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Transport of Charged Particle Beams in Nonlinear Periodic Fields

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In this paper, the transport of charged particle beams in nonlinear periodic fields is studied. By means of mapping technique the emittance plots of beams matched to the nonlinear periodic fields can be obtained. The variance of emittance plots and the beam envelopes are periodic, and their periods are equal to the period of the field.

1. INTRODUCTION

A perfect theory on the transport of charged particle beams in linear periodic fields has been developed [1-3]. Knowing the transfer matrix of whole period in a periodic field system, we can obtain the emittance plots of the beams matched to the system, whose shapes are superellipsoidal. When the beams with these emittance plots are transported in the periodic field, the variance of emittance plots and the envelopes are periodic and their periods are equal to the field period. But sometimes we must consider the effects of nonlinear periodic fields. For example, in some large-diameter intense beam transport systems, the nonlinearity of field is not negligible.

In nonlinear fields, for the beams with initial superellipsoidal emittance plots, the emittance plots will be distorted during transport process. In this case above-mentioned method encounters trouble.

Recently, a mapping technique was used to discuss the transport of the beam in nonlinear periodic fields (in [4], for example). We will take advantage of this technique to determine the emittance plots matched to the nonlinear field, although they are not superellipsoidal. The coupling between the transverse and the longitudinal motions and the space-charge effects are not taken into account.

For simplicity, we only discuss the axisymmetrical beams, although the results can be generalized to other cases.

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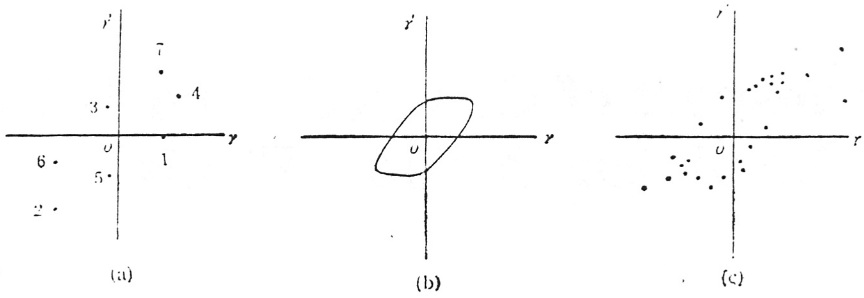

Fig. 1
Three types of motion of particles in periodic fields. (a) Periodic motion; (b) Quasiperiodic motion; (c) Nonperiodic motion.

2. MOTION OF PARTICLES IN NONLINEAR PERIODIC FIELDS

Here we are not concerned about the motion of particles within a period but only the evolution of the position and the direction of motion of particles every period, i.e., r_i and r'_i, the values of r and r' at $z = iL$. Here z is the longitudinal coordinate, r is the radial coordinate, $r' = dr/dz$, L is the field period and i is an integer.

The periodic field is described by means of mapping. Introducing a column vector x,

$$x = \begin{pmatrix} r \\ r' \end{pmatrix}. \tag{1}$$

The relation between x_{i+1} and x_i can be represented by a two-dimensional map F_p:

$$x_{i+1} = F_p(x_i). \tag{2}$$

The map F_p is determined by the field parameter p. Let $F_p^2(x)$ denote $F_p(F_p(x))$, $F_p^3(x)$ denote $F_p(F_p(F_p(x)))$ and so on. We have

$$x_i = F_p^i(x_0). \tag{3}$$

where x_0 is the initial value of x.

Giving p and x_0, we can determine x_1, x_2, x_3 $\cdots$ x_i $\cdots$, with Eq. (3) and mark the corresponding points on the phase plane (r,r'). It is found that there are three types of the motion of particles.

1) The periodic motion. The period of motion is n times the field period, where n is an integer. The particle jumps through n points on the phase plane in a certain sequence, then repeats periodically (see Fig. 1(a)). These points are the periodic points of the map F_p.

2) The quasiperiodic motion. The points corresponding to x_i ($i = 0, 1, 2, 3 \cdots, \infty$) form the "orbit" of a particle, which is one or several closed curve(s) on the phase plane. Note that the "orbit" is not the real trajectory of the particle on the phase plane because we are not concerned about the motion of particles inside a period. When a particle pass through a field period, it jumps from a point on the "orbit" to other point (see Fig. 1(b)). If the particle is located at point A on the phase plane at $z = iL$, we can find such an integer k that the distance between the position B of the particle at $z = (i + k)L$ and A is smaller than a given small positive number ε.

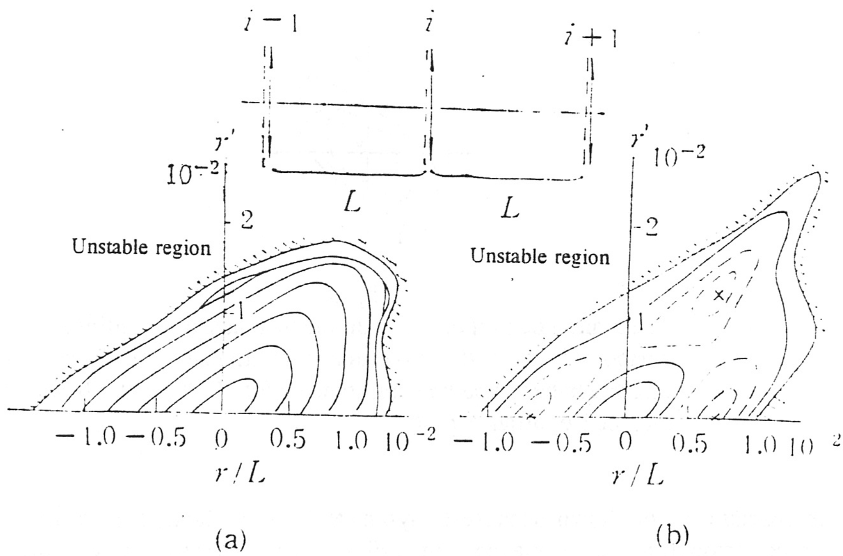

Fig. 2
The periodic field system consisting of axisymmetric thin lenses and the "orbits" of particles on the phase plane. The regions surrounded by "orbits" are the emittance plots matched to the system. (a) $L\alpha = 2$, $\alpha/\beta L^2 = -4 \times 10^{-4}$; (b) $L\alpha = 1.8$, $\alpha/\beta L^2 = 4 \times 10^{-4}$.

3) The nonperiodic motion. The particles can escape from the system at last (see Fig. 1(c)).

Obviously, the third case is unstable, the first is a special case, and we are interested in the second one.

As an example, let us consider a periodic field system consisting of axisymmetrical thin lenses [5], in which all the thin lenses are the same and are selected as the starting points of the periods. Then the map F_p can be written as

$$\begin{cases} r_{i+1} = r_i + L r'_{i+1}, \\ r'_{i+1} = r'_i - \alpha r_i - \beta r_i^3, \end{cases} \tag{4}$$

where L is the distance between the lenses, and α and β are lens parameters with $\alpha > 0$ reflecting the linear focusing power of the lenses and β the nonlinear property. Note r'_i is the value of r' at the incident side of the i-th thin lens. The "orbits" of particles for different lens parameters can be obtained by computation. From Fig. 2, we can see some characteristics of nonlinear periodic field systems.

1) The linear periodic field systems are stable when the values of parameters are in a certain range. However, for nonlinear periodic fields, the stability dependents not only on the field parameters but also the initial conditions of the motion of particles. The systems with different parameters have different stable regions. Some systems do not have a stable region.

2) For the stable linear periodic field systems, all the quasiperiodic "orbits" are the similar ellipses. But for nonlinear systems the "orbits" have different shapes. Generally speaking, the "orbits" approach ellipses in the regions with weak nonlinearity (as the region near the origin in Fig. 2), but are distorted significantly in the regions with strong nonlinearity.

3) For the linear periodic field systems, all the particles are in a periodic motion when the field parameters take some special values. But nonlinear systems differ greatly from it. For example, in Fig. 2(b), the particles at the resonance points marked by x are in a periodic motion (the resonance point

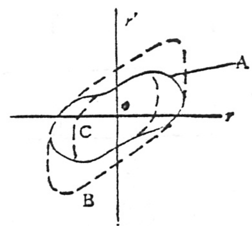

Fig. 3
The emittance plot of the beam mismatched to a nonlinear periodic field. A the mismatched emittance plot; B the circumscribed matched emittance plot; C the inscribed matched emittance plot.

under the abscissa is not shown because of symmetry). Near them, the "orbits" consist of several closed curves surrounding the resonance points. Figure 2(b) shows two dashed "orbits", each of which consists of four closed curves (the curves under the abscissa are not shown). When the particle pass through a period it jumps from a closed curve to other closed curve in a certain sequence and repeats periodically. Such regions are called the resonance regions. Farther from the resonance points, the "orbits" are still closed curves surrounding the origin. The exist of resonance region is a special phenomenon in nonlinear system.

Generally speaking, in fringes of the stable region or the resonance regions, a small variance of the field parameters or the initial conditions can result in a significant change of the "orbit".

The further discussions on the above-mentioned characteristics of nonlinear periodic field systems will be given in other papers.

3. MATCHED EMITTANCE PLOTS IN NONLINEAR PERIODIC FIELDS

If a nonlinear periodic field system is described with the two-dimensional map F_p, any point on the "orbit" of a particle in the system must be mapped onto the same "orbit", which means that any "orbits" are mapped on it.

As the "orbits" cannot intersect each other, the phase-plane region surrounded by any "orbit" is mapped on itself. Hence the regions surrounded by any "orbits" are the emittance plots matched to this nonlinear periodic field system. If the beams with these emittance plots enter the periodic field, the variance of the emittance plots in transport process and the beam envelopes are periodic, and their periods are equal to the period of the periodic field. It is different from linear periodic field systems that these emittance plots are not elliptical. Figure 2 shows that some emittance plots are simpler and approximate ellipses or parallelograms. But other emittance plots have more complex shapes, and some of them consist of several closed curves.

If the parameters are chosen properly, nonlinear periodic fields can also be used to transport the charged particle beams. For example, such systems as Fig. 2(a) can be used to transport the beams with some parallelogramic emittance plots.

If the emittance plot A of a beam is not matched to the nonlinear periodic field system, its circumscribed emittance plot B and inscribed emittance plot C matched to the system can be determined (see Fig. 3). For the beam with an initial emittance plot A, its emittance plot is distorted in transport process and its envelope is not periodic, but its effective emittance cannot be larger than that of the beam with the emittance plot B, its envelope cannot go beyond the envelope of the latter and the phase region overlapped with the emittance plot C is not distorted.

4. GENERALIZATION OF THEORY

We have discussed the transport of the axisymmetric beams. For other beams, if there is no coupling between two transverse motion (x and y directions), the above-mentioned theory can also be used to study transverse motion of the beam in nonlinear periodic field directly. On the contrary, if there is coupling between two transverse motion, the "orbits" of particles become "supersurfaces". Then the computation is very complex, while the idea remains applicable.

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