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Mass Splitting of $L = 1$ States of Quarkonium and Lorentz Structure of Long-range Confinement Potential

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The Lorentz nature of the linear confinement potential between the quark-antiquark pair is analyzed by using the fine mass splitting data of $L = 1$ spin triplets of the quarkonium. It is found that the linear confinement potential has a mixed nature of the Lorentz scalar and vector ($0 \leq \eta \leq 29\%$, and $\eta = 0$ represents the pure Lorentz scalar nature). By analyzing tentatively measured $\chi(1^1P_1)$ and $\gamma(1^1P_1)$ masses, it is shown that in the linear confinement potential, a pure Lorentz scaler nature is required for the former meson, while the mixture of 79% Lorentz scalar and 21% Lorentz vector is required for the latter.

1. INTRODUCTION

In the potential model, by solving the Schrödinger equation, the spectroscopy of $J/\psi(\gamma)$ meson is understood as the bound state of the quark-antiquark pair $c\bar{c}$ ($b\bar{b}$). The interaction between the quark and antiquark is usually calculated [1] by quantum chromodynamics. It is known that in the short-range, this potential behaves like the color-electric interaction, and in the long-range it has the linear confinement behavior, i.e., the so-called Funnel potential [2]:

$$U_0 = -K/r + ar \quad (a > 0). \tag{1}$$

In order to analyze the fine and hyperfine structure [3] of the $q\bar{q}$ bound-state spectrum, the spin-dependent effective potential should be employed. This effective potential can be obtained from

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the Bethe-Salpater equation in the nonrelativistic approximation, and written as [4]:

$$U_{spin} = U_{LS}(L \cdot S) + U_T S_{12} + U_{SS}(S_1 \cdot S_2). \tag{2}$$

where the tensor operator $S_{12} = 12(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - 4(S_1 \cdot S_2)$, and coefficients are

$$\begin{cases} U_{LS} = \dfrac{\hbar^2}{2m^2c^2}\left(\dfrac{3V'}{r} - \dfrac{S'}{r}\right), \\ U_T = \dfrac{\hbar^2}{12m^2c^2}\left(-V'' + \dfrac{V'}{r}\right), \\ U_{SS} = \dfrac{2\hbar^2}{3m^2c^2}\nabla^2 V. \end{cases} \tag{3}$$

with V being the Lorentz vector and S the Lorentz scalar.

The spin-dependent potential U_{spin} depends on the exact Lorentz structure of $q\bar{q}$ interaction U_0. Unfortunately, it is impossible to find the latter from the first principle (e.g., lattice gauge theory): the only alternative is identifying the Lorentz structure by using the phenomenological method to fit the well-proved theories and experimental results. Thus, physicists proposed different Lorentz structures of the long-range confinement potential to calculate the fine structure of the spectrum of $c\bar{c}$ ($b\bar{b}$) bound states (the color-electric part of the potential is provided by the one-gluon exchange and is a Lorentz vector). For example, Barik and Jena [5] used a potential where the linear confinement part is modified by the long-range vacuum polarization and found that by employing the structure with half scalar and half vector nature, the fine mass splitting of $c\bar{c}$ bound states can be well described, while Olsson and Suchyta [6] took the Funnel potential and believed that the linear confinement part of the potential is a pure Lorentz scalar. Recently, Lucha and Schöberl [7] analyzed the Lorentz structure of the Funnel potential from the data of the fine mass splitting and concluded that the linear confinement part of the potential is a pure Lorentz scalar.

Besides the available data of fine mass splitting of $L = 1$ states of $c\bar{c}$ and $b\bar{b}$ mesons, two possible mass candidates, $\chi(1^1P_1)$ and $\gamma(1^1P_1)$ mesons whose masses are 3525.4 ± 0.8 MeV and 9894.8±1.5 MeV, respectively, were announced by R704 [8] and CLEO [9] groups. Since the difference between the weighted center of $L = 1$ spin triplets and $M(1^1P_1)$ state depends only on the expectation value of the spin-spin term, by analyzing these two data in detail, the Lorentz structure of the linear confinement potential can be clearly identified. However, it was found that in the perturbative potential model, these two mass data are inconsistent, as pointed out previously [10,11] (e.g., in Ref. [10] the $q\bar{q}$ potential was obtained from the perturbative two-loop calculation and modified by the additional long-range term).

In order to understand the Lorentz structure of the linear confinement potential better, we first provide the formula of the mass splitting induced by U_{spin} (for a certain orbital angular momentum) and then analyze the available fine mass splitting data of $L = 1$ states. Next, we study the Lorentz structure from the masses of the $\chi(1^1P_1)$ and $\gamma(1^1P_1)$ mesons given by R704 and CLEO groups, by using the energy-dependent effective potential.

2. MASS FORMULA

Using

$$\begin{cases} \langle \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \rangle = \dfrac{1}{2}\left[S(S+1) - \dfrac{3}{2}\right], \\ \langle \boldsymbol{L} \cdot \boldsymbol{S} \rangle = \dfrac{1}{2}[J(J+1) - L(L+1) - S(S+1)], \\ \langle S_{12} \rangle = \dfrac{4\left[\langle L^2\rangle\langle S^2\rangle - \dfrac{3}{2}\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle - 3\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle^2\right]}{(2L-1)(2L+3)}. \end{cases} \tag{4}$$

where < > represents the expectation value in the zero-order wave function state with a fixed angular momentum $L > 0$, under the perturbative theory, we may write the masses of spin triplets $^3L_J(J = L-1, L, L+1)$ and spin singlet ^1L_L as

$$\begin{cases} M_{L-1} = E - (L+1)\langle U_{LS}\rangle - \dfrac{2L+2}{2L-1}\langle U_T\rangle + \dfrac{1}{4}\langle U_{SS}\rangle, \\ M_L = E - \langle U_{LS}\rangle + 2\langle U_T\rangle + \dfrac{1}{4}\langle U_{SS}\rangle, \\ M_{L+1} = E + L\langle U_{LS}\rangle - \dfrac{2L}{2L+3}\langle U_T\rangle + \dfrac{1}{4}\langle U_{SS}\rangle, \\ M'_L = E - \dfrac{3}{4}\langle U_{SS}\rangle. \end{cases} \tag{5}$$

where E is the zero-order energy of the state with $L > 0$ (i.e., the spin-averaged value of the masses of four levels, $^3L_{L-1}$, ^3L_L, $^3L_{L+1}$ and ^1L_L).

From Eq.(5) we obtain the ratio of fine mass splitting of the state with $L > 0$

$$\rho = \frac{M_{L+1} - M_L}{M_L - M_{L-1}} = \frac{(L+1)\langle U_{LS}\rangle - \dfrac{6L+6}{2L+3}\langle U_T\rangle}{\langle U_{LS}\rangle + \dfrac{6L}{2L-1}\langle U_T\rangle}. \tag{6}$$

and the weighted center of spin triplets

$$\begin{aligned} \overline{M}_L &= \frac{(2L-1)M_{L-1} + (2L+1)M_L + (2L+3)M_{L+1}}{3(2L+1)} \\ &= E + \frac{1}{4}\langle U_{SS}\rangle. \end{aligned} \tag{7}$$

The difference between $\overline{M}_L$ and M'_L is

$$\overline{M}_L - M'_L = \langle U_{SS}\rangle. \tag{8}$$

Thus, the fine mass splitting of $q\bar{q}$ bound state with $L > 0$ depends only on the spin-orbital and tensor terms, while the difference between the spin-weighted center and the mass of the spin singlet (hyperfine mass splitting) depends on the spin-spin term.

Table 1

Meson mass data [3] and the corresponding values of ρ_{exp}.

	$M(^3P_0)$	$M(^3P_1)$	$M(^3P_2)$	ρ_{exp}
$c\bar{c}$	3.4151	3.5106	3.5563	0.479
$b\bar{b}(n = 1)$	9.8598	9.8919	9.9132	0.664
$b\bar{b}(n = 2)$	10.2353	10.2552	10.269	0.693

Note: n is the radial quantum number; the unit of mass is GeV.

3. FINE MASS SPLITTING OF $L = 1$ STATE AND LORENTZ NATURE OF LINEAR CONFINEMENT POTENTIAL

Assume that the Funnel potential is the sum of a Lorentz vector and a Lorentz scalar,

$$V = -K/r + \eta a r, \quad S = (1 - \eta) a r \quad (0 \leqslant \eta \leqslant 1). \tag{9}$$

For the $L = 1$ state, Eq.(6) becomes

$$\rho = \frac{2\langle U_{LS}\rangle - \frac{12}{5}\langle U_T\rangle}{\langle U_{LS}\rangle + 6\langle U_T\rangle}. \tag{10}$$

Using Eqs.(3) and (9), we have

$$\rho = \frac{\frac{12}{5}\left\langle\frac{K}{r^3}\right\rangle + \left(\frac{19}{5}\eta - 1\right)\left\langle\frac{a}{r}\right\rangle}{3\left\langle\frac{K}{r^3}\right\rangle + \left(\frac{5}{2}\eta - \frac{1}{2}\right)\left\langle\frac{a}{r}\right\rangle} = \frac{2}{5}\cdot\frac{12x + (19\eta - 5)}{6x + (5\eta - 1)}. \tag{11}$$

where $x = \langle K/r^3\rangle / \langle a/r\rangle$. It is obvious that $0 < x < +\infty$.

The fine mass splitting data of $\chi(1^3P_J)$, $\gamma(1^3P_J)$ and $\gamma(2^3P_J)$ masons and corresponding values of ρ_{exp} are given in Table 1. The possible value of η, which determines the Lorentz structure, should be in such a region that three values of ρ_{exp} obtained from the experimental data are consistent.

We then analyze the Lorentz nature with different values (regions) of η.

1) $\eta = 1$, i.e., the linear confinement potential is a pure Lorentz vector. We find

$$\rho = \frac{2}{5}\cdot\frac{6x + 7}{3x + 2}. \tag{12}$$

is a monotonously decreasing function of x and $0.8 < \rho < 1.4$.

2) $\eta = 0$, i.e., the linear confinement potential is a pure Lorentz scalar. In this case,

$$\rho = \frac{2}{5}\frac{12x - 5}{6x - 1}. \tag{13}$$

Table 2
Fine mass splitting of mesons (ρ) and Lorentz nature of $q\bar{q}$ linear confinement potential (η).

Theoretical analysis			Data
η value	Fine structure		
$\eta = 1$	$0.8<\rho<1.4$		
$0.292<\eta<1$	$0.479<\rho<1.4$		
$0.2\leqslant\eta\leqslant0.292$	$-\infty<\rho<0.8$		$0.479\leqslant\rho\leqslant0.693$
$0<\eta<0.2$	$-\infty<\rho<0.8$	$2<\rho<+\infty$	
$\eta = 0$	$-\infty<\rho<0.8$	$2<\rho<+\infty$	

where a singularity appears at $x = 1/6$. For $0 < x < 1/6$, or $1/6 < x < +\infty$, ρ is a monotonously increasing function of x. Since we have $\rho(0) = 2$, $\rho(1/6^-) = +\infty$, $\rho(1/6^+) = -\infty$ and $\rho(+\infty) = 0.8$, the value of ρ falls into the region $2 < \rho < +\infty$ or $-\infty < \rho < 0.8$.

3) $0 < \eta < 1$, i.e., the linear confinement potential is a mixture of the Lorentz scalar and Lorentz vector.

For $0.2 < \eta < 1$, $\rho(x)$ is analytical.

(a) For $1/3 < \eta < 1$, $\rho(x)$ is a monotonously decreasing function of x and in the range of $0.8 < \rho < 1.4$.

(b) For $\eta = 1/3$, $\rho(x) = 0.8$.

(c) For $0.2 < \eta < 1/3$, $\rho(x)$ is a monotonously increasing function of x and has its minimum value at $x = 0$. We find that at $\eta = 0.292$, $\rho(0)$ approaches the smallest value of ρ_{exp} in Table 1, $\rho_{exp} = 0.479$, and becomes even smaller as $\eta \to 0.2$. Therefore, only when η is in the sub-region $0.2 < \eta \leq 0.292$ can the three ρ_{exp} values in Table 1 be consistent with one another.

For $0 < \eta \leq 0.2$, $\rho(x)$ has a singularity.

(a) At $\eta = 0.2$, $\rho(x)$, $\rho(x) = (24x - 2.4)/30x$ and is in the range of $-\infty < \rho < 0.8$.

(b) For $0 < \eta < 0.2$, $\rho(x)$ is a monotonously increasing function of x when $0 < x < x_0 = 1/6 - (5/6)\eta$ or $x_0 < x < +\infty$. In this case, $2 < \rho(0) < +\infty$, $\rho(x_0^-) = +\infty$, $\rho(x_0^+) = -\infty$, and $\rho(+\infty) = 0.8$. Thus, for $0 < \eta < 0.2$, we have $2 < \rho < +\infty$ or $-\infty < \rho < 0.8$.

The above results are summarized in Table 2. It is clearly shown that the possible value of η, with which three ρ_{exp} values obtained from the experimental data of the fine mass splitting of $L = 1$ mesons are consistent, is limited within the region

$$0 \leqslant \eta \leqslant 0.292. \tag{14}$$

That is to say, the $q\bar{q}$ linear confinement potential is a mixture of the Lorentz scalar and small Lorentz vector ($0 \leq \eta \leq 0.292$) (the linear confinement potential is a pure Lorentz scalar if $\eta = 0$). Since Lucha and Schöber [7] did not consider the mixture of the Lorentz scalar and vector, they concluded that the linear confinement potential is a pure Lorentz scalar.

4. $M(1^1P_1)$ AND LORENTZ STRUCTURE OF LINEAR CONFINEMENT POTENTIAL

In this section, we shall give an analysis in terms of the effective potential obtained from the relativistic two-body Dirac equation in the nonrelativistic where the expansion is approximation in the order of the inverse energy (E^{-1}). It is clear that an improved description for the hyperfine mass splittings of $L = 0$ states can be achieved with this energy-dependent potential [12]. The potential used here takes the following form

$$\begin{cases} U_{LS} = \dfrac{2\hbar^2 c^2}{E^2}\left(\dfrac{3V'}{r} - \dfrac{S'}{r}\right), \\ U_T = -\dfrac{2\hbar^2 c^2}{3E(E+2mc^2)} V'' + \dfrac{\hbar^2 c^2}{E^2}\left[1 - \dfrac{4E}{3(E+2mc^2)}\right]\dfrac{V'}{r}, \\ U_{SS} = \dfrac{16\hbar^2 c^2}{3E(E+2mc^2)} \nabla^2 V. \end{cases} \tag{15}$$

where one minor term in U_{SS} is omitted [12]. Let $E = 2mc^2 + E_n$. In the limit $E_n << 2mc^2$ Eq.(15) becomes Eq.(3).

Since the relation $<\nabla^2 1/r> = -4\pi<\delta(\boldsymbol{r})> = 0$ holds for $L = 1$ states (the zero-order wave function is zero at $r = 0$), we obtain from Eqs.(8), (9) and (15) ($\hbar = c = 1$):

$$\bar{M}_1 - M'_1 = \frac{16}{3E(E+2m)}\left\langle \nabla^2\left(-\frac{K}{r} + \eta a r\right)\right\rangle = \frac{32\eta a}{3E(E+2m)}\left\langle\frac{1}{r}\right\rangle. \tag{16}$$

When $\eta = 0$, i.e., the linear confinement potential is a pure Lorentz scalar, we have $\bar{M}_L - M'_L = 0$. When $\eta > 0$, i.e., the mentioned potential is a mixture of the Lorentz scalar and vector, $\bar{M}_L - M'_L > 0$. However, the experimental data are

$$\begin{cases} (\bar{M}_1 - M'_1)^{\text{Exp}}_{c\bar{c}} = 0 \pm 1.1\text{MeV}, \\ (\bar{M}_1 - M'_1)^{\text{Exp}}_{b\bar{b}} = 5.4 \pm 1.7\text{MeV}. \end{cases} \tag{17}$$

It is obvious that these data are inconsistent under Eq.(16). However, the measurements of R704's and CLEO's are statistically poor [8,9] and the assignments of energy-levels are only tentative [11]. If 3525.4 MeV is the correct mass of $\chi(1^1P_1)$ meson, one obtains $\eta = 0$ from Eq.(26) i.e., the linear confinement potential is a pure Lorentz scalar. If we believe $M(\gamma(1^1P_1)) = 9894.8$ MeV, then $\eta > 0$, i.e., the linear confinement potential is a mixture of the Lorentz scalar and vector. Next, we calculate the Lorentz structure of the linear confinement potential by using the fine mass splitting data of $\gamma(1^3P_J)$ meson and $M_{b\bar{b}}(1^1P_1) = 9894.8$ MeV.

Substituting Eq.(15) into Eq.(5), we have

$$\begin{cases} M_0 = E - \dfrac{16}{E^2} f_1 + \dfrac{(-12E - 40m)\eta + 4(E+2m)}{E^2(E+2m)} f_2, \\ M_1 = E - \dfrac{4}{E^2} f_1 + \dfrac{2 - 6\eta}{E^2} f_2, \\ M_2 = E + \dfrac{28}{5E^2} f_1 + \dfrac{(162E + 228m)\eta - 30(E+2m)}{15E^2(E+2m)} f_2, \\ M'_1 = E - \dfrac{8\eta}{E(E+2m)} f_2. \end{cases} \tag{18}$$

for $L = 1$ states, where $f_1 = <K/r^3>$, $f_2 = <a/r>$, and the quark masses are $(m_c, m_b) = (1600, 4938)$ MeV, with which the hyperfine mass splitting of $L = 0$ states [12] and the mass spectrum of vector mesons [13] are well described. Solving Eq.(18), we can obtain the values of E, f_1, f_2, η and find $\eta = 0.2075$, i.e., the $q\bar{q}$ linear confinement potential is a mixture of 79% Lorentz scalar and 21% Lorentz vector.

Table 3
Masses of $^1P_1$ levels (unit: MeV) and Lorentz structure of $q\bar{q}$ linear confinement potential.

	Theoretical prediction		Data
	$\eta = 0$	$\eta = 0.2075$	
$c\bar{c}$: M'	3525.4	3484.4	3525.4
$\bar{M} - M'$	0	41	0
$b\bar{b}(n = 1)$: M'	9900.2	9894.8	9894.8
$\bar{M} - M'$	0	5.4	5.4
$b\bar{b}(n = 2)$: M'	10260.7	10257.5	
$\bar{M} - M'$	0	3.2	

Using the mass data of $\chi(1^3P_J)$ and (2^3P_J) (J = 0, 1, 2) mesons, we solve the first three equations of Eq.(18) for E, f_1 and f_2, and obtain the masses of $\chi(1^1P_1)$ and $\gamma(1^1P_1)$ mesons from the fourth equation. They are 3484.4 MeV and 10257.5 MeV, respectively.

Since both η = 0 and 0.2075 fall into the allowed region found in Section 3, it is not certain whether $M_{c\bar{c}}(1^1P_1) = 3525.4$ MeV· or $M_{b\bar{b}}(1^1P_1) = 9894.8$ MeV is more reasonable from the above analysis. If both mass data and the assignments of corresponding energy-levels are correct, we have to modify the first-order perturbative potential model. By using the $q\bar{q}$ potential, Igi and Ono [10] pointed out that one cannot describe these two mass data in the potential model consistently. Thus, it is necessary to measure the masses of $\chi(1^1P_1)$ and $\gamma(1^1P_1)$ mesons with higher accuracy.

Table 3 shows the masses of $\chi(1^1P_1)$, $\gamma(1^1P_1)$ and $\gamma(2^1P_1)$ mesons calculated from Eq.(18) at η = 0 and η = 0.2075, respectively. The accurate measurements of these meson masses will provide us with the experimental evidence of the Lorentz structure of the $q\bar{q}$ linear confinement potential.

5. CONCLUSIONS

In this study, we use the Funnel $q\bar{q}$ potential where its long-range part is a linear confinement potential. This is because this potential not only successfully describes the bound-state spectrum of the heavy $q\bar{q}$ pair [2], but also agrees with the lattice gauge calculation [14]. Moreover, by employing the linear confinement potential, we can perform a simple calculation to distinguish the Lorentz structure from the available fine mass splitting data of mesons.

Finally, we summarize the above results as follows:

1) The analysis from the fine mass splitting data of mesons indicates that the linear confinement potential is a pure Lorentz scalar (η = 0), or a mixture of the Lorentz scalar and small Lorentz vector ($\eta \leq 29\%$).

2) In the perturbative potential model, the measured masses of $\chi(1^1P_1)$ and $\gamma(1^1P_1)$ mesons by R704 and CLEO groups are inconsistent. If the measured $M(\chi(1^1P_1))$ by the R704 group is reliable, the linear confinement potential is pure Lorentz scalar. On the other hand, if the measured $M(\gamma(1^1P_1))$ by the CLEO group is correct, the linear confinement potential is a mixture of 79% Lorentz scalar and 21% Lorentz vector.

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