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A New Search for 1^{-+} Exotic State

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For the purpose of data analysis with BES (Beijing Spectrometer), some new relations, which can be used to search for the 1^{-+} exotic state in $J/\psi \to \omega X$, $X \to K\bar{K}\pi$, are obtained. How to search for the 1^{-+} exotic state in the three-step two-body hadronic decay process $J/\psi \to \omega X$, $X \to K\bar{K}^*$, $K^* \to K\pi$ is also discussed.

1. INTRODUCTION

We discussed previously [1] how to determine whether a state X is a 1^{-+} exotic state or 1^{++} ordinary meson in the process $e^+e^- \to J/\psi \to \omega + X$, $\omega \to 2\pi$ or 3π, $X \to K\bar{K}\pi$ by using the generalized moment analysis [2], and gave some relations for vector meson angular distributions of moments $H_J(\theta_v, LMlm)$ and moments $M_J(LMlm)$ (the spin-parity of X is J^P). We noted that these relations include some unknown quantities, e.g., the decay parameter $|R_\mu|^2$, and helicity amplitude ratios x_1, z_1 and z_1', and that the common normalization factor is still included. This makes data analysis difficult.

The experimental results of Mark III [3] indicate that the J/ψ hadronic decays $J/\psi \to \omega + X$, $X \to K\bar{K}\pi$ may proceed through the following three-step two-body decay process:

$$
\begin{array}{l}
J/\psi \to \omega + X \\
\qquad\qquad\quad \lfloor\!\to \bar{K} + K^* \\
\qquad\qquad\qquad\qquad \lfloor\!\to K + \pi
\end{array}
$$

In this paper, we introduce a new quantity — forward moment $M_J(LMlm)$, give some new relations which will be useful for data analysis, discuss how to search for the 1^{-+} exotic state in the three-step two-body decay process $e^+e^- \to J/\psi \to \omega + X$, $\omega \to 2\pi$ or 3π, $X \to K\bar{K}^*$, $K^* \to K\pi$ and obtain some relations.

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2. NEW RELATIONS

On the basis of the vector meson angular distribution of the moment $H_J(\theta_v, LMlm)$, we have the corresponding moment

$$M_J(LMlm) = \int_0^{\pi} H_J(\theta_V, LMlm)\sin\theta_V d\theta_V,$$

We also can define the forward moment as follows:

$$\overline{M}_J(LMlm) = \int_0^{\pi/2} H_J(\theta_V, LMlm)\sin\theta_V d\theta_V, \tag{1}$$

where θ_v is the angle between the incident positron beam direction and the emitting vector meson direction in the J/ψ rest frame. In addition to the 11 independent, non-zero and real moments, we have 17 independent, non-zero and real forward moments for the $J = 1$ case. We have the following new relations for the ratios of moments:

$$[2M(0022) - 5M(2022)]/M(2022) = \begin{cases} 0 & J^P = 1^- \\ -15 & J^P = 1^+ \end{cases}, \tag{2}$$

$$[2\overline{M}(2100) - 5\overline{M}(2120)]/\overline{M}(212-2) = \begin{cases} 5\sqrt{6}/2 & J^P = 1^- \\ -5\sqrt{6}/2 & J^P = 1^+ \end{cases}, \quad (x_1 \neq 0), \tag{3}$$

$$[-5/\sqrt{3}\,\overline{M}(2100) + 25/2\sqrt{3}\,\overline{M}(2120) + 25/2\sqrt{2}\,\overline{M}(212-2)]/\overline{M}(2120) = \begin{cases} 0 & J^P = 1^- \\ 25\sqrt{3} & J^P = 1^+ \end{cases}, \quad (x_1 \neq 0), \tag{4}$$

$$[-5/\sqrt{3}\,\overline{M}(2100) + 25/2\sqrt{3}\,\overline{M}(2120) - 25/2\sqrt{2}\,\overline{M}(212-2)]/\overline{M}(212-2) = \begin{cases} -25/\sqrt{2} & J^P = 1^- \\ 0 & J^P = 1^+ \end{cases}, \quad (x_1 \neq 0), \tag{5}$$

Since the unknown quantities $|R_\mu|^2$, x_1, z_1, z_1' and the normalization factor are all eliminated in these relations, it is easier and more effective to use them in identifying the 1^{-+} exotic state and 1^{++} ordinary state.

The helicity amplitude ratios (their definitions are the same as in Ref.[1]) are obtained directly by using these forward moments and moments. For $J^{PC} = 1^{-+}$ case we have

$$\begin{aligned} x_1 &= 2\sqrt{2}\,\overline{M}(212-2)/[\sqrt{3}\,M(2022)], \\ z_1 &= [-2\overline{M}(0021) - 10\overline{M}(2021)]/[3M(0022)], \\ z_1' &= 5M(2121)/[\sqrt{6}\,M(0022)]. \end{aligned} \tag{6}$$

For $J^{PC} = 1^{-+}$ case, we have

$$x_1 = 4\bar{M}(2100)/M(0022),$$
$$z_1' = 10\, M(2121)/[\sqrt{6}\, M(0022)]. \tag{7}$$

The helicity amplitudes are related directly to the dynamical mechanism of the process $J/\psi \to \omega + X$ and properties of the X particle. So it is very important to measure precisely the helicity amplitude ratios — the polarization parameters. Eqs.(6) and (7) are convenient for measuring precisely the polarization parameters.

3. HOW TO SEARCH FOR THE 1^{-+} EXOTIC STATE IN THE THREE-STEP TWO-BODY HADRONIC DECAY PROCESS OF J/ψ

The analysis of DM2 [4] and Mark III [5] shows the following three-step, two-body J/ψ radiative decay processes $J/\psi \to \omega + X$, $X \to K\bar{K}\pi$:

$$J/\psi \to \gamma + X(J^{PC} = 0^{-+}),\quad m_X = 1416\,{}^{+8}_{-8}\,{}^{+7}_{-5}\,\mathrm{MeV}$$
$$\hookrightarrow \pi + a_0(980)$$
$$\hookrightarrow K\bar{K},$$

$$J/\psi \to \gamma + X(J^{PC} = 0^{-+}),\, m_X = 1490\,{}^{+14}_{-8}\,{}^{+3}_{-16}\,\mathrm{MeV}$$
$$\hookrightarrow \bar{K} + K^*$$
$$\hookrightarrow K\pi,$$

$$J/\psi \to \gamma + X(J^{PC} = 1^{++}),\, m_X = 1443\,{}^{+7}_{-6}\,{}^{+3}_{-2}\,\mathrm{MeV}$$
$$\hookrightarrow \bar{K} + K^*$$
$$\hookrightarrow K\pi.$$

and the experimental result of Mark III shows the following three-step, two-body J/ψ hadronic decay processes $J/\psi \to \omega + X$, $X \to K\bar{K}\pi$:

$$J/\psi \to \omega + X$$
$$\hookrightarrow \bar{K} + K^*$$
$$\hookrightarrow K + \pi.$$

For the process $e^+e^- \to J/\psi \to V_1 + X$, $V_1 \to P_1P_2$ (or + P_3) and $X \to P_4 + V_2$, $V_2 \to P_5P_6$ (P_i is the pseudoscalar meson, V_j represents the vector meson), the helicity formalism of the angular distribution is

$$W_J(\theta_{V_1};\theta_1,\phi_1;\theta_2,\phi_2;\theta_3,\phi_3) \approx \sum_{\substack{\lambda_1,\lambda_1',\lambda_2,\\ \lambda_2',\lambda_X,\lambda_X'}} I_{\lambda_J,\lambda_J'}(\theta_{V_1})$$

$$A_{\lambda_1,\lambda_X} A^{*}_{\lambda'_1,\lambda'_X} \cdot B_{0,\lambda_2} B^{*}_{0,\lambda'_2} D^{1*}_{\lambda_1,0}(\phi_1,\theta_1,0)$$
$$D^{1}_{\lambda'_1,0}(\phi_1,\theta_1,0) D^{J*}_{-\lambda_X,-\lambda_2}(\phi_2,\theta_2,0) D^{J}_{-\lambda'_X,-\lambda'_2}(\phi_2,\theta_2,0)$$
$$D^{1*}_{\lambda_2,0}(\phi_3,\theta_3,0) D^{1}_{\lambda'_2,0}(\phi_3,\theta_3,0). \tag{8}$$

where $I_{\lambda_J,\lambda'_J}(\theta_{V_1}) = \frac{1}{4}\sum_{rr'}\langle\psi_{\lambda_J}|T|e^+_r e^-_{r'}\rangle\langle\psi_{\lambda'_J}|T|e^+_r e^-_{r'}\rangle^*$; θ_{V_1} is the angle between the incident positron beam direction and the emitting vector meson V_1 direction; (θ_1, ϕ_1) and (θ_2, ϕ_2) describe the direction of the momentum of P_1 (for the two-body decay of V_1) or the normal of the decay plane (for the three-body decay of V_1) in the rest frame of V_1 and P_4 in the rest frame of X, respectively (we choose a coordinate system in which the z axis is parallel to the V_1 direction in the rest frame of J/ψ); (θ_3, ϕ_3) describe the direction of the P_5 in the rest frame of V_2 (the z axis is taken as the V direction in the rest frame of X); λ_J, λ_1, λ_2 and λ_X are helicities of J/ψ, V_1, V_2 and X, respectively. A_{λ_1,λ_X} is the helicity amplitude of the process $J/\psi \rightarrow V_1 + X$. In general, we can define the helicity amplitude ratios as follows:

$$x = \frac{A_{1,1}}{A_{1,0}}, y = \frac{A_{1,2}}{A_{1,0}}, z = \frac{A_{0,0}}{A_{1,0}}, z' = \frac{A_{0,1}}{A_{1,0}}. \tag{9}$$

We only have x, z, z' for particle X with $J^{PC} = 1^{-+}$ and x, z' for particle X with $J^{PC} = 1^{++}$. B_{0,λ_2} is the helicity amplitude of the process $X \rightarrow P_4 + V_2$. We only have $B_{0,1}$ for particle X with $J^P = 1^-$ whereas the independent helicity amplitudes are $B_{0,1}$, $B_{0,0}$ for particle X with $J^P = 1^+$. We have used the above parity conservation condition:

$$A_{\lambda_1,\lambda_X} = P(-1)^J A_{-\lambda_1,-\lambda_X},$$
$$B_{0,\lambda_2} = P(-1)^{J+1} B_{0,-\lambda_2}. \tag{10}$$

The time reversal invariance requires that both A_{λ_1,λ_X} and B_{0,λ_2} be relatively real.

By using the generalized moment analysis [2], the vector meson angular distribution of moment is defined as

$$H_J(\theta_{V_1}, LMl_1m_1l_2m_2) = \int W_J(\theta_{V_1};\theta_1,\phi_1;\theta_2,\phi_2;\theta_3,\phi_3)$$
$$D^{L}_{M,m}(\phi_2,\theta_2,0) D^{l_1}_{m_1,0}(\phi_1,\theta_1,0) D^{l_2}_{m_2,0}(\phi_3,\theta_3,0)\sin\theta_1 d\theta_1$$
$$\sin\theta_2 d\theta_2 \sin\theta_3 d\theta_3 d\phi_1 d\phi_2 d\phi_3$$
$$\approx \sum_{\substack{\lambda_1\lambda'_1\lambda_2\\ \lambda'_2\lambda_X\lambda'_X}} I_{\lambda_J,\lambda'_J}(\theta_{V_1}) A_{\lambda_1,\lambda_X} A_{\lambda'_1,\lambda'_X} B_{0,\lambda_2} B_{0,\lambda'_2} (1\lambda'_1 l_1 m_1 | 1\lambda_1)$$
$$(10l_10|10)(J-\lambda'_X LM|J-\lambda_X)(J-\lambda'_2 Lm|J-\lambda_2)$$
$$(1\lambda'_2 l_2 m_2|1\lambda_2)(10l_20|10), \tag{11}$$

where $M = \lambda'_X - \lambda_X, m = \lambda'_2 - \lambda_2 = -m_2, m_1 = \lambda_1 - \lambda'_1$. The corresponding moment is

$$M_J(LMl_1m_1l_2m_2) = \int_0^{\pi} H_J(\theta_{V_1}, LMl_1m_1l_2m_2)\sin\theta_{V_1} d\theta_{V_1}. \tag{12}$$

The corresponding forward moment is

$$\overline{M}_J(LMl_1m_1l_2m_2) = \int_0^{\pi/2} H_J(\theta_{V_1}, LMl_1m_1l_2m_2)\sin\theta_{V_1}d\theta_{V_1} \tag{13}$$

How can we determine whether $P = -$ or $+$ for $J = 1$ case? This is the key point for determining whether 1^{-+} exotic state exists or not. For simplicity, we consider the $l_2 = m_2 = 0$ case. So we have 17 independent, non-zero real $\overline{M}_J(LMl_1m_100)$ and 11 independent non-zero real $M_J(LMl_1m_100)$. The difference from Ref.[1] and the first part of this paper is that when $L = 0$ and 2, we must make the following substitutions in the corresponding equations of the moments and the forward moments for $J^{PC} = 1^{-+}$ case:

$$|R_0|^2 \rightarrow \begin{cases} 2B^2_{0,1} & L = 0, \\ -B^2_{0,1} & L = 2, \end{cases} \tag{14}$$

and for $J^{PC} = 1^{++}$ case

$$|R_{+1}|^2 + |R_{-1}|^2 \rightarrow \begin{cases} 2B^2_{0,1} + B^2_{0,0} & L = 0, \\ 2B^2_{0,1} - 2B^2_{0,0} & L = 2. \end{cases} \tag{15}$$

Therefore we must find new relations. As a result, when $B^2_{0,1} \neq B^2_{0,0}$ for both $J^{PC} = 1^{-+}$ and 1^{++}, we have

$$x_1^2 = -M_1(222-200)/[\sqrt{6}M_1(202200)]. \tag{16}$$

and if $x_1^2 \neq 2$, we have

$$\begin{aligned}&[5M_1(202000) - 2M_1(200000) + 10\sqrt{6}M_1(202200) \\ &+ 5M_1(222-200)]/[5M_1(202000) - 2M_1(200000)] \\ &= \begin{cases} 2 & \mathrm{P} = - \\ 0 & \mathrm{P} = +, \end{cases}\end{aligned} \tag{17}$$

$$\begin{aligned}&[5M_1(202000) - 2M_1(200000) - 10\sqrt{6}M_1(202200) \\ &- 5M_1(222-200)]/[5M_1(202000) - 2M_1(200000)] \\ &= \begin{cases} 0 & \mathrm{P} = - \\ 2 & \mathrm{P} = +. \end{cases}\end{aligned} \tag{18}$$

if $x_1^2 = 2$ we have

$$\begin{aligned}&[2\overline{M}_1(210000) - 5\overline{M}_1(212000)]/\overline{M}_1(212-200) \\ &= \begin{cases} 5\sqrt{6}/2 & \mathrm{P} = -, \\ -5\sqrt{6}/2 & \mathrm{P} = +. \end{cases}\end{aligned} \tag{19}$$

When $B^2_{0,1} = B^2_{0,0}$, we have

$$\begin{aligned}&5M_1(202200)/M_1(002200) \\ &= \begin{cases} -1 & \mathrm{P} = -, \\ 0 & \mathrm{P} = +. \end{cases}\end{aligned} \tag{20}$$

In any event, we can always use one or more relations in Eqs.(17)-(20) to determine whether X is a 1^{-+} exotic state or 1^{++} ordinary state.

We can also obtain the helicity amplitude ratios directly by using the moments and the forward moments of the process. For $J^{PC} = 1^{-+}$, we have

$$x_1 = 2\sqrt{2}\bar{M}_1\ (212 - 200)/[\sqrt{3}M_1(202200),$$
$$z_1 = [-2\bar{M}_1(002100) + 20\bar{M}_1(202100)]/[3M_1(002200)],$$
$$z_1' = -5\sqrt{2}M_1(212100)/[\sqrt{3}M_1(002200)]. \tag{21}$$

For $J^{PC} = 1^{++}$, if $B^2_{0,1} \neq B^2_{0,0}$, we have

$$x_1 = -4\bar{M}_1(210000)/[5M_1(202200)],$$
$$z_1' = M_1(212 - 100)/[\sqrt{6}M_1(202200)], \tag{22}$$

and if $B^2_{0,1} = B^2_{0,0}$, we obtain

$$x_1^2 = \frac{\sqrt{6}}{60}\,\frac{2M_1(000000) - 5M_1(002000)}{M_1(002200)} - 1,$$

$$z_1'^2 = \frac{\sqrt{6}}{60}\,\frac{M_1(000000) + 5M_1(002000)}{M_1(002200)}. \tag{23}$$

The relative sign of x_1 and z_1' can be determined by the following equation

$$x_1 z_1' = -2\bar{M}_1(002100)/M_1(002200). \tag{24}$$

4. CONCLUSION

The three-body decay ($X \to K\bar{K}\pi$) and two-step two-body decays ($X \to K\bar{K}^*$, $K^* \to K\pi$) of the resonance X produced in the J/ψ hadronic decay process $J/\psi \to \omega K\bar{K}\pi$ are discussed respectively in this paper. Our aim is to determine whether the resonance X at 1420 MeV is a 1^{-+} exotic state or a 1^{++} ordinary state. We give some relations from the generalized moment analysis which are simple, clear, and convenient for identifying the 1^{-+} exotic state from 1^{++} ordinary state. In addition, we also give important helicity amplitude ratios.

Of course, as far as BES data analysis is concerned, we must correct the acceptance efficiency of BES and conduct the Monte Carlo simulation. We believe that if we have 10^7 perfect J/ψ events, we can definitely tell whether the resonance X is a 1^{-+} exotic state or not.

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