相对论 Boltzmann–Uehling–Uhlenbeck 方程

余自强　茅广军
(南开大学物理系,天津 300071)

卓益忠*　李祝霞　孙諎民
(中国原子能研究院,北京 102413)

摘　要

在 Walecka 的 QHD-I 和 QHD-II 关于核子-核子相互作用模型的基础上，采用闭合时间回路的格林函数方法并假设格林函数和自能项是质心坐标的缓变函数,推得由核子分布函数所满足的 BUU 方程,它包括 Hartree 和 Fock 自能项及 Born 碰撞项和它的交换项，结果表明质子和中子分布函数所满足的 BUU 方程是相互联立的.

一、引　言

当前相对论重离子碰撞已引起人们广泛兴趣，面对已经发现及即将出现的大量实验结果，如何去解释这些实验现象进而探讨相对论重离子碰撞的机制已成为核理论的一个重要前沿课题． 正如我们在前文[1,2]中已经指出的：Boltzmann-Uehling-Uhlenbeck (BUU)方程的求解和应用在描述重离子碰撞方面已取得可喜的成功．应该指出，近年来有关相对论 BUU 方程已经发表过一些文章[3],但还是不大完善．例如在平均场部分仅考虑 Hartree 项而没有作出它的相应交换项-Fock 项，且平均场的碰撞项中的截面不是由一个统一拉氏量导出，而是把自由核子-核子截面放进碰撞项中，或虽给出形式但没有具体地推导出截面的表达式，因而也无法进行具体计算．此外，至今为止推导仅限于 QHD (Quantum-Hadron-Dynamic)-I,即仅考虑 σ 与 ω 介子，这是不够的．针对这些问题，本工作继先前文章[1,2]，仍采用闭合时间回路格林函数方法，从 QHD-I 与 QHD-II 出发，即不仅考虑 σ 与 ω 介子，还考虑了 π 和 ρ 介子，引进了核子同位旋，将质子和中子分布函数加以区分，它们分别所满足的相对论 BUU 方程是联立的． 在我们所推导的 BUU 方程中完全自洽地包括了 Hartree-Fock 平均场和 Born 碰撞项中的等效微分截面，这就自动顾及介质效应，并且有解析表达式，物理图象清楚，易于计算．

本文所建立的相对论 BUU 方程适用于在质心系中能量为几百 MeV 到 1～2GeV/核子的重离子碰撞研究，这时相对论效应已显得重要但夸克和胶子自由度尚未显露出来，

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* 中国科学院理论所客座研究人员.

可以不必考虑。

二、理 论 框 架

1. 核子-介子系统的拉氏量密度

$$\mathscr{L} = \mathscr{L}_D + \mathscr{L}_I, \tag{1}$$

其中 $\mathscr{L}_D$ 是自由核子场的拉氏量密度[4]

$$\mathscr{L}_D = \frac{i}{2}\bar{\psi}(x)\gamma^\mu\overleftrightarrow{\partial}_\mu\psi(x) - M\bar{\psi}(x)\psi(x). \tag{2}$$

$\mathscr{L}_I$ 是核子-介子耦合的相互作用拉氏量密度，按 Walecka 的 QHD-I 和 QHD-II 模型[5]为：

$$\begin{aligned}\mathscr{L}_I &= g_s\bar{\psi}(x)\phi(x)\psi(x) - g_v\bar{\psi}(x)\gamma_\mu\psi(x)V^\mu(x)\\ &\quad - ig_\pi\bar{\psi}(x)\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\psi(x) - \frac{1}{2}g_\rho\bar{\psi}(x)\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{b}^\mu\psi(x)\\ &= \sum_{a=s,v,\pi,\rho}\bar{\psi}(x)U_a(x)\psi(x),\end{aligned} \tag{3}$$

其中 $U_a(x)=\Gamma_a\phi_a(x)$，$\psi(x)$是核子场，$\phi_a(x)$是介子场，Γ_a 是核子-介子耦合顶角，a=s，v，π，ρ 分别代表标量 σ、矢量 ω、π 和 ρ 介子。

2. 闭合时间回路格林函数[1,6,7]

我们知道，基于 Gellmann 和 Low 定理所得算符的期望值不能用于非静止状态的期望值，而闭合时间回路格林函数允许我们去研究多粒子量子系统的时间演化过程。

在相互作用表象中，沿闭合回路的格林函数为：

$$iG_{12} = \left\langle T\left[\exp\left(-i\oint \mathrm{d}x\mathscr{H}_I(x)\psi(1)\bar{\psi}(2)\right)\right]\right\rangle, \tag{4}$$

其中 $\oint \mathrm{d}x \equiv \oint \mathrm{d}t\int \mathrm{d}\boldsymbol{x}$，$\oint \mathrm{d}t$表示对时间的积分是沿整个闭合回路进行的。根据场算符在回路中的不同位置，我们可以有四种格林函数：G_{12}^{--}，G_{12}^{++}，$G_{12}^{+-}\equiv G_{12}^{>}$，$G_{12}^{-+}\equiv G_{12}^{<}$，它们组成一个矩阵。相应于玻色子的格林函数及各项自能均可写成类似的矩阵形式，这正是时间回路格林函数与通常格林函数最主要的差别。基于这点，照样可以作微扰展开并建立 Dyson 方程。下面首先按 Walecka 模型把我们所推导出的零级格林函数写出以便作微扰展开及计算平均场和碰撞项。

(1) 核子场：

$$\langle T[\psi(1)\bar{\psi}(2)]\rangle = iG_{12}^0\delta_{t_1t_2'}, \tag{5}$$

其中 t_1、t_2' 分别是 $\psi(1)$ 和 $\bar{\psi}(2)$ 的同位旋第三分量。

$$iG_{12}^0 = i\int\frac{\mathrm{d}^4K}{(2\pi)^4}G^0(X,K)\mathrm{e}^{-iK(x_1-x_2)}, \tag{6}$$

$$G^{0\mp\mp}(X,K) = (\not{K}+M)\left[\frac{\pm 1}{K^2-M^2\pm i\varepsilon} + \frac{i\pi}{E(K)}\delta(K_0-E(K))f_t(X,K)\right], \tag{7}$$

$$G^{0+-}(X,K) = -\frac{\pi i}{E(K)}\delta(K_0 - E(K))[1 - f_t(X,K)](\not{K} + M), \tag{8}$$

$$G^{0-+}(X,K) = \frac{\pi i}{E(K)}\delta(K_0 - E(K))f_t(X,K)(\not{K} + M), \tag{9}$$

其中 $X = \frac{1}{2}(x_1 + x_2)$是质心坐标。设 $a_t^+(X,K)$ 和 $a_t(X,K)$ 是核子的产生和湮没算符，则 $f_t(X,K) = \langle | a_t^+(X,K)a_t(X,K)| \rangle$.

(2) 介子场:

$$\langle T[\phi_a(1)\phi_b(2)]\rangle = iD_a\Delta_a^0(1,2)\delta_{ab}, \tag{10}$$

其中 $D_a = D_a^\mu D_a^i$,

$$D_a^\mu = \begin{cases} 1, & a = s,\pi \\ -g_{\mu\nu}, & a = v,\rho \end{cases}; \qquad D_a^i = \begin{cases} 1, & a = s,v. \\ \delta_{ij}, & a = \pi,\rho. \end{cases} \tag{11}$$

$$i\Delta_a^0(1,2) = i\int\frac{d^4K}{(2\pi)^4}\Delta_a^0(X,K)e^{-iK(X_1-X_2)}, \tag{12}$$

$$\Delta_a^{0\mp\mp}(X,K) = \frac{\pm 1}{K^2 - m_a^2 \pm i\varepsilon} - 2\pi i\delta(K^2 - m_a^2)f_a(X,K), \tag{13}$$

$$\Delta_a^{0\pm\mp}(X,K) = -2\pi i\delta(K^2 - m_a^2)[\theta(\pm K_0) + f_a(X,K)], \tag{14}$$

$$f_a(X,K) = \langle | C_a^+(X,K)C_a(X,K)| \rangle, \tag{15}$$

其中 $C_a^+(X,K)$ 和 $C_a(X,K)$ 是 a 类介子的产生和湮没算符.

3. 微扰展开及 Dyson 方程

将(4)按 Wick 定理展开并可得到 Dyson 方程

$$iG_{12} = iG_{12}^0 + \int d3\int d4 G_{14}^0 \sum(4,3)iG_{32}, \tag{16}$$

在四阶微扰近似下

$$\sum(4,3) = \sum^{HF}(4,3) + \sum^{(2)}(4,3). \tag{17}$$

自能项 $\sum^{HF}(4,3)$ 包括 Hartree 项及其交换项-Fock 项

$$\sum^{HF}(4,3) = \sum^{H}(4,3) + \sum^{F}(4,3), \tag{18}$$

碰撞项 $\sum^{(2)}(4,3)$ 包括 Born 项及其交换项

$$\sum^{(2)}(4,3) = \sum^{d}(4,3) + \sum^{e}(4,3), \tag{19}$$

其中:

$$\sum^{H}(4,3) = \sum_a \left\{-\delta(3,4)\langle t_1|\Gamma_a|t_1\rangle\int d3'\mathrm{tr}\left(\sum_{t_3'}\langle t_3'|\Gamma_a|t_3'\rangle G_{3'3'}^0\right)iD_a\Delta_a^0(4,3'), \right. \tag{20}$$

$$\overset{F}{\sum}(4,3)=\sum_{a}\sum_{t_3'}\langle t_1|\Gamma_a|t_3'\rangle G_{43}^0\langle t_3'|\Gamma_a|t_1\rangle iD_a\Delta_a^0(4,3), \tag{21}$$

$$\overset{d}{\sum}(4,3)=\sum_{a,b}\sum_{t_4,t_5,t_6}\int d5\int d6\{\langle t_1|\Gamma_a|t_4\rangle G_{43}^0\langle t_4|\Gamma_b|t_2'\rangle \mathrm{tr}(\langle t_6|\Gamma_b|t_5\rangle \times G_{56}^0\langle t_5|\Gamma_a|t_6\rangle G_{65}^0)D_a\Delta_a^0(4,6)D_b\Delta_b^0(3,5)\delta_{t_1t_2'}, \tag{22}$$

$$\overset{e}{\sum}(4,3)=\sum_{a,b}\sum_{t_4,t_5,t_6}\int d5\int d6\{-\langle t_1|\Gamma_a|t_4\rangle G_{45}^0\langle t_4|\Gamma_b|t_5\rangle G_{56}^0\langle t_5|\Gamma_a|t_b\rangle \times G_{63}^0\langle t_6|\Gamma_b|t_2'\rangle D_a\Delta_a^0(4,6)D_b\Delta_b^0(3,5)\delta_{t_1t_2'}. \tag{23}$$

4. 格林函数的运动方程

将算符 $\hat{G}_{01}^{-1}=i\gamma\cdot\partial_{x_1}-M$ 作用到 Dyson 方程(16)两边并利用 Dirac 方程即得到格林函数的运动方程。

$$\hat{G}_{01}^{-1}iG_{12}=i\delta(1,2)+\int d3\sum(1,3)iG_{32}. \tag{24}$$

我们最感兴趣的是 $G_{12}^{\lessgtr}$ 的运动方程：Kadanoff-Baym 方程

$$\left[i\gamma\cdot\partial_{x_1}-M-\overset{H}{\sum}(1)\right]iG_{12}^{\lessgtr}=\int_{t_0}^{t}d3\overset{F}{\sum}{}^{--}(1,3)iG_{32}^{\lessgtr}+\int_{t_0}^{t_1}d3\left[\overset{(2)}{\sum}{}^{>}(1,3)-\overset{(2)}{\sum}{}^{<}(1,3)\right]iG_{32}^{\lessgtr}-\int_{t_0}^{t_2}d3\overset{(2)}{\sum}{}^{<}(1,3)[iG_{32}^{>}-iG_{32}^{<}], \tag{25}$$

其中

$$\overset{H}{\sum}(1)=-\sum_{a}\langle t|\Gamma_a|t\rangle\left\{\int_{t_0}^{t}d3'\mathrm{tr}\left(\sum_{t_3'}\langle t_3'|\Gamma_a|t_3'\rangle G_{3'3'}^{0--}\right)iD_a\Delta_a^{0--}(3',1)+\int_{t}^{t_0}d3'\mathrm{tr}\left(\sum_{t_3'}\langle t_3'|\Gamma_a|t_3'\rangle G_{3'3'}^{0++}\right)iD_a\Delta_a^{0+-}(3',1)\right\}, \tag{26}$$

$$\overset{F}{\sum}{}^{--}(1,3)=\sum_{a}\sum_{t_3'}\langle t|\Gamma_a|t_3'\rangle G_{13}^{0--}\langle t_3'|\Gamma_a|t\rangle iD_a\Delta_a^{0--}(3,1), \tag{27}$$

$$\overset{(2)}{\sum}{}^{\gtrless}(1,3)=\overset{e}{\sum}{}^{\gtrless}(1,3)+\overset{d}{\sum}{}^{\gtrless}(1,3), \tag{28}$$

$$\begin{aligned}\overset{e}{\sum}{}^{\gtrless}(1,3)=-\sum_{a,b}\sum_{t_2,t_3,t_4}\int_{t_0}^{t}d5\int_{t_0}^{t}d6\\
&\cdot\{\langle\Gamma_a(1)\rangle G_{15}^{0\gtrless}\langle\Gamma_b(5)\rangle G_{56}^{0\mp\mp}\langle\Gamma_a(6)\rangle G_{63}^{0\mp\mp}\langle\Gamma_b(3)\rangle\\
&\cdot D_a(1,6)\Delta_a^{0\gtrless}(1,6)D_b(3,5)\Delta_b^{0\mp\mp}(3,5)\\
&+\langle\Gamma_a(1)\rangle G_{15}^{0\pm\pm}\langle\Gamma_b(5)\rangle G_{56}^{0\pm\pm}\langle\Gamma_a(6)\rangle G_{63}^{0\gtrless}\langle\Gamma_b(3)\rangle\\
&\cdot D_a(1,6)\Delta_a^{0\pm\pm}(1,6)D_b(3,5)\Delta_b^{0\gtrless}(3,5)\\
&-\langle\Gamma_a(1)\rangle G_{15}^{0\gtrless}\langle\Gamma_b(5)\rangle G_{56}^{0\lessgtr}\langle\Gamma_a(6)\rangle G_{63}^{0\gtrless}\langle\Gamma_b(3)\rangle\\
&\cdot D_a(1,6)\Delta_a^{0\pm\pm}(1,6)D_b(3,5)\Delta_b^{0\mp\mp}(3,5)\\
&-\langle\Gamma_a(1)\rangle G_{15}^{0\pm\pm}\langle\Gamma_b(5)\rangle G_{56}^{0\gtrless}\langle\Gamma_a(6)\rangle G_{63}^{0\mp\mp}\langle\Gamma_b(3)\rangle\end{aligned}$$

$$\cdot\ D_a(1,6)\Delta_a^{0\gtrless}(1,6)D_b(3,5)\Delta_b^{0\lessgtr}(3,5)\},\tag{29}$$

其中顶角表式为:

$$\left.\begin{aligned}&\langle\Gamma_a(1)\rangle=\langle t|\Gamma_a(1)|t_4\rangle,\quad\langle\Gamma_a(6)\rangle=\langle t_2|\Gamma_a(6)|t_3\rangle.\\&\langle\Gamma_b(3)\rangle=\langle t_3|\Gamma_b(3)|t\rangle,\quad\langle\Gamma_b(5)\rangle=\langle t_4|\Gamma_b(5)|t_2\rangle.\end{aligned}\right\}\tag{30}$$

$$\begin{aligned}\sum^{d}{}^{\gtrless}(1,3)=&\sum_{a,b}\sum_{t_2,t_3,t_4}\int_{t_0}^{t}d5\int_{t_0}^{t}d6\langle\Gamma_a(1)\rangle G_{13}^{0\gtrless}\langle\Gamma_b(3)\rangle\\&\cdot\{\mathrm{tr}(\langle\Gamma_b(5)\rangle G_{56}^{0\mp\mp}\langle\Gamma_a(6)\rangle G_{65}^{0\mp\mp})D_a\Delta_a^{0\gtrless}(1,6)D_b\Delta_b^{0\mp\mp}(3,5)\\&+\mathrm{tr}(\langle\Gamma_b(5)\rangle G_{56}^{0\pm\pm}\langle\Gamma_a(b)\rangle G_{65}^{0\pm\pm})D_a\Delta_a^{0\pm\pm}(1,6)D_b\Delta_b^{0\lessgtr}(3,5)\\&-\mathrm{tr}(\langle\Gamma_b(5)\rangle G_{56}^{0\lessgtr}\langle\Gamma_a(b)\rangle G_{65}^{0\gtrless})D_a\Delta_a^{0\pm\pm}(1,6)D_b\Delta_b^{0\mp\mp}(3,5)\\&-\mathrm{tr}(\langle\Gamma_b(5)\rangle G_{56}^{0\gtrless}\langle\Gamma_a(6)\rangle G_{65}^{0\lessgtr})D_a\Delta_a^{0\gtrless}(1,6)D_b\Delta_b^{0\lessgtr}(3,5)\}.\end{aligned}\tag{31}$$

$$\left.\begin{aligned}&\langle\Gamma_a(1)\rangle=\langle t|\Gamma_a(1)|t_3\rangle,\langle\Gamma_a(6)\rangle=\langle t_2|\Gamma_a(6)|t_4\rangle\\&\langle\Gamma_b(3)\rangle=\langle t_3|\Gamma_b(3)|t\rangle,\langle\Gamma_b(5)\rangle=\langle t_4|\Gamma_b(5)|t_2\rangle.\end{aligned}\right\}\tag{32}$$

5. BUU 方程

作坐标变换:

$$X=\frac{1}{2}(x_1+x_2),\ x=x_1-x_2,\ x'=x_3-x_2.\tag{33}$$

将(25)式两边对 x 坐标作傅氏变换,假设 G 和 Σ 随 X 坐标变化非常缓慢,并将时间积分区域扩充为 $t_0\to-\infty,t_1,t_2,t\to\infty$. 于是得到

$$\begin{aligned}&\left\{\frac{i}{2}\gamma\cdot\partial_X+\gamma\cdot p-M-e^{-\frac{i}{2}\partial_X\partial_P}\sum^{H}(X)-e^{-\frac{i}{2}\partial_X\partial_P}e^{\frac{i}{2}\partial_P\partial_X}\sum^{F}{}^{--}(X,P)\right\}iG^{<}(X,P)\\&=e^{-\frac{i}{2}\partial_X\partial_P}e^{\frac{i}{2}\partial_X\partial_P}\left[\sum^{(2)}{}^{>}(X,P)iG^{<}(X,P)-\sum^{(2)}{}^{<}(X,P)iG^{>}(X,P)\right].\end{aligned}\tag{3,4}$$

基于 $G(X,P)$ 和 $\Sigma(X,P)$ 是 X 的缓变函数,在 $e^{\pm\frac{i}{2}\partial_X\partial_P}$ 的展开中可仅保留低阶项. (34)可简化为:

$$\begin{aligned}&\left\{\frac{i}{2}\gamma\cdot\partial_X+\gamma\cdot p-M-\sum^{HF}(X,p,t)+\frac{i}{2}\partial_X\sum^{HF}(X,p,t)\partial_P\right.\\&\left.-\frac{i}{2}\partial_P\sum^{F}{}^{--}(X,p,t)\partial_X\right\}iG_t^{<}(X,p)=F_c(X,p,t),\end{aligned}\tag{35}$$

其中指标 t 表示被考虑的核子的同位旋第三分量.

$$\sum^{HF}(X,p,t)=\sum^{H}(X,t)+\sum^{F}{}^{--}(X,p,t).\tag{36}$$

$$F_c(X,p,t)=\sum^{(2)}{}^{>}(X,p,t)iG_t^{<}(X,p)-\sum^{(2)}{}^{<}(X,p,t)iG_t^{>}(X,p).\tag{37}$$

$$\sum^{H}(X,t)=\sum^{H}_{s}(X)+\gamma_\mu\sum^{H}{}^{\mu}(X,t).\tag{38}$$

$$\sum^{H}{}^{\mu}(X,t)=\sum^{H}{}_{v}^{\mu}(X)+\sum^{H}{}_{\rho 0}^{\mu}(X,t).\tag{39}$$

$$\sum^{H}_{s}(X)=-\frac{g_s^2}{m_s^2}\frac{1}{(2\pi)^3}\int d\boldsymbol{p}_3\frac{M}{(\boldsymbol{p}_3^2+M^2)^{1/2}}[f_p(X,p_3)+f_n(X,p_3)].\tag{40}$$

$$\overset{H}{\Sigma}{}^{\mu}_{v}(X)=\frac{g_v^2}{m_v^2}\frac{1}{(2\pi)^3}\int d\boldsymbol{p}_3\,\frac{p_3^\mu}{(\boldsymbol{p}_3^2+M^2)^{1/2}}[f_p(X,p_3)+f_n(X,p_3)].\tag{41}$$

$$\overset{H}{\Sigma}{}^{\mu}_{\rho 0}\left(X,t=\pm\frac{1}{2}\right)=\pm\frac{1}{2}\frac{g_\rho^2}{m_\rho^2}\frac{1}{(2\pi)^3}\int d\boldsymbol{p}_3\,\frac{p_3^\mu}{(\boldsymbol{p}_3^2+M^2)^{1/2}}\cdot[f_p(X,p_3)-f_n(X,p_3)].\tag{42}$$

$\overset{H}{\Sigma}(X,t)$ 的物理意义如图 1 所示．它表明赝标量 π 介子的引入对 Hartree 自能项没有贡献．在计算中已考虑到在核物质中 $G^{0--}(X,p)$ 的自由传播项可略去[8]．

$$\overset{F}{\Sigma}{}^{--}(X,p,t)=\overset{F}{\Sigma}(X,p,t)+\gamma_\mu\overset{F}{\Sigma}{}^{\mu}(X,p,t).\tag{43}$$

$$\begin{aligned}\overset{F}{\Sigma}\left(X,p,\ \pm\frac{1}{2}\right)=&\ g_s^2\overset{F}{\Sigma}\left(X,p,s,\pm\frac{1}{2}\right)-4g_v^2\overset{F}{\Sigma}\left(X,p,v,\pm\frac{1}{2}\right)\\&-g_\pi^2\overset{F}{\Sigma}\left(X,p,\pi^0,\pm\frac{1}{2}\right)-2g_\pi^2\overset{F}{\Sigma}\left(X,p,\pi^\pm,\mp\frac{1}{2}\right)\\&-g_\rho^2\overset{F}{\Sigma}\left(X,p,\rho^0,\pm\frac{1}{2}\right)-2g_\rho^2\overset{F}{\Sigma}\left(X,p,\rho^\pm,\mp\frac{1}{2}\right).\end{aligned}\tag{44}$$

$$\begin{aligned}\overset{F}{\Sigma}{}^{\mu}\left(X,p,\ \pm\frac{1}{2}\right)=&\ g_s^2\overset{F}{\Sigma}{}^{\mu}\left(X,p,s,\pm\frac{1}{2}\right)+2g_v^2\overset{F}{\Sigma}{}^{\mu}\left(X,p,v,\ \pm\frac{1}{2}\right)\\&+g_\pi^2\overset{F}{\Sigma}{}^{\mu}\left(X,p,\pi^0,\pm\frac{1}{2}\right)+2g_\pi^2\overset{F}{\Sigma}{}^{\mu}\left(X,p,\pi^\pm,\mp\frac{1}{2}\right)\\&+\frac{1}{2}g_\rho^2\overset{F}{\Sigma}{}^{\mu}\left(X,p,\rho^0,\pm\frac{1}{2}\right)+g_\rho^2\overset{F}{\Sigma}{}^{\mu}\left(X,\rho,\rho^\pm,\mp\frac{1}{2}\right).\end{aligned}\tag{45}$$

$$\overset{F}{\Sigma}(X,p,a,t_3')=-\frac{1}{(2\pi)^3}\int d\boldsymbol{p}_3\,\frac{M}{(\boldsymbol{p}_3^2+M^2)^{1/2}}\frac{1}{(p-p_3)^2-m_a^2}f_{t_3'}(X,p_3).\tag{46}$$

$$\overset{F}{\Sigma}{}^{\mu}(X,p,a,t_3')=-\frac{1}{(2\pi)^3}\int d\boldsymbol{p}_3\,\frac{p_3^\mu}{(\boldsymbol{p}_3^2+M^2)^{1/2}}\frac{1}{(p-p_3)^2-m_a^2}f_{t_3'}(X,p_3).\tag{47}$$

其中考虑到在壳条件 $\delta((p-p_3)^2-m_a^2)$ 不被满足，故已将介子格林函数中的虚部项去掉，因此 Fock 项也只有实部． $\overset{F}{\Sigma}{}^{--}(X,p,t)$的物理意义如图 2 所示．

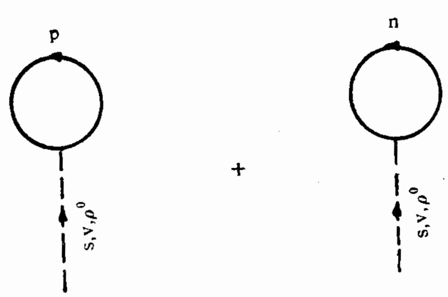


图 1　Hartree 自能项费曼图

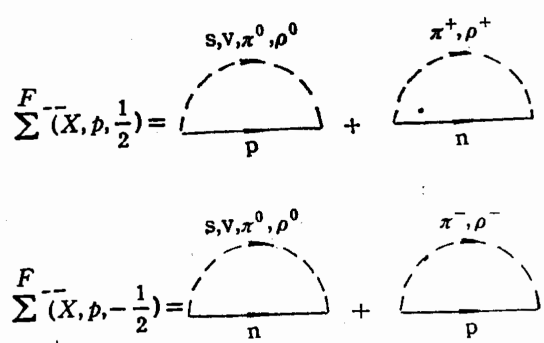


图 2　Fock 自能项费曼图

将 Wigner 函数 $iG^{<}(X,p)$ 作自旋分解，对于自旋同位旋饱和系统

$$iG^{<}(X,p)=F(X,p)+\gamma_\mu V^\mu(X,p)+\frac{1}{2}\sigma_{\mu\nu}S^{\mu\nu}(X,p), \tag{48}$$

在经典极限下，$S^{\mu\nu}=0$[9]. 将(48)代入(35)并经过取阵迹运算和将实、虚部分开. 可得到在壳条件及 F 满足的运动方程. 其中在壳条件为

$$\bar{p}^\mu\bar{p}_\mu=M_R^{*2} \tag{49}$$

在仅保留对 $F(X,\bar{p})$ 的一阶导数条件下，在壳 4 动量

$$\bar{p}^\mu(X,p,t)=p^\mu-\overset{H}{\sum}{}^\mu(X,t)-\overset{F}{\sum}{}^\mu(X,p,t) \tag{50}$$

有效质量实部

$$M_R^*(X,p,t)=M+\overset{H}{\sum}{}_s(X)+\overset{F}{\sum}(X,p,t), \tag{51}$$

而在壳的核子分布函数 $f_t(X,\bar{p})$ 所满足的运动方程(即 BUU 方程)为

$$\begin{aligned}&\Big\{\Big[\partial_X^\mu-\overset{HF}{\sum}{}^{\mu\nu}\Big(X,p,\ \pm\frac{1}{2}\Big)\partial_\nu^{\bar{p}}-\partial_{\bar{p}}^\nu\overset{F}{\sum}{}^\mu\Big(X,p,\ \pm\frac{1}{2}\Big)\partial_\nu^X\Big]\frac{M}{M_R^*}\frac{\bar{p}_\mu}{E(\bar{p})}\\&\quad+\Big[\partial_\nu^X\Big(\overset{H}{\sum}(X)+\overset{F}{\sum}\Big(X,p,\ \pm\frac{1}{2}\Big)\Big)\partial_{\bar{p}}^\nu\\&\quad-\partial_{\bar{p}}^\nu\overset{F}{\sum}\Big(X,p,\ \pm\frac{1}{2}\Big)\partial_\nu^X\Big]\frac{M}{E(\bar{p})}\Big\}f_{\pm\frac{1}{2}}(X,\bar{p})\\&=\int d\bar{p}_2\sigma_{\pm\frac{1}{2}-\pm\frac{1}{2}}(s,t)v\ \Big[F_2^0\Big(\pm\frac{1}{2}\Big)-F_1^0\Big(\pm\frac{1}{2}\Big)\Big]d\Omega\\&\quad+\int d\bar{p}_2\sigma_{\pm\frac{1}{2}-\mp\frac{1}{2}}(s,t)v\ \Big[F_2^1\Big(\pm\frac{1}{2}\Big)-F_1^1\Big(\pm\frac{1}{2}\Big)\Big]d\Omega,\end{aligned} \tag{52}$$

其中

$$\begin{aligned}\overset{HF}{\sum}{}^{\mu\nu}(X,p,t)&=\partial_X^\mu\Big[\overset{H}{\sum}{}^\nu(X,t)+\overset{F}{\sum}{}^\nu(X,p,t)\Big]\\&\quad-\partial_X^\nu\Big[\overset{H}{\sum}{}^\mu(X,t)+\overset{F}{\sum}{}^\mu(X,p,t)\Big],\end{aligned} \tag{53}$$

而 $\sigma_{t_1-t_2}(s,t)$ 是在质心系中同位旋第三分量分别为 t_1 和 t_2 的核子-核子弹性散射的有效微分截面. 它们的显式是:

$$\sigma_{p-p}(s,t)=\sigma_{n-n}(s,t)\doteq\frac{1}{(2\pi)^5 s}\Big[\sum_{i=1}^{4}\sigma_i(g_a,m_a)+\sum_{i=5}^{10}\sigma_i(g_a,m_a;g_b,m_b)\Big]. \tag{54}$$

$$\sigma_1(g_s,m_s)=g_s^4[D_1(t,m_s)+c_1(t,m_s)+E_1(t,m_s)]. \tag{55}$$

$$\sigma_2(g_v,m_v)=g_v^4[D_2(t,m_v)+c_2(t,m_v)+E_2(t,m_v)]. \tag{56}$$

$$\sigma_3(g_\pi,m_\pi)=g_\pi^4[D_3(t,m_\pi)+c_3(t,m_\pi)+E_3(t,m_\pi)]. \tag{57}$$

$$\sigma_4(g_\rho,m_\rho)=\Big(\frac{g_\rho}{2}\Big)^4[D_2(t,m_\rho)+c_2(t,m_\rho)+E_2(t,m_\rho)]. \tag{58}$$

$$\sigma_5(g_s,m_s;g_v,m_v)=g_s^2g_v^2[D_5(t,m_s,m_v)+c_5(t,m_s,m_v)+E_5(t,m_s,m_v)]. \tag{59}$$

$$\sigma_6(g_s,m_s;g_\pi,m_\pi)=g_s^2g_\pi^2[c_6(t,m_s,m_\pi)]. \tag{60}$$

$$\sigma_7(g_s,m_s;g_\rho,m_\rho)=g_s^2\left(\frac{g_\rho}{2}\right)^2[D_5(t,m_s,m_\rho)+c_5(t,m_s,m_\rho)+E_5(t,m_s,m_\rho)]. \quad (61)$$

$$\sigma_8(g_v,m_v;g_\pi,m_\pi)=g_v^2g_\pi^2[C_8(t,m_v,m_\pi)]. \quad (62)$$

$$\sigma_9(g_v,m_v;g_\rho,m_\rho)=g_v^2g_\rho^2[D_9(t,m_v,m_\rho)+C_9(t,m_v,m_\rho)+E_9(t,m_v,m_\rho)]. \quad (63)$$

$$\sigma_{10}(g_\rho,m_\rho;g_\pi,m_\pi)=\left(\frac{g_\rho}{2}\right)^2g_\pi^2[C_8(t,m_\rho,m_\pi)]. \quad (64)$$

$$\sigma_{p-n}(s,t)=\sigma_{n-p}(s,t)=\frac{1}{(2\pi)^5s}\left[\sum_{i=1}^{4}\sigma_i'(g_a,m_a)+\sum_{i=5}^{10}\sigma_i'(g_a,m_a;g_b,m_b)\right]. \quad (65)$$

$$\sigma_1'(g_s,m_s)=g_s^4[D_1(t,m_s)+E_1(t,m_s)]. \quad (66)$$

$$\sigma_2'(g_v,m_v)=g_v^4[D_2(t,m_v)+E_2(t,m_v)]. \quad (67)$$

$$\sigma_3'(g_\pi,m_\pi)=g_\pi^4[5D_3(t,m_\pi)-4C_3(t,m_\pi)+5E_3(t,m_\pi)]. \quad (68)$$

$$\sigma_4'(g_\rho,m_\rho)=\left(\frac{g_\rho}{2}\right)^4[5D_2(t,m_\rho)-4C_2(t,m_\rho)+5E_2(t,m_\rho)]. \quad (69)$$

$$\sigma_5'(g_s,m_s;g_v,m_v)=g_s^2g_v^2[D_5(t,m_s,m_v)+E_5(t,m_s,m_v)]. \quad (70)$$

$$\sigma_6'(g_s,m_s;g_\pi,m_\pi)=g_s^2g_\pi^2[2C_6(t,m_s,m_\pi)]. \quad (71)$$

$$\sigma_7'(g_s,m_s;g_\rho,m_\rho)=g_s^2\left(\frac{g_\rho}{2}\right)^2[-D_5(t,m_s,m_\rho)+2C_5(t,m_s,m_\rho)-E_5(t,m_s,m_\rho)]. \quad (72)$$

$$\sigma_8'(g_v,m_v;g_\pi,m_\pi)=g_v^2g_\pi^2[2C_8(t,m_v,m_\pi)]. \quad (73)$$

$$\sigma_9'(g_v,m_v;g_\rho,m_\rho)=g_v^2g_\rho^2[-D_9(t,m_v,m_\rho)+2C_9(t,m_v,m_\rho)-E_9(t,m_v,m_\rho)]. \quad (74)$$

$$\sigma_{10}'(g_\rho,m_\rho;g_\pi,m_\pi)=g_\pi^2g_\rho^2[-C_8(t,m_\rho,m_\pi)]. \quad (75)$$

其中：

$$\left.\begin{aligned}&D_1(t,m_a)=\frac{(t-4M^2)^2}{8(t-m_a^2)^2},\quad C_1(t,m_a)=-\frac{\frac{1}{2}(t^2+st)+2M^2(s-t)}{4(t-m_a^2)(4M^2-s-t-m_a^2)},\\&E_1(t,m_a)=D_1(u,m_a).\end{aligned}\right\} \quad (76)$$

$$\left.\begin{aligned}&D_2(t,m_a)=\frac{(s-2M^2)^2+t\left(s-\frac{t}{4}\right)}{2(t-m_a^2)^2},\\&C_2(t,m_a)=\frac{(s-2M^2)(s-6M^2)}{2(t-m_a^2)(4M^2-s-t-m_a^2)},\\&E_2(t,m_a)=D_2(u,m_a).\end{aligned}\right\} \quad (77)$$

$$\left.\begin{aligned}&D_3(t,m_a)=\frac{t^2}{8(t-m_a^2)^2},\quad C_3(t,m_a)=-\frac{(t-4M^2+s)t}{8(t-m_a^2)(4M^2-s-t-m_a^2)},\\&E_3(t,m_a)=D_3(u,m_a).\end{aligned}\right\} \quad (78)$$

$$\left.\begin{aligned}&D_5(t,m_a,m_b)=\frac{(2s+t-4M^2)M^2}{(t-m_a^2)(t-m_b^2)},\\&C_5(t,m_a,m_b)=-\frac{(t+s)^2-\frac{7}{2}M^2s+\frac{1}{2}M^2t}{4(t-m_a^2)(4M^2-s-t-m_b^2)}\end{aligned}\right\} \quad (79)$$

$$-\frac{t^2-4M^2s-\frac{17}{2}M^2t+18M^4}{4(t-m_b^2)(4M^2-s-t-m_a^2)},$$

$$E_5(t,m_a,m_b)=D_5(u,m_a,m_b).$$

$$C_6(t,m_a,m_b)=-\frac{t(t+s)}{8(t-m_a^2)(4M^2-s-t-m_b^2)}$$

$$-\frac{\frac{1}{2}t^2+\frac{1}{2}st-4M^2t-2M^2s+8M^4}{4(t-m_b^2)(4M^2-s-t-m_a^2)}. \tag{80}$$

$$C_8(t,m_a,m_b)=\frac{t^2-\frac{7}{2}M^2t}{4(t-m_a^2)(4M^2-s-t-m_b^2)}$$

$$+\frac{(t+s)^2-\frac{9}{2}M^2(t+s)+2M^4}{4(t-m_b^2)(4M^2-s-t-m_a^2)}. \tag{81}$$

$$\left.\begin{aligned}
D_9(t,m_a,m_b)&=\frac{(s-2M^2)^2+t\left(s-\frac{t}{4}\right)}{4(t-m_a^2)(t-m_b^2)},\\
C_9(t,m_a,m_b)&=\frac{(s-2M^2)(s-6M^2)}{8}\left(\frac{1}{(t-m_a^2)(4M^2-s-t-m_b^2)}\right.\\
&\left.\quad+\frac{1}{(t-m_b^2)(4M^2-s-t-m_a^2)}\right),\\
E_9(t,m_a,m_b)&=D_9(u,m_a,m_b).
\end{aligned}\right\} \tag{82}$$

其中:

$$\left.\begin{aligned}
&s=(\bar{p}+p_2)^2=4E^{*2},E^*=[(\bar{\boldsymbol{p}})^2+M_R^{*2}]^{1/2},\\
&t=(\bar{p}-p_3)^2=\frac{1}{2}(s-4M^2)(\cos\theta-1),\theta=\bar{\boldsymbol{p}}\wedge\bar{\boldsymbol{p}}_3,u=4M^2-s-t.
\end{aligned}\right\} \tag{83}$$

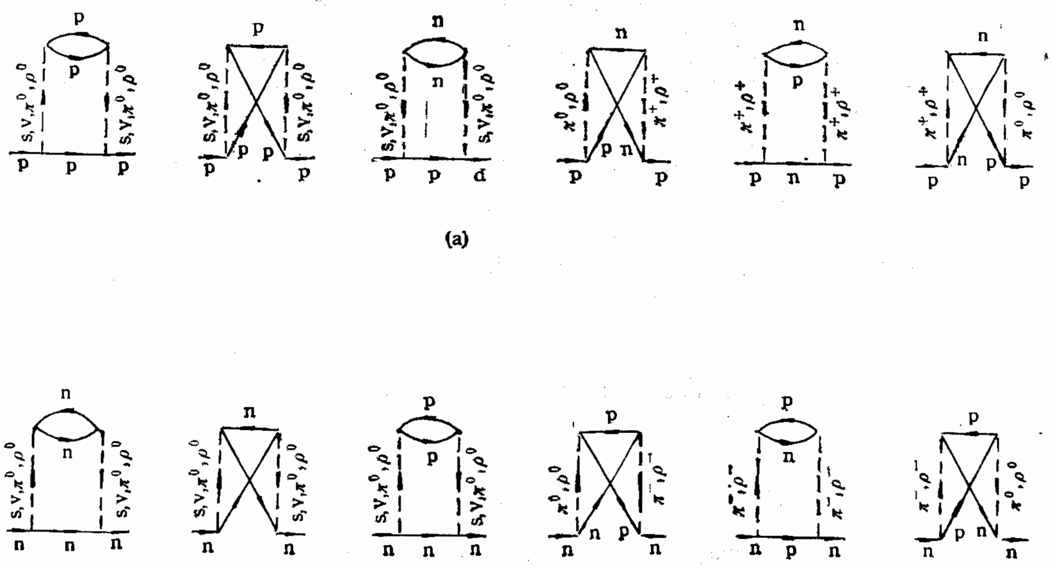
(76)—(82)式中的 $M\equiv M_R^*$,M_R^* 由(51)式给出,$\bar{p}$ 由(50)式给出. 函数 D_i 表示直接项贡献,E_i 表示交义项贡献,C_i 表示二者相乘贡献. 质心系中二粒子的相对速度 v 为:

$$v=\left|\frac{\bar{\boldsymbol{p}}}{\bar{p}^0}-\frac{\boldsymbol{p}_2}{p_2^0}\right|=\frac{F}{\bar{p}_0p_2^0},\quad F=[(\bar{p}^\mu p_{2\mu})^2-M_R^{*4}]^{1/2}. \tag{84}$$

$$\left.\begin{aligned}
F_1^0\left(\pm\frac{1}{2}\right)&=f_{\pm\frac{1}{2}}(X,p_2)[1-f_{\pm\frac{1}{2}}(X,p_3)][1-f_{\pm\frac{1}{2}}(X,p_4)]f_{\pm\frac{1}{2}}(X,\bar{p}),\\
F_2^0\left(\pm\frac{1}{2}\right)&=[1-f_{\pm\frac{1}{2}}(X,p_2)]f_{\pm\frac{1}{2}}(X,p_3)f_{\pm\frac{1}{2}}(X,p_4)[1-f_{\pm\frac{1}{2}}(X,\bar{p})],\\
F_1^1\left(\pm\frac{1}{2}\right)&=f_{\mp\frac{1}{2}}(X,p_2)[1-f_{\pm\frac{1}{2}}(X,p_3)][1-f_{\mp\frac{1}{2}}(X,p_4)]f_{\pm\frac{1}{2}}(X,\bar{p}),\\
F_2^1\left(\pm\frac{1}{2}\right)&=[1-f_{\mp\frac{1}{2}}(X,p_2)]f_{\pm\frac{1}{2}}(X,p_3)f_{\mp\frac{1}{2}}(X,p_4)[1-f_{\pm\frac{1}{2}}(X,\bar{p})].
\end{aligned}\right\} \tag{85}$$

其中 $f_{\frac{1}{2}}$ 和 $f_{-\frac{1}{2}}$ 分别表示质子和中子的分布函数. 在得到(52)式的碰撞项时,正如在它

论 Fock 项时已指出那样，由于 $\delta[(\bar{p}-p_i)^2-m_*^2](i=3,4)$ 在壳条件不满足，因而已将(29)和(31)中包含 $\Delta_*^{0\gtrless}(\bar{p}-p_i,X)$ 的项去掉. 质子和中子的碰撞项的物理意义如图 3 所示.



(a)

(b)

图 3　碰撞项费曼图

(a) 质子引起的碰撞项　(b) 中子引起的碰撞项

三、小　　结

本文从 QHD-I 推广到 QHD-II，即在相互作用拉氏量中进一步引入交换同位旋矢量的 π 介子和 ρ 介子，这就导致核子同位旋自由度的引入. 其结果是：〈i〉导出质子和中子分布函数所分别满足的 BUU 方程，它们之间是相互耦合的. 〈ii〉Hartree 和 Fock 项也是与核子同位旋相关，但赝标量 π 介子的引入对 Hartree 项无贡献，Fock 项对核子的有效质量和动量都提供修正. 〈iii〉在碰撞项中也分为两项，一项是正比于质子-质子(中子-中子)散射的有效微分截面，另一项是正比于质子-中子(中子-质子)散射有效微分截面，即包括了质子(中子)所引起的核子-核子散射的全过程.

我们所建立的相对论 BUU 方程是基于等效拉氏量，其参数已在核物质中确定，因此对于输运过程来说再没有自由参数了. 当然这一套自洽理论是否能符合实验还有待于进一步进行理论计算并与实验数据进行比较. 但我们在此强调的是建立了一个完整的理论框架，等价于 G 矩阵和 BUU 耦合的理论框架，自动顾及介质效应，但又比之简单得多，便于计算，今后我们还应把非弹性道包括进来进一步完善我们的理论.

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Relativistic BUU Equation

Yu Ziqing Mao Guangjun
(*Department of Physics, Nankai University, Tianjin* 300071)

Zhuo Yizhong Li Zhuxia Sun Zhemin
(*Institute of Atomic Energy, Beijing* 102413)

Abstract

Based on the Waleck's models QHD-I and QHD-II describing the nucleon-nucleon interaction, the Boltzmann-Uehling-Uhlenbeck (BUU) equation, which is the time evolution of the nucleon distribution function including the Hartree and Fock self-energy terms as well as the Born collision term and its exchange term, has been derived by using the closed-time path Green's function technique and assuming that the Green's functions and the self-energy terms are slowly varying functions of the centre-of-mass coordinates. Our result shows that the BUU equation for proton and that for neutron are simultaneous each other.