta}(x)} + \frac{\delta W}{\delta\eta(x)}\eta(x)\right] = 0. \tag{2.11a}$$

$$\int d^3x\left[\bar{\eta}(x)\frac{i}{2}\gamma_5\frac{\delta W}{\delta\bar{\eta}(x)} - \frac{\delta W}{\delta\eta(x)}\frac{i}{2}\gamma_5\eta(x) + \frac{\delta W}{\delta K_5(x)}K(x) - \frac{\delta W}{\delta K(x)}K_5(x)\right] = 0. \tag{2.11b}$$

These are the chiral Ward-Takahashi identities for the generating functional of connected Green's functions. Taking several derivatives of Eq.(2.11) with respect to external sources, we can obtain some Ward-Takahashi identities between Green's functions in the absence of external sources.

In order to derive mass spectra, we express Eq.(2.11) in the form of the effective action. After using Eq.(2.6-7) and (2.9), Eq.(2.11) is rewritten as

$$\int d^3x\left[\frac{\delta\Gamma}{\delta\psi_c(x)}\psi_c(x) + \bar{\psi}_c(x)\frac{\delta\Gamma}{\delta\bar{\psi}_c(x)}\right] = 0, \tag{2.12a}$$

$$\int d^3x\left[\bar{\psi}_c(x)\frac{i}{2}\gamma_5\frac{\delta\Gamma}{\delta\bar{\psi}_c(x)} - \frac{\delta\Gamma}{\delta\psi_c(x)}\frac{i}{2}\gamma_5\psi_c(x) + \frac{\delta\Gamma}{\delta G(x)}G_5(x) - \frac{\delta\Gamma}{\delta G_5(x)}G(x)\right] = 0. \tag{2.12b}$$

which are the chiral Ward-Takahashi identities for the effective action. Differentiating Eq.(2.12) several times with respect to classical fields, we can obtain some Ward-Takahashi identities for the proper vertexes. With the aid of these identities, we can determine the mass spectra when chiral symmetry is spontaneously broken.

3. MASS SPECTRA

After symmetry realization, the axial symmetry is spontaneously broken and the fermion mass is dynamically generated. Therefore, the axial Ward-Takahashi identity plays an important role in deriving the mass spectra.

At first, we determine the fermion mass. Taking derivatives of Eq.(2.12b) with respect to $\psi_c(y)$ and $\bar{\psi}_c(z)$, we have

$$\int d^3x \left[\bar{\psi}_c(x)\frac{i}{2}\gamma_5\frac{\delta^3\Gamma}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\bar{\psi}_c(x)} - \delta^3(x-z)\frac{i}{2}\gamma_5\frac{\delta^2\Gamma}{\delta\psi_c(y)\delta\bar{\psi}_c(x)}\right.$$
$$- \frac{\delta^3\Gamma}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\psi_c(x)}\frac{i}{2}\gamma_5\psi_c(x) + \frac{\delta^2\Gamma}{\delta\bar{\psi}_c(z)\delta\psi_c(x)}\frac{i}{2}\gamma_5\delta^3(x-y)$$
$$\left. + \frac{\delta^3\Gamma}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta G(x)}G_5(x) - \frac{\delta^3\Gamma}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta G_5(x)}G(x)\right] = 0. \tag{3.1}$$

In the absence of external sources,

$$\psi_c(x)|_{J\to 0} = \bar{\psi}_c(x)|_{J\to 0} = 0. \tag{3.2}$$

where J denotes all external sources. Using Eqs.(2.7a-b), we can express Eq.(3.1) as

$$\frac{i}{2}\gamma_5\Gamma^{(2)}_{\psi,\bar{\psi}}(y,z) + \Gamma^{(2)}_{\psi,\bar{\psi}}(y,z)\frac{i}{2}\gamma_5$$
$$= \int d^3x[\Gamma^{(3)}_{\bar{\psi},\psi;G}(y,z;x)\langle\bar{\psi}i\gamma_5\psi\rangle - \Gamma^{(3)}_{\bar{\psi},\psi;G_5}(y,z;x)\langle\bar{\psi}\psi\rangle]. \tag{3.3}$$

According to the properties of chiral symmetry, we can obtain the relation between the vertexes

$$\Gamma^{(3)}_{\bar{\psi},\psi;G}(y,z;x) = -i\gamma_5\Gamma^{(3)}_{\bar{\psi},\psi;G_5}(y,z;x). \tag{3.4}$$

Using Eq.(3.4) and performing the Fourier transformation, we can rewrite Eq.(3.3) as

$$i\frac{1}{2}\gamma_5\Gamma^{(2)}_{\psi,\bar{\psi}}(p) + \Gamma^{(2)}_{\psi,\bar{\psi}}(p)\frac{i}{2}\gamma_5 = -i\gamma_5\Gamma^{(3)}_{\bar{\psi},\psi;G}(p,-p;0)(\langle\bar{\psi}\psi\rangle + i\gamma_5\langle\bar{\psi}i\gamma_5\psi\rangle), \tag{3.5}$$

where we have used the properties that the vacuum expectation values of $\bar{\psi}(x)\psi(x)$, $\bar{\psi}(x)i\gamma_5\psi(x)$ are independent of x.

Since the general form of $\Gamma^{(2)}_{\psi,\bar{\psi}}(p)$ can be expressed as

$$\Gamma^{(2)}_{\psi,\bar{\psi}}(p) = A(p^2)\gamma\cdot p + B(p^2). \tag{3.6}$$

on the right side of Eq.(3.5), the kinetic energy terms are cancelled with each other and only the self-energy term is left, i.e.,

$$\frac{i}{2}\gamma_5\Gamma^{(2)}_{\psi,\bar{\psi}}(p) + \Gamma^{(2)}_{\psi,\bar{\psi}}(p)\frac{i}{2}\gamma_5 = i\gamma_5 B(p^2). \tag{3.7}$$

From the above discussion, in the limit $p \to 0$, we obtain the fermion mass

$$m_f = \Gamma^{(3)}_{\bar{\psi},\psi;G}(0,0;0)(\langle\bar{\psi}\psi\rangle + i\gamma_5\langle\bar{\psi}i\gamma_5\psi\rangle). \tag{3.8}$$

This shows that if there is no fermion pair condensate, the fermion remains massless; if there are fermion pair condensates, the fermion will obtain a mass after the symmetry realization.

In the theory with dynamical breaking, there exist bound states which consist of composite fields. The bound states can be written as

$$\sigma(x) = a\bar{\psi}(x)\psi(x), \tag{3.9a}$$

$$\pi(x) = b\bar{\psi}(x)i\gamma_5\psi(x). \tag{3.9b}$$

Due to the properties of the bound states, in the chiral transformation, it is easy to obtain $a = b$. Taking the vacuum expectation value of Eq.(3.9a), we find

$$a = \frac{\langle\sigma\rangle}{\langle\bar{\psi}\psi\rangle}. \tag{3.10}$$

Differentiating Eq.(2.12b) with respect to $G_5(y)$, we have

$$\int d^3x \left[\bar{\psi}_c(x)\frac{i}{2}\gamma_5\frac{\delta^2\Gamma}{\delta G_5(y)\delta\bar{\psi}_c(x)} - \frac{\delta^2\Gamma}{\delta G_5(y)\delta\psi_c(x)}\frac{i}{2}\gamma_5\psi_c(x)\right.$$
$$\left. + \frac{\delta\Gamma}{\delta G(x)}\delta^3(x-y) + \frac{\delta^2\Gamma}{\delta G_5(y)\delta G(x)}G_5(x) - \frac{\delta^2\Gamma}{\delta G_5(y)\delta G_5(x)}G(x)\right] = 0. \tag{3.11}$$

Taking a derivative of Eq.(3.11) with respect to $G(z)$, we can obtain

$$\int d^3x \left[\bar{\psi}_c(x)\frac{i}{2}\gamma_5\frac{\delta^3\Gamma}{\delta G(z)\delta G_5(y)\delta\bar{\psi}_c(x)} - \frac{\delta^3\Gamma}{\delta G(z)\delta G_5(y)\delta\psi_c(x)}\frac{i}{2}\gamma_5\psi_c(x)\right.$$
$$+ \frac{\delta^2\Gamma}{\delta G(z)\delta G(x)}\delta^3(x-y) + \frac{\delta^3\Gamma}{\delta G(z)\delta G_5(y)\delta G(x)}G_5(x)$$
$$\left. - \frac{\delta^3\Gamma}{\delta G(z)\delta G_5(y)\delta G_5(x)}G(x) - \frac{\delta^2\Gamma}{\delta G_5(x)\delta G_5(y)}\delta^3(x-z)\right] = 0. \tag{3.12}$$

Using Eq.(3.2), we can reduce Eqs.(3.11) and (3.12) as

$$\Gamma^{(2)}_{G_5}(x,y) = \Gamma^{(2)}_{G,G_5}(x,y)\frac{\langle\bar{\psi}i\gamma_5\psi\rangle}{\langle\bar{\psi}\psi\rangle} \tag{3.13}$$

$$\Gamma^{(2)}_{G_5}(y,z) - \Gamma^{(2)}_{G}(y,z) = \int d^3x[\Gamma^{(3)}_{G,G_5;G_5}(z,y;x)\langle\bar{\psi}\psi\rangle - \Gamma^{(3)}_{G,G_5;G}(z,y;x)\langle\bar{\psi}i\gamma_5\psi\rangle]. \tag{3.14}$$

Taking the Fourier transformation and using Eq.(3.9), in the limit $p \to 0$, from Eqs.(3.13-14) we obtain the mass spectra of bound states σ and π

$$m_\pi^2 = -\Gamma^{(2)}_{G,G_5}(0)\langle\psi\psi\rangle\langle\bar{\psi}i\gamma_5\psi\rangle/\langle\sigma\rangle^2. \tag{3.15}$$

$$m_\sigma^2 = m_\pi^2 + \Gamma^{(3)}_{G,G_5;G_5}(0,0;0)\langle\bar{\psi}\psi\rangle^3/\langle\sigma\rangle^2$$
$$- \Gamma^{(3)}_{G,G_5;G}(0,0;0)\langle\bar{\psi}i\gamma_5\psi\rangle\langle\bar{\psi}\psi\rangle^2/\langle\sigma\rangle^2. \tag{3.16}$$

In the following, we discuss the fermion mass generation and the mass spectra of the bound states as well as Goldstone boson after the chiral symmetry realization.

From Eq.(3.8), we find that $\Gamma^{(3)}_{\bar{\psi},\psi;G}(p,-p;0)$ is an exact vertex factor, which contains the summation of all order in $1/N$, as shown in Fig. 1. In Fig. 1, × denotes the vertex connected with fermion pair condensate $<\bar{\psi}\psi>$, and the fermion lines are truncated. In the lowest approximation,

$$\Gamma^{(3)}_{\bar{\psi},\psi;G}(0,0;0) = g/N. \tag{3.17}$$

the fermion mass is

$$m_f = \frac{g}{N}[\langle\bar{\psi}\psi\rangle + i\gamma_5\langle\bar{\psi}i\gamma_5\psi\rangle]. \tag{3.18}$$

which is the same as the result of large-N expansion [4]. It should be noted that in the theory with dynamical breaking, the Lagrangian is divided into a free lagrangian and an interaction Lagrangian which are quite different from those in the perturbative theory [11,12]. For example, in the chiral Gross-Neveu model,

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_1 = \mathscr{L}'_0 + \mathscr{L}'_1. \tag{3.19}$$

where

$$\mathscr{L}_0 = -\bar{\psi}\gamma\cdot\partial\psi, \tag{3.20a}$$

$$\mathscr{L}_I = -\frac{g}{2N}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2]. \tag{3.20b}$$

$$\mathscr{L}'_0 = \mathscr{L}_0 - m\bar{\psi}\psi, \tag{3.20c}$$

$$\mathscr{L}'_I = \mathscr{L}_I + m\bar{\psi}\psi. \tag{3.20d}$$

Usually, in the perturbative expansion $\mathscr{L}_I$ is expanded based on $\mathscr{L}_0$; however, the lowest approximation in Eq.(3.17) corresponds to that where $\mathscr{L}'_I$ is expanded based on $\mathscr{L}'_0$.
If the broken direction is chosen such that

$$\langle\bar{\psi}i\gamma_5\psi\rangle = 0 \tag{3.21}$$

the mass spectra are

$$m_f = \Gamma^{(3)}_{\bar{\psi},\psi;\sigma}(0,0;0)\langle\sigma\rangle, \tag{3.22a}$$

$$m_\pi^2 = 0, \tag{3.22b}$$

$$m_\sigma^2 = \Gamma^{(3)}_{G,G_5;G_5}(0,0;0)\langle\bar{\psi}\psi\rangle^3/\langle\sigma\rangle^2. \tag{3.22c}$$

In Eq.(3.22a), we used Eq.(3.9a) to rewrite $\Gamma^{(3)}_{\bar{\psi},\psi:G}$ as $\Gamma^{(3)}_{\bar{\psi},\psi;\sigma}$. Eq.(3.22a) is the Goldberger-Treiman relation [13]. From Eq.(3.22b), we see that π meson is massless. From the commutators of composite fields

$$\langle 0|[Q_5,\bar{\psi}(x)\psi(x)]|0\rangle = 0, \tag{3.23a}$$

$$\langle 0|[Q_5,\bar{\psi}(x)i\gamma_5\psi(x)]|0\rangle \neq 0. \tag{3.23b}$$

and according to Goldstone's theorem [14], we see that the bound state π or the composite field $\bar{\psi}i\gamma_5\psi$ corresponds to the Goldstone boson after symmetry breaking.

According to the properties of $m_f>0$ and $<\bar{\psi}(x)\psi(x)> = -\mathrm{tr}S_F(0)$, both $<\bar{\psi}i\gamma_5\psi>$ and $<\bar{\psi}\psi>$ are negative. Thus, Eq.(3.8) requires

$$\Gamma^{(3)}_{\bar{\psi},\psi;G}(0,0;0) \leqslant 0. \tag{3.24}$$

which indicates that there is dynamical breaking only if the four-fermi interaction is attractive. Thus, the fermion acquires a mass.

Since (2 + 1) dimensional Gross-Neveu model is non-perturbative renormalizable, we can calculate the fermion mass in any order of $1/N$ from Eq.(3.8). Incidentally, it should be noted that the mass spectra are valid even if N is not very large, which is different from the large-N expansion.

4. MASS SPECTRA OF MASSIVE FERMION AND PCAC

In the previous sections, we have investigated the case of explicit chiral symmetry. However, quarks in QCD are massive and chiral symmetry is not explicit, but an approximative one. Therefore, we discuss the case where fermion is massive before the symmetry is broken.

Correspondingly, the Lagrangian with massive fermion is

$$\mathscr{L} = -\bar{\psi}(\gamma\cdot\partial + m_0)\psi - \frac{g}{2N}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2]. \tag{4.1}$$

where m_0 is a fermion mass before symmetry breaking. Because the fermion mass can be taken as a composite external source, we rewrite the Lagrangian as

$$\mathscr{L} = \mathscr{L}_s - m_0\bar{\psi}\psi. \tag{4.2}$$

where the subscript s denotes the case of explicit symmetry.

The generating functional is $Z[\bar{\eta},\eta;K,K_5] = e^{iW[\bar{\eta},\eta;K,K_5]}$

$$= \frac{\int D[\bar{\psi},\psi]\exp\left(i\int d^3x[\mathscr{L} + \bar{\eta}\psi + \bar{\psi}\eta + (K - m_0)\bar{\psi}\psi + K_5\bar{\psi}i\gamma_5\psi]\right)}{\int D[\bar{\psi},\psi]\exp\left(i\int d^3x[\mathscr{L} - m_0\bar{\psi}\psi]\right)}. \tag{4.3}$$

In terms of symmetric generating functional, the generating functional of the connected Green's function in the case of massive fermion can be re-expressed as

$$W[\bar{\eta},\eta;K,K_5] = W_s[\bar{\eta},\eta;K - m_0,K_5] - W_s[0,0;-m_0,0]. \tag{4.4}$$

After Legendre transformation, we have

$$\Gamma[\bar{\psi}_c,\psi_c;G,G_5] = \Gamma_s[\bar{\psi}_c,\psi_c;G,G_5] - m_0\int d^3x(\bar{\psi}_c(x)\psi_c(x) + G(x)) - W_s[0,0;-m_0,0]. \tag{4.5}$$

With the aid of chiral Ward-Takahashi identities for the symmetric effective action $\Gamma_s[\bar{\psi}_c(x), \psi_c(x); G, G_5]$, we can obtain those identities of the effective action $\Gamma[\bar{\psi}_c(x), \psi_c(x); G, G_5]$

$$\int d^3x\left[\frac{\delta\Gamma}{\delta\psi_c(x)}\psi_c(x) + \bar{\psi}_c(x)\frac{\delta\Gamma}{\delta\bar{\psi}_c(x)}\right] = 0. \tag{4.6a}$$

$$\int d^3x \left[\bar{\psi}_c(x) \frac{i}{2} \gamma_5 \frac{\delta \Gamma}{\delta \bar{\psi}_c(x)} - \frac{\delta \Gamma}{\delta \psi_c(x)} \frac{i}{2} \gamma_5 \psi_c(x) + m_0 (\bar{\psi}_c(x) i\gamma_5 \psi_c(x) + G_5(x)) + \frac{\delta \Gamma}{\delta G(x)} G_5(x) - \frac{\delta \Gamma}{\delta G_5(x)} G(x) \right] = 0. \tag{4.6b}$$

Similar to section 3, differentiating Eq.(4.6b) with respect to $\bar{\psi}_c(y)$ and $\psi_c(z)$, we have

$$\frac{i}{2} \gamma_5 \Gamma^{(2)}_{\psi,\bar{\psi}}(z, y) + \Gamma^{(2)}_{\psi,\bar{\psi}}(z, y) \frac{i}{2} \gamma_5 = \int d^3x [\Gamma^{(3)}_{\bar{\psi},\psi;G}(y, z; x) \langle \bar{\psi} i\gamma_5 \psi \rangle - \Gamma^{(3)}_{\bar{\psi},\psi;G_5}(y, z; x) \langle \bar{\psi}\psi \rangle] - i\gamma_5 m_0. \tag{4.7}$$

Since the explicit breaking results from the fermion mass term, the interaction Lagrangian remains invariant under the chiral transformation. Therefore,

$$\Gamma^{(3)}_{\bar{\psi},\psi;G_5}(y, z; x) = i\gamma_5 \Gamma^{(3)}_{\bar{\psi},\psi;G}(y, z; x). \tag{4.8}$$

Applying Eq.(4.8) and performing Fourier transformation, in the limit $p \to 0$, we can reduce Eq.(4.7) as

$$m_f = m_0 + \Gamma^{(3)}_{\bar{\psi},\psi;G}(0, 0; 0)(\langle \bar{\psi}\psi \rangle + i\gamma_5 \langle \bar{\psi} i\gamma_5 \psi \rangle). \tag{4.9}$$

Correspondingly, the bound state spectra are

$$m_\pi^2 = -(m_0 + \Gamma^{(2)}_{G_5;G}(0) \langle \bar{\psi} i\gamma_5 \psi \rangle) \langle \bar{\psi}\psi \rangle / \langle \sigma \rangle^2. \tag{4.10}$$

$$m_\sigma^2 = m_\pi^2 + \Gamma^{(3)}_{G,G_5;G_5}(0, 0; 0) \langle \bar{\psi}\psi \rangle^3 / \langle \sigma \rangle^2 - \Gamma^{(3)}_{G,G_5;G}(0, 0; 0) \langle \bar{\psi} i\gamma_5 \psi \rangle \langle \bar{\psi}\psi \rangle^2 / \langle \sigma \rangle^2. \tag{4.11}$$

According to Eq.(4.10), if the breaking direction is chosen such that $<\bar{\psi} i\gamma_5 \psi> = 0$, the mass spectrum of bound state π is not zero, but

$$m_\pi^2 = -m_0 \langle \bar{\psi}\psi \rangle / \langle \sigma \rangle^2. \tag{4.12}$$

which is in agreement with the result of Current Algebra [15]. Eq.(4.12) indicates that if fermion is massive, the bound state π obtains a mass; correspondingly, the axial current A_μ is not a conserved current.

From the definition of the axial current A_μ and the equation of motion, we have

$$\partial_\mu A_\mu(x) = f_\pi m_\pi^2 \pi(x). \tag{4.13}$$

where we used the definition and the mass spectrum of bound state π, and $f_\pi = -<\sigma>$. This shows that if the chiral symmetry is explicitly broken the axial current is partially conserved, which is very similar to PCAC.

It should be pointed out that usually the mass spectrum of bound state π and partially conserved axial current are derived under the asymptotic condition. However, those obtained here are valid and independent of the approximative method. Even if m_0 is not small and the chiral perturbative expansion cannot be applied, the mass spectra and the partially conserved axial current are still valid.

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