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Multi-particle Azimuthal Correlations and the Nuclear Equation of State

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Based on the method of Beckmann *et al.*, a new quantitative method is proposed for the study of multi-particle azimuthal correlations in relativistic heavy ion collisions. Multiparticle azimuthal correlations of nucleons for collisions of 1.2 *A* GeV Ar + KCl in the Bevalac streamer chamber are analyzed by this new method and Beckmann's. The experimental results are compared with the VUU model predictions for different nuclear equations of state. The incompressibility of nuclear matter can be estimated by the new method.

1. INTRODUCTION

One of the main objectives of studying relativistic heavy ion collisions is to gain some insight into the properties of nuclear matter under high temperature and high density, and to determine the nuclear equation of state (EOS). So far, theoretical predictions and experimental results show that collective flow parameters of particles in the final state are appropriate experimental observed quantities for the phenomenological study of compressive properties of nuclear matter in intermediate energy nucleus-nucleus collisions [1]. In order to study collective flow, it is necessary to make full use of the information measured in the final state of a collision. Sphericity analysis [2], transverse momentum analysis [3], azimuthal distribution function analysis [4] and particle pair

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correlation analysis [5] have been used to analyze collective flow. In these methods, collective flow parameters are flow angle, average transverse momentum per particle in the reaction plane and maximum azimuthal anisotropy, which display the characteristics of collective flow from different aspects. The method proposed by Beckmann *et al.*[6] reveals collective flow via the study of multi-particle azimuthal correlations in collision events. While it makes full use of the multi-particle transverse momentum information measured in the final state of a collision, this method does not provide us with a well-defined flow parameter. Therefore, it has not been applied to obtaining information about the EOS. In this paper, we first investigate the multi-particle azimuthal correlations of nucleons in the collision of 1.2 *A* GeV Ar + KCl by the method of Beckmann *et al.*, and compare the experimental results with VUU model predictions for different EOS. Then the method of Beckmann *et al.* is developed into a new quantitative method, with which the stiffness of the EOS is estimated.

2. OUTLINE OF THE VUU MODEL AND THE EXPERIMENT

The VUU model [7] used in this study is based on the Vlasov-Uehling-Uhlenbeck (VUU) equation. It is not an analytic solution of the VUU equation, but is a microscopic Monte-Carlo simulation. The core of the VUU model is the INC model [8]. The nucleon is assumed to be accelerated by the nuclear mean field, and the collisions between nucleons take place according to the experimental cross sections for free particles and are corrected by a factor for Pauli blocking. We use a version of the VUU code in which momentum-dependent interactions are ignored. We assume that the local nucleon density-dependent potential, U, is of the functional form:

$$U(\rho) = a\rho + b\rho^c, \tag{1}$$

where ρ is the density of the nucleons, and a, b and c are constants. The parameter c is determined by incompressibility, K, and the other two are constrained by the nuclear equilibrium conditions. $c = 2$ corresponds to incompressibility K = 380 MeV and lies in the range of what is normally considered a "stiff" EOS, while $c = 7/6$ corresponds to K = 200 MeV, usually described as a "soft" EOS.

The experimental data for collisions of 1.2 *A* GeV Ar + KCl come from the Bevalac streamer chamber. The samples under consideration contain 571 events with charged multiplicity $M \geq 30$. Assuming a simple geometrical picture, the impact parameters are between 0 and 3.6 fm. Further experimental details can be found elsewhere [9]. For each of the two values of EOS stiffness mentioned above, we have generated model statistics amounting to typically five times the experimental samples. In order to obtain meaningful conclusions, the VUU model predictions must be filtered before being compared with experimental data. The filtering process includes distortions resulting from particle misidentification, observational losses, absorption in the target, and energy losses [3,5].

3. METHOD OF BECKMANN *ET AL.* AND CORRESPONDING ANALYSIS

With the method of Beckmann *et al.*, possible collective effects in a relativistic heavy ion collision can be revealed through the study of multi-particle azimuthal correlations within the group of particles emitted forward or backward in the CMS of the collision, between these two groups of particles, and in the global event. In this method, the following multi-particle variables are introduced:

$$W_s = \frac{\sum_i^{m_s} p_i^t}{\sum_i^{m_s} |p_i^t|}. \tag{2}$$

$$W_s = |W_s|, \tag{3}$$

$$\alpha = \cos^{-1}\left[\frac{W_f \cdot W_b}{W_f \cdot W_b}\right], \tag{4}$$

where $s = f$, $s = b$ and $s = g$ represent the forward-emitted group of particles, the backward-emitted group of particles, and the global event, respectively. m_s is the number of charged particles in the corresponding particle group. $p_i^t(\mu)$ is the transverse momentum for the μth observed particle in the group being considered. Then correlation functions $\Delta(W_s)$ and $R(W_s)$, and a distribution function F are defined:

$$\Delta(W_s) = D(W_s) - B(W_s), \tag{5}$$

$$R(W_s) = \frac{D(W_s) - B(W_s)}{D(W_s) + B(W_s)}, \tag{6}$$

$$F = D(\alpha), \tag{7}$$

where $D(W_s)$ and $B(W_s)$ stand for the distribution of W_s for the observed events and the background events, respectively; $D(\alpha)$ is the distribution of α for the observed events. For every observed event, its background event is constructed by randomizing azimuthal angles, but preserving the polar angle and the magnitude of the momentum vector of every charged particle in it. So in $B(W_s)$, possible azimuthal correlations caused by the flow are removed [6].

Figure 1 shows the $D(W_f)$ and $B(W_f)$ of the collisions. The experimental data for the $D(W_f)$ and $B(W_f)$ are denoted by the solid circles and open circles, respectively. The best-fitted curves for the $D(W_f)$ and $B(W_f)$ are represented by dotted and solid lines, respectively. According to Eqs.(2) and (3), the magnitude of W_f is a variable that reflects the extent to which the number and the magnitude of transverse momentum vectors for the particles in the forward-emitted group preferentially lie in the direction defined by W_f. The bigger the value of W_f is, the more effective this group of emitted particles is, i.e., the stronger the multi-particle azimuthal correlations become, the more evident the collective effect is, and the larger the most probable value of W_f for $D(W_f)$ is. Because the calculation of W_f both for the collision event and the background event is influenced by many factors such as particle misidentification, observational losses, absorption in the target and energy losses [3,5], we cannot conclude exactly if there is collective effect in a collision merely through the calculation of $D(W_f)$ (the most probable value of W_f). This is the very reason why the most probable value of W_f for the $B(W_f)$ shown in Fig. 1 is not zero. The difference between the $D(W_f)$ and $B(W_f)$ of Fig. 1 indicates that within the forward-emitted group of particles in the CMS of the collision, there are multi-particle azimuthal correlations caused by the flow. The collective flow is produced in this reaction.

Figure 2 shows the correlation function $\Delta(W_f)$ in the experimental data, as well as in the VUU model predictions for the soft (shown in dotted histogram), and the hard EOS (signified by solid histogram). From Fig. 2, one can conclude that the VUU model can satisfactorily reproduce the multi-particle azimuthal correlations caused by the flow as seen in the experiment; but with the

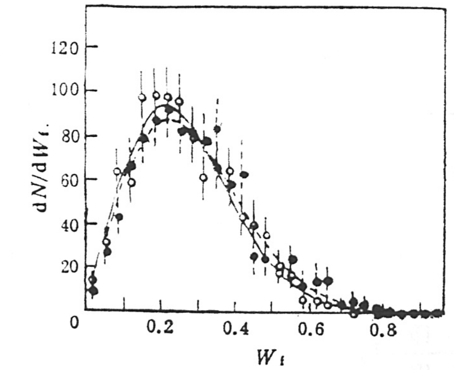

Fig. 1
Experimental results of $D(W_t)$ and $B(W_t)$ for collisions of 1.2 A GeV Ar + KCl.
—O—O— $B(W_t)$; —●—●— $D(W_t)$.

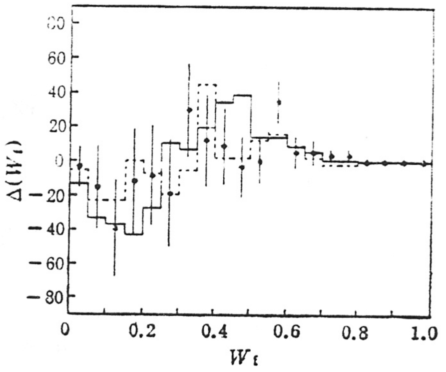

Fig. 2
Multi-particle azimuthal correlation function $\Delta(W_t)$ for collisions of 1.2 A GeV Ar + KCl.
—— Hard; - - - - Soft.

current statistics, the predictions for different EOS cannot be differentiated from the experimental results. We have calculated the multi-particle azimuthal correlation function $\Delta(W_g)$ for the global events, and the distribution function F of the multi-particle azimuthal correlation between the forward-emitted and backward-emitted groups of particles in the CMS of the collision. We also have made comparisons between the theoretical predictions and the experimental results. Our conclusion is that collective flow effect has been found in the reaction 1.2 A GeV Ar + KCl with the method of Beckmann *et al.*, but quantitative information about the EOS is difficult to obtain because of the poor statistics.

4. THE DEVELOPED BECKMANN'S METHOD AND CORRESPONDING ANALYSIS

With the current statistics, in order to study the collective flow from the point of view of multi-particle azimuthal correlations and then obtain some quantitative information about the EOS, we proceed as follows.

First of all, in order to describe the contribution of the multi-particle azimuthal correlations to the variable proposed by Beckmann *et al.*, a variable ν is introduced as

$$\nu = W_s^E - W_s^B \tag{8}$$

where W_s^E and W_s^B stand for the experimental events and corresponding background events, respectively. Because both W_s^E and W_s^B belong to [0, 1], the variable ν is in the range from —1 to +1. To test the statistical characteristic of the variable ν, we define

$$N(\nu) = \kappa \frac{dn}{d\nu}, \tag{9}$$

where $N(\nu)$ is the distribution function of the variable ν for the observed events, and κ is a normalization factor.

Because the Gaussian distribution, which is the shape of the curve for $N(\nu)$ for the experimental data, is a common one for statistical events, we assume that $N(\nu)$ is a Gaussian function:

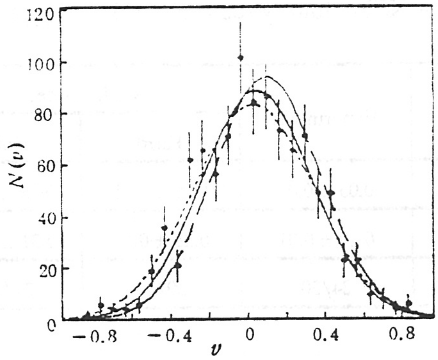

Fig. 3
The fitted curves for the distribution function $N(\nu)$ for collisions of 1.2 A GeV Ar + KCl in the rapidity region $y > 0.75\ y_{beam}$. ——— Hard; – – – – – – Soft.

$$N(\nu) = C_0 \cdot \exp\left(-\frac{(\nu-\lambda)^2}{2\sigma^2}\right), \tag{10}$$

where λ and σ represent the most probable value and the mean square deviation of the variable ν, and C_0 is a normalization factor. By fitting the experimental $N(\nu)$ with Eq.(10), λ and σ can be inferred. Relative to the background events, λ and σ, as the characteristic values of the distribution function $N(\nu)$, indicate the most probable effect and the dispersion of the contribution arised from the multi-particle azimuthal correlations to the Beckmann variable W_s.

Previous investigations [3-5] show that characteristic parameters have higher sensitivity to both the collective flow and the EOS in the rapidity region $y \geq 0.75\ y_{beam}$. Furthermore, in this rapidity region, the VUU model has produced an inclusive distribution that conforms to the experimental data, and the detector efficiency is high. Therefore, we study the multi-particle azimuthal correlations among the charged particle using $N(\nu)$ in this rapidity region, which might provide valuable information about the EOS of nuclear matter. Fig. 3 shows the fitted curve of the experimental distribution function $N(\nu)$ (solid line), the fitted curves of the VUU model predictions for both the soft (dotted line) and the hard (dashed-dotted line) EOS. The corresponding fitted parameters are listed in Table 1. The three χ^2/NDF for the fitting listed in Table 1 are about unity, which indicates that the phenomenological Gaussian assumption of the distribution function about the multi-particle azimuthal correlations is reasonable with regard to the current statistics. The three λ values corresponding to the three curves in Fig. 3 are all greater than zero. This fact indicates that there are multi-particle azimuthal correlations among the particles in this rapidity region, which force the particles to be preferentially emitted, and that the VUU model can predict the existence of the multi-particle azimuthal correlations. Fig. 3 and the values of λ and σ listed in Table 1 show that the experimental values of λ and σ are between the two theoretical predictions with the frame work of VUU model. With the current statistics, the developed new method can be used to infer the information about the EOS. When K increases from 200 MeV to 380 MeV, the value of λ increases about twice, hence λ is a sensitive variable to the EOS.

Table 1
The fitted values of λ and σ for collisions of 1.2 A GeV Ar + KCl (rapidity region $y > 0.75\, y_{beam}$).

	Experiment	VUU model	
		Hard	Soft
λ	0.05 ± 0.01	0.10 ± 0.01	0.03 ± 0.01
σ	0.29 ± 0.01	0.28 ± 0.01	0.31 ± 0.01
χ^2/NDF	24/30	29/39	27/30

5. CONCLUSIONS

With the method of Beckmann *et al.* and the new method developed in this paper, experimental data in the Bevalac streamer chamber and the VUU model simulation for the collisions of 1.2 A GeV Ar + KCl are analyzed. With the current statistics, our conclusions are as follows: (1) By using the method of Beckmann *et al.*, the analyses show that the collective flow exists in this reaction, but quantitative information about the EOS cannot be obtained because of the poor statistics. (2) By the method proposed in this paper, two fitted variables, λ and σ, reflect the multi-particle azimuthal correlation properties, and provide us with a quantitative description of sideward collective flow. By this new method, we find that λ is a parameter sensitive to the EOS, and the incompressibility of nuclear matter is in the range of 200 MeV -- 380 MeV within the framework of the VUU model.

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