High Energy Physics and Nuclear Physics

Volume 15, Number 4

An Analysis of Energy Spectra and Determination of Spins for Superdeformed Bands in the A ~ 190 Region

Chen Xingqu and Xing Zheng

Department of Modern Physics, Lanzhou University, Lanzhou, Gansu, China

Fifteen superdeformed bands in seven nuclei of the A ~ 190 region are analyzed based on a phenomenological model. An overall and excellent agreement between the calculated and observed spectra E_γ, kinematic moment of the inertia $\mathscr{I}^{(1)}$ and dynamic moment of the inertia $\mathscr{I}^{(2)}$ is obtained for all superdeformed bands in the A ~ 190 region.

Recently, it was found that the rotational bands in different superdeformed (SD) nuclei in A ~ 150 and A ~ 190 regions have nearly identical transition energies (with differences within 1-2 keV). This is by no means accidental. It may have profound physical implications. Stephens *et al.* [1,2] pointed out that this feature can be characterized as a quantized alignment effect ($1/2\hbar$ or $1\hbar$). On the other hand, Wu *et al.* [3,4] mentioned that the quantized alignment proposed by Stephens *et al.* relies essentially on the accurate determination of the spins for SD bands. However, the spins are not yet measured for such bands and the tentative spin assignments have uncertainties. Therefore, if the spin values in these bands are shifted properly (e.g., by $1\hbar$), the original non-zero quantized spin alignments will disappear. Thus, whether or not the quantized alignment is a real fact depends entirely on the accurate determination of the spins in such bands. However, up to now the spin values in SD bands can not be measured yet. Thus, it is important whether the spin values in SD bands can be determined from γ-transition energies.

We have pointed out [5] that the SD band structure can be analyzed and the spin values of the SD band levels can be precisely determined from the observed transition energies by using W-Z expression [6-9]. In this paper our purpose is to perform the phenomenological analysis of the SD bands for all the present data in A ~ 190 region. First, the spin values of the SD bands are determined. Then, the energy spectra of these bands are fitted. Finally, the kinematic moments of

---

Supported by the National Natural Science Foundation and the Science Foundation of the State Education Commission of China.
Received on February 2, 1991.

Table 1
Various calculated results.

		$I_{exit}(\hbar)$	a(keV)	b	c	Δ(keV)	$(I_{exit})_{exp}(\hbar)$
^{192}Hg		8	46382	2.422×10^{-4}	5.768×10^{-10}	2.5	8[22]
^{191}Hg	$b1$	15.5	103906	1.008×10^{-4}	-2.469×10^{-10}	0.97	14.5[4]
	$b2$	12.5	68513	1.547×10^{-4}	1.906×10^{-10}	0.37	12.5[11]
	$b3$	13.5	65003	1.628×10^{-4}	-1.177×10^{-10}	0.38	13.5[11]
^{193}Hg	$b1$	7.5	24375	4.469×10^{-4}	1.533×10^{-8}	5.6	7.5[15]
	$b2$	10.5	73023	1.457×10^{-4}	-6.595×10^{-10}	1.0	10.5[15]
	$b3$	9.5	60109	1.774×10^{-4}	1.332×10^{-11}	1.1	9.5[15]
	$b4$	13.5	136870	7.127×10^{-5}	9.002×10^{-10}	4.2	13.5[15]
^{194}Hg	$b1$	10	41639	2.697×10^{-4}	1.467×10^{-9}	3.9	10[2]
	$b2$	11	71373	1.483×10^{-4}	-3.668×10^{-10}	0.91	11[2]
	$b3$	8	63116	1.691×10^{-4}	-2.885×10^{-10}	0.71	8[2]
^{194}Tl	$b1$	12	125729	7.952×10^{-5}	-3.308×10^{-10}	0.71	12[18]
	$b2$	10	64633	1.622×10^{-4}	6.197×10^{-10}	0.97	10[18]
^{194}Pb		6	41422	2.730×10^{-4}	4.140×10^{-10}	0.79	6[19,20]
^{196}Pb		8	58609	1.949×10^{-4}	-5.608×10^{-10}	0.60	Not indicated

the inertia $\mathscr{I}^{(1)}$ and the dynamic moments of the inertia $\mathscr{I}^{(2)}$ are given.

In the $A\sim190$ region, 15 SD bands in ^{191}Hg [10,11], ^{192}Hg [12,13], ^{193}Hg [14,15], ^{194}Hg [16,17], ^{194}Tl [18], ^{194}Pb [19,20] and ^{196}Pb [20] nuclei have been discovered. These bands involve even-even, odd-A and odd-odd nuclei, including both Yrast and excited bands. The analysis of all these bands has been performed by using W-Z expression. The two-parameter expression for the rotational band is

$$E(I)=a[\sqrt{1+bI(I+1)}-1], \tag{1}$$

Thus,

$$E_\gamma(I)=a[\sqrt{1+bI(I+1)}-\sqrt{1+b(I-2)(I-1)}]. \tag{2}$$

Since the change of the spin values can only be integers and the transition energy spectra are rather sensitive to the assigned spin values, it is easy to determine the spin values of the SD bands based on the transition energies [5]. The procedure to determine the spins of an SD band is as follows: suppose the spins of the SD band have been assigned. The parameters a and b in Eq.(2) are obtained in terms of the two lowest observed γ-transition energies. When the spin assignment for the SD band is correct, the calculated transition energies from Eq.(2) should be optimally fit to all observed ones; if the spin assignment is not correct, the obtained parameters a and b cannot describe the experimental data quite well. In this case, we readjust the spin values within the experimental uncertainty and recalculate a and b until the calculated transition energies best fit to all measured ones. Then, the spin values of the SD band are obtained correctly. The calculated results are given in Table 1, where I_{exit} (the exit spin) is the lowest spin value of the SD band we assigned, while

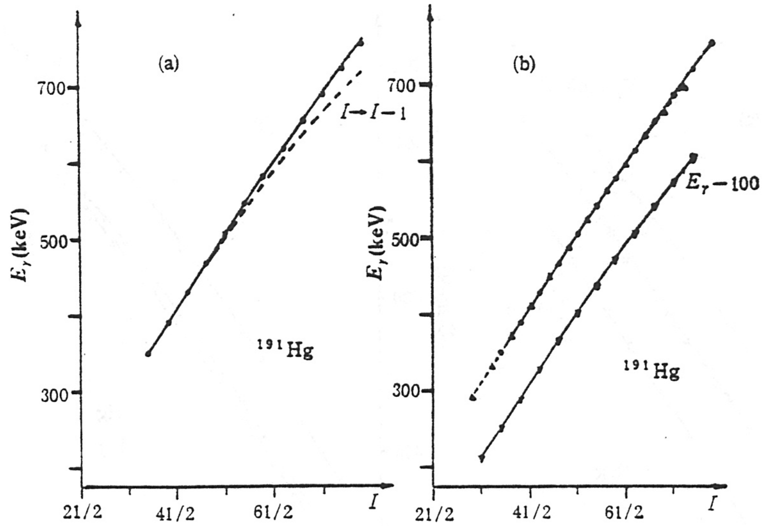

Fig. 1

(a) Determination of spin for the SD bands 1 of ^{191}Hg. The solid line corresponds to theoretical results. The calculated values corresponding to $I \to I - 1$ are shown by the dashed line. In the case of $I \to I - 1$, the abscissa scale is changed from I to $I - 1$. The experimental data are marked by filled circles. The parameters used here: a = 111175 keV, $b = 9.4 \times 10^{-5}$ for solid line and a = 35616 keV, $b = 3.2 \times 10^{-4}$ for dashed line. (b) Comparison between calculated and observed γ-transition energies $E_\gamma(I)$ for three SD bands of ^{191}Hg. The curves show the calculated values. The experimental data are marked by ● for band 1, ▲ for band 2 and ▼ for band 3. To make the Figure clear, the curve of band 3 is shifted down by 100 keV.

$(I_{exit})_{exp}$ is a result assigned by using the method suggested by Becker *et al.* [12]. This table shows that all I_{exit}, except ^{191}Hg (band 1), are the same as those assigned by experimentalists. However, our analysis shows that the lowest spin value should be $15.5\hbar$ for ^{191}Hg(b1) rather than $14.5\hbar$ in order to fit to the observed data optimally. The calculated results are given in Fig. 1(a). This figure shows that the agreement between the calculated and observed transition energies is better for $I_{exit} = 15.5\hbar$, compared with that of $I_{exit} = 14.5\hbar$. The root-mean-square deviation is defined as

$$\Delta = \sqrt{\frac{1}{N-n}\sum_I |E_\gamma(I,\mathrm{cacl}) - E_\gamma(I,\exp)|^2}, \tag{3}$$

where N is the number of the fitted transition energies, n is the number of the parameters. In the case of $I_{exit} = 15.5\hbar$, Δ = 3.9 keV, while for $I_{exit} = 14.5\hbar$, Δ = 19.9 keV. In the latter case, the calculated transition energies deviate obviously from the observed ones. After the spin values of the SD bands have been assigned, the three-parameter expression [5]

$$E(I) = a[\sqrt{1 + bI(I+1) + cI^2(I+1)^2} - 1], \tag{4}$$

are further used to analyze the measured data. Both the used parameters and the root-mean-square deviations for all 15 SD bands are given in Table 1. In Figs. 1(b), 2 and 3, the comparison between the calculated and measured γ-transition energies $E_\gamma(I)$ is given, where the curves are calculated

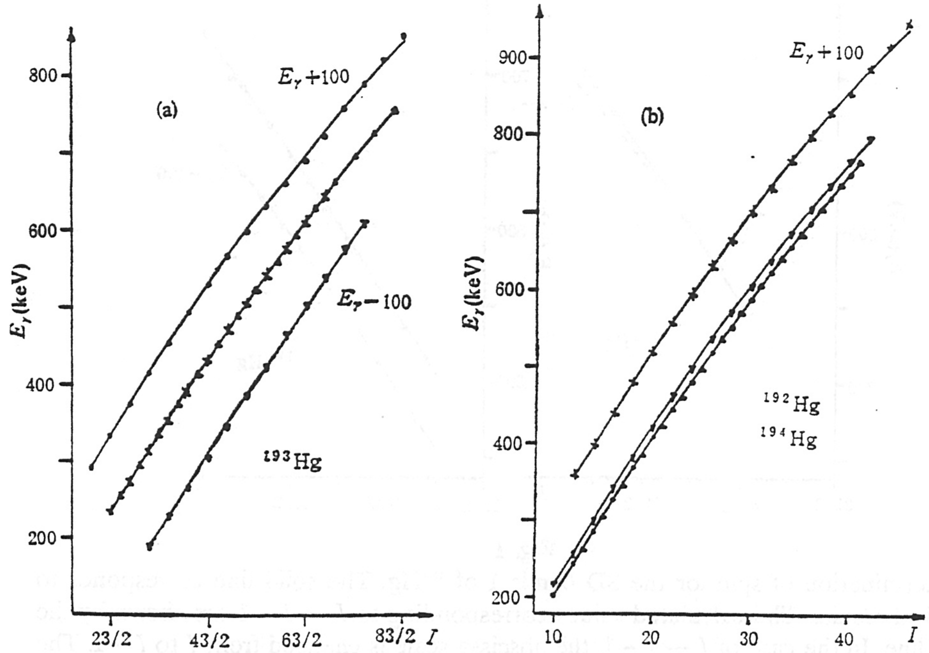

Fig. 2

Comparison between calculated and observed γ-transition energies $E_\gamma(I)$ for the SD bands of ^{193}Hg, ^{192}Hg and ^{194}Hg. Used parameters are given in Table 1. To make the Figure clear, some curves are shifted down by 100 keV. (a) ^{193}Hg. The experimental data are marked by ● for band 1, ▲ for band 2, × for band 3 and ▼ for band 4. (b) ^{192}Hg and ^{194}Hg. The experimental data are marked by ▼ for ^{192}Hg, while ×, ▲ and ● correspond to band 1, band 2 and band 3 of ^{194}Hg, respectively.

from Eq.(4) and the experimental data are denoted by different symbols. The values of the parameters are listed in Table 1. From these figures and Table 1 one can see that the calculated values of the transition energies are in excellent agreement with the observed data. The root-mean-square deviations Δ are less than 1 keV for 11 of the 15 SD bands considered. The two largest deviations occur in the band 1 and band 4 of ^{193}Hg, which have anomalous behaviors in $\mathcal{J}^{(2)}$ and probably have band crossing [15].

The following conclusions can be drawn from our analysis:

1) The W-Z expression is successfully applied to analyze all SD bands in $A \sim 190$ region (including odd-A and odd-odd nuclei) and the determination of the spin values for SD bands on the basis of the observed γ-transition energies $E_\gamma(I)$ is reliable and unique with no uncertainties.

2) The exit spins I_{exit} assigned by us for the SD bands in the $A \sim 190$ region are the same as those used in Refs.[1,2]. Therefore, it is not appropriate to shift the exit spins I_{exit} by $1\hbar$ in Refs.[3,4]. Therefore, a natural conclusion is that the analysis of the observed data in Refs.[1,2] is correct, namely, there is a ^{192}Hg family in the $A \sim 190$ region, which consists of nine SD bands. The spin alignment for the member of the family relative to the Yrast band of ^{192}Hg is quantized over a wide range of rotational frequency [1,2].

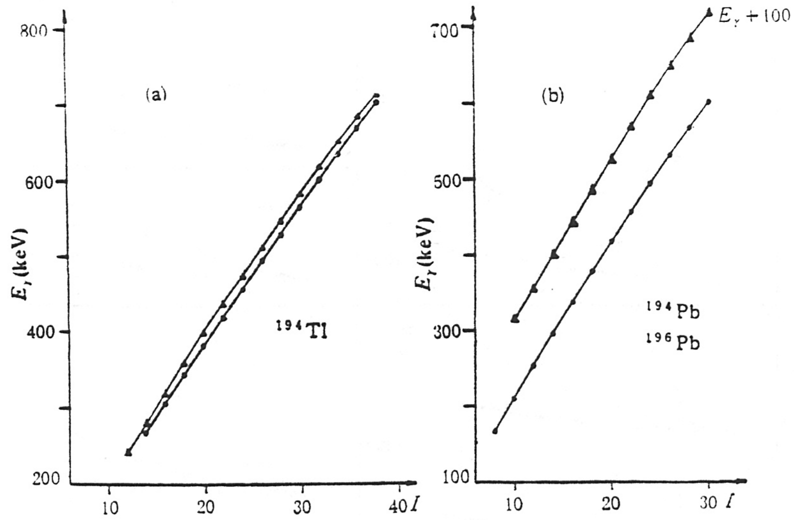

Fig. 3

Comparison between calculated and observed γ-transition energies $E_\gamma(I)$ for the SD bands of ^{194}Tl, ^{194}Pb and ^{196}Pb. The parameters used here are given in Table 1. To make the Figure clear, the curve of ^{196}Pb is shifted up by 100 keV. (a) ^{194}Tl. The experimental data are marked by ● for band 1 and ▲ for band 2. (b) ^{194}Pb and ^{196}Pb. The experimental data are marked by ● for ^{194}Pb and ▲ for ^{196}Pb.

3) A method has been developed by Becker *et al.* to determine the spins of the SD bands [12]. In this method the dynamic moment of inertia $\mathscr{I}^{(2)}$ is expanded by a power-series in ω^2. The expansion coefficients are determined from the least-square fit to the observed data I (2) within a certain range of frequency (for example, $[\omega_{exit}, \omega_2]$). Although there exist some uncertainties in this method, the assigned exit spins of the SD bands are still correct for even-even nuclei because the observed exit rotational frequency is rather low and I (2) is a smooth function of the rotational frequency ω in the $A \sim 190$ region. However, in our opinion, one should be cautious when dealing with odd-A and odd-odd nuclei. As for the $A \sim 150$ region, because the exit rotational frequency of the SD band is much higher, it must be carefully reconsidered whether the method suggested by Becker *et al.* can be applied to this region.

From Eq.(4), the rotational frequency is defined as

$$\omega(I) = \frac{1}{\hbar}\frac{dE}{dI_z}, \quad (5)$$

where

$$I_z = \sqrt{I(I+1) - K^2}, \quad (6)$$

Thus,

$$\hbar\omega(I) = \frac{a[b + 2cI(I+1)]I_z}{\sqrt{1 + bI(I+1) + cI^2(I+1)^2}}. \quad (7)$$

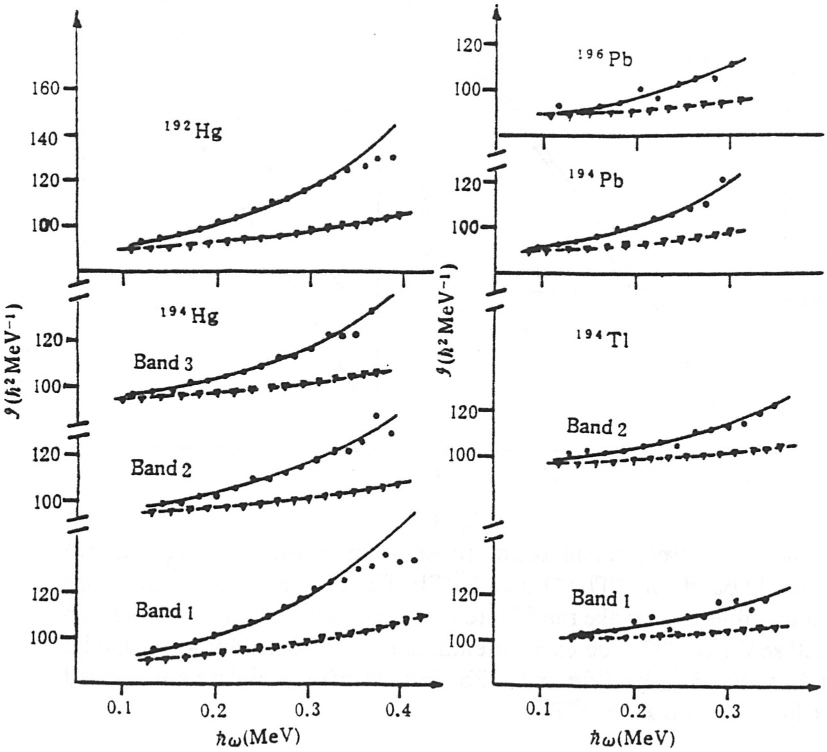

Fig. 4

Comparison between calculated and observed kinematic and dynamic moments of inertia $I(1)$ and $I(2)$ for ^{192}Hg, ^{194}Hg, ^{194}Tl, ^{194}Pb and ^{196}Pb. The solid lines correspond to the calculated values of I (2), while the dashed lines denote the calculated values of I. The parameters used here are given in Table 1. The experimental data are marked by ▼ for $\mathcal{J}^{(1)}$ and ● for $\mathcal{J}^{(2)}$.

If $I \gg K$, then

$$\hbar\omega(I) = \frac{a[b + 2cI(I+1)]\sqrt{I(I+1)}}{\sqrt{1 + bI(I+1) + cI^2(I+1)^2}}. \tag{8}$$

Furthermore, the kinematic moment of inertia $\mathcal{J}^{(1)}$ is

$$\mathcal{J}^{(1)} = \frac{\hbar I_x}{\omega(I)} = \frac{\hbar^2}{a(b + 2cI(I+1))}\sqrt{1 + bI(I+1) + cI^2(I+1)^2}, \tag{9}$$

and the dynamic moment of inertia $\mathcal{J}^{(2)}$ is

$$\mathcal{J}^{(2)} = \hbar\left(\frac{d\omega(I)}{dI_x}\right)^{-1} = \frac{\hbar^2}{a} \cdot \frac{[1 + bI(I+1) + cI^2(I+1)^2]^{3/2}}{a + 6cI(I+1) + 3bcI^2(I+1)^2 + 2c^2I^3(I+1)^3}. \tag{10}$$

In Figs. 4 and 5, the calculated kinematic and dynamic moments of inertia $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ are plotted against the rotational frequency ω and compared with the experimental data. The observed kinematic moment of inertia $\mathcal{J}^{(1)}$ is extracted from $\mathcal{J}^{(1)} = (2I-1)/E_\gamma$, assuming that the exit spin of the SD band has been determined as given in Table 1. These figures show that the calculated $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ curves reproduce the observed data for all SD bands in $A \sim 190$ region satisfactorily and

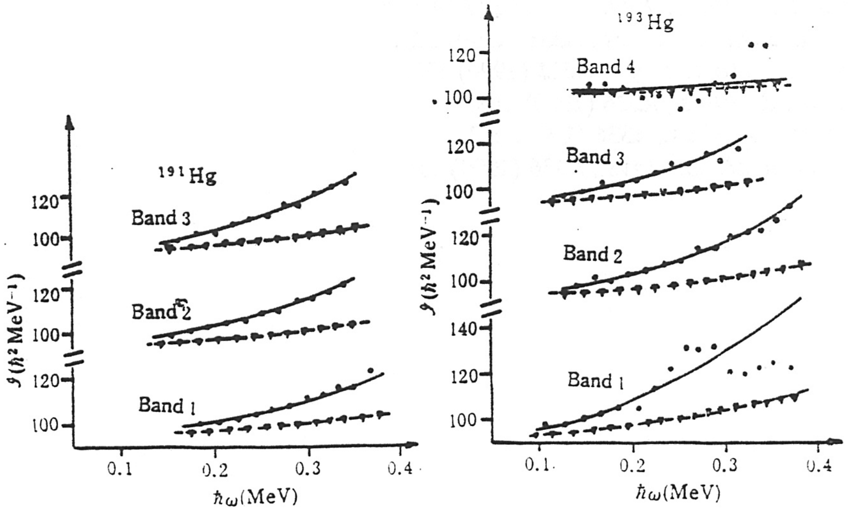

Fig. 5

Comparison between calculated and observed kinematic and dynamic moments of inertia $I(1)$ and $I(2)$ for ^{191}Hg and ^{193}Hg. The symbols are the same as those in Fig. 4.

give the general feature of moments of inertia, which increase with the rotational frequency ω. For two anomalous bands ^{193}Hg(b1) and ^{193}Hg(b4), the calculated $\mathcal{I}^{(2)}$ also show the average properties of the observed data.

In brief, it is a great success that the W-Z formula can be used to analyze the SD band structure in $A \sim 190$ region. Thus, the spin values of the SD bands can be determined by using the W-Z formula correctly. Using three-parameter formula, the calculated γ-transition energy spectra $E_\gamma(I)$, the moments of inertia $\mathcal{I}^{(1)}$ and $\mathcal{I}^{(2)}$ are in excellent agreement with the observed data for all SD bands in the $A \sim 190$ region.

REFERENCES

[1] F. S. Stephens *et al.*, *Phys. Rev. Lett.*, **64** (1990) 2623.
[2] F. S. Stephens *et al.*, *Phys. Rev. Lett.*, **65** (1990) 301.
[3] C-L. Wu *et al.*, *Phys. Rev. Lett.*, **66** (1991) 1377.
[4] C-L. Wu *et al.*, Is There Non-Zero Quantized Spin Alignment in Superdeformed Nuclei, Submitted to *Phys. Rev. Lett.*
[5] Xing Zheng and Chen Xingqu, *High Energy Phys. Nucl. Phys.* (in Chinese), **15** (1991) 1020.
[6] C. S. Wu and J. Y. Zeng, *Physica Energiae Fortis Et Physica Nuclearis*, **8** (1984) 219; **8** (1984) 445; **9** (1985) 77; **9** (1985) 214.
[7] C. S. Wu and J. Y. Zeng, *Commun. in Theor. Phys.*, **8** (1987) 51.
[8] H. X. Huang, C. S. Wu and J. Y. Zeng, *Phys. Rev.*, **C39** (1989) 1617.
[9] F. X. Xu, C. S. Wu and J. Y. Zeng, *Phys. Rev.*, **C40** (1989) 2337.
[10] E. F. Moore *et al.*, *Phys. Rev. Lett.*, **63**(1989)360.
[11] M. P. Carpenter *et al.*, *Phys. Lett.*, **B240** (1990) 44.
[12] J. A. Becker *et al.*, *Phys. Rev.*, **C41** (1990) R9.
[13] D. Ye *et al.*, *Phys. Rev.*, **C41** (1990) R13.

[14] E. A. Henry *et al.*, *Z. Phys.*, **A335** (1990) 361.
[15] D. M. Cullen *et al.*, *Phys. Rev. Lett.*, **65** (1990) 1547.
[16] C. W. Beausang *et al.*, *Z. Phys.*, **A335** (1990) 325.
[17] M. A. Riley *et al.*, *Nucl. Phys.*, **A512** (1990) 178.
[18] F. Azaiez *et al.*, *Z. Phys.*, **A336** (1990) 243.
[19] K. Theine *et al.*, *Z. Phys.*, **A336** (1990) 113.
[20] M. J. Brinkman *et al.*, *Z. Phys.*, **A336** (1990) 115.