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Average Yields of Long-Lived Charged Particles in High Energy e^+e^- Annihilation

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Based on the quark production rule and quark combination rule, the average yields of long-lived charged particles and average charged multiplicities in high energy e^+e^- annihilation have been studied. The calculated yields of hadrons, average charged multiplicities and their energy dependences are all consistent with the available data.

1. INTRODUCTION

In [1], the hadronization model in e^+e^- annihilation, the quark production rule and combination rule had been studied. Based on this model the average charged multiplicities $<n_{ch}>$ and various hadron yields in e^+e^- annihilation were calculated. The results are consistent with the data available at that time. According t_0 this model, certain authors have also studied the baryon-antibaryon flavor correlations, the semi-inclusive distribution, the gluon fragmentation and the charmed meson yields, and all obtained satisfying results [2], indicating that the hadronization model in [1] is successful. However, in [1] only the electromagnetic interaction between e^+ and e^- was taken into account during calculating the production cross section of the primary quark pair $q_i\bar{q}_i$, while the weak interaction was not. Using the electroweak unified standard model, one can calculated the production cross section $\sigma(e^+e^- \to \gamma, Z^0 \to q_i\bar{q}_i)$ for the primary quark pair $q_i\bar{q}_i$. The results show that for the cms energy. $\sqrt{s} \leq 40$ GeV the contribution from the weak interaction to this cross section can be neglected comparing

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to that from the electromagnetic interaction, but for $\sqrt{s} > 40$ GeV the former must be taken into account. For $\sqrt{s} \leq 40$ GeV the ratios among the production probabilities for the primary quark pair $q_i\bar{q}_i$, $P_i(i = u,d,s,c,b)$, are $P_u:P_d:P_s:P_c:P_b = 4:1:1:4:1$, which do not depend on $\sqrt{s}$. For $\sqrt{s} > 40$ GeV the ratios among P_i, depend on $\sqrt{s}$ and for $\sqrt{s} = m_z$ they become $P_u:P_d:P_s:P_c:P_b = 7:9:9:7:9$. The probabilities P_i are the fundamental parameters for calculating the hadron yields. Since the weak interaction is not taken into account, the calculated hadron yields in [1] are only suitable for $\sqrt{s} \leq$ 40 GeV. Recently the ALEPH, DELPHI and TOPAZ etc. collaborations have published the data for the average charged multiplicities $<n_{ch}>$ at $\sqrt{s} = 91$ and 50-60 GeV. These collaborations will also publish the data for various hadron yields in the corresponding energy regions. In order to examine the hadronization mechanism in wider energy region, it is necessary to calculate $<n_{ch}>$ and various hadron yields at $\sqrt{s} > 40$ GeV based on the hadronization model in [1] and the electroweak unified standard model.

Many experiments have revealed that the $(3/2)^+$ baryon productions suffer very strong suppression comparing to the $(1/2)^+$ baryon production e^+e^- annihilation. According to $SU(6)$ symmetry the production probability of the particle with a spin j is proportional to $2j + 1$. Experiments have verified that the ratio of 1^- meson 0^- mesons produced directly in e^+e^- annihilation consistent with the relation of 3:1 given by $SU(6)$ symmetry, but the measured ratio of the directly produced $(3/2)^+$ baryons to $(1/2)^+$ baryons is about 0.3:1. which is not consistent with the relation of 2:1 given by $SU(6)$ symmetry. The data of [3] point out that the "spin suppression" of baryon results from the flavor conservation and the ratio of $(3/2)^+$ baryons to $(1/2)^+$ baryons is $\beta = (\lambda + 1)/(2\lambda + 1)$, here λ is the suppression factor for the strange quark production. Take $\lambda = 0.3$ and put it into the above mentioned formula of β, giving $\beta = 0.36$, which is consistent with the experimental data. In this paper we take $\beta = 0.36$ in our calculation of hadron yields.

As the decay branch ratios of the particles containing heavy quark $c(\bar{c})$ or $b(\bar{b})$ are not fully know at present, it becomes an important problem how to take the contributions of the heavy-particle decays when calculating into account the final-state particle yields. In this paper we will consider this problem in detail.

2. THE PRODUCTION PROBABILITIES OF VARIOUS FLAVOR QUARK JETS

The production of hadrons in e^+e^- annihilation is a complex process. First of all e^+e^- are converted into a primary quark pair $q_i\bar{q}_i$ by the electroweak interaction. Then many new quark pairs $q_i\bar{q}_i$ are produced by the strong interaction between q_i and $\bar{q}_i$ through vacuum excitation when q_i and $\bar{q}_i$ separate from each other. Finally all these quarks and antiquarks are combined into various hadrons in virtue of the strong interaction. There is a certain probability that the primary quark pair q_i and $\bar{q}_i$ first radiate a hard gluon, then carry out the above-mentioned processes to form two hadron jets. According to the quantum chromodynamics (QCD), the hard gluon produced by q_i or $\bar{q}_i$ will continuous radiate many new gluons and all these gluons will finally convert into many quark pairs $q_i\bar{q}_i$ to form the third hadron jet. That is the cause for a few three-jet events emerge in e^+e^- annihilation. The two or three jets formed by the primary i-flavor quark $q_i\bar{q}_i$ are known as the i-jets. The production probability P_i of the i-jets is just that of the primary quark pair $q_i\bar{q}_i$ which is determined by the cross section $\sigma(e^+e^- \to \gamma, Z^\circ \to q_i\bar{q}_i g)$, where g stands for the radiated gluon. The normalized cross section $R(q_i)$ is defined as

$$R(q_i) = \frac{\sigma(e^+e^- \to \gamma, Z^0 \to q_i\bar{q}_i g)}{\sigma(e^+e^- \to \gamma \to \mu^+\mu^-)}, \quad (1)$$

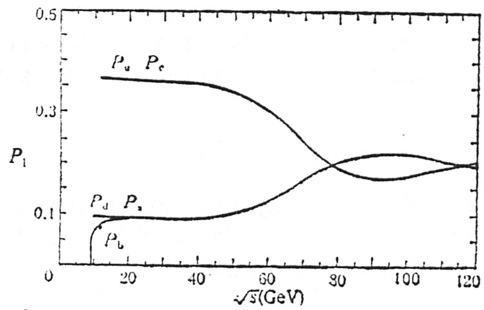

Fig. 1
The production probabilities for five kinds of quark jets.

where

$$\sigma(e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3S} \tag{2}$$

is the lowest QED point-like cross section for $e^+e^- \rightarrow \mu^+\mu^-$. In Eq. (2), α is the fine-structure constant, and s is the square of the cms energy of the e^+e^- system. According to the unified electroweak model and taking the third order QCD corrections and the mass effect of the quark q_i into account, we can write

$$R(q_i) = 3 \sum_{\mu=V,A} F_\mu \left[1 + C_1^\mu \frac{\alpha_S}{\pi} + C_2^\mu \left(\frac{\alpha_S}{\pi}\right)^2 + C_3^\mu \left(\frac{\alpha_S}{\pi}\right)^3\right] \theta(\sqrt{s} - 2m_i), \tag{3}$$

In Eq. (3), the factor 3 accounts for the three colors of the quarks; m_i is the construction mass of quark q_i; F_μ ($\mu = V,A$) ia related to the energy $\sqrt{s}$, the charge Q_i and velocity β_i of quark q_i, the coupling constant G_F of the weak interaction, the mass m_Z and decay with Γ_Z of the particle Z^0 and Weinberg angle θ_W; $C_k^\mu(k = 1, 2, 3)$ is related to β_i and the flavor number N_f of the primary quarks in e^+e^- annihilation at the given energy $\sqrt{s}$; α_S is the strong coupling constant which is related to $\sqrt{s}$, N_f and the QCD parameter $\Lambda\frac{(N_f)}{M^S}$. In [4], we introduced the methods of how to calculate F_μ, C_k^μ and α_S, and to determine the values of the parameters in formulas of F_μ, C_k^μ and α_S. The production probability of i-jets can be written as

$$P_i = R(q_i) \Big/ \sum_{i=udscb} R(q_i). \tag{4}$$

Figure 1 shows the calculated P_i in the energy range $\sqrt{s}$ = 10-120 GeV. The parameters in formulas of P_i are taken as follows: m_Z = 91.16 GeV, Γ_Z = 2.534 GeV, $\sin^2\theta_W$ = 0.230, $\Lambda\frac{(5)}{MS}$ = 0.22 GeV, $m_u = m_d$ = 0.34 GeV, m_s = 0.45 GeV, m_c = 1.5 GeV, m_b = 4.73 GeV and $G_F = 1.1653 \times 10^{-5}$ GeV^{-2}. The calculated probabilities of c-jet and b-jet in e^+e^- annihilation at $\sqrt{s}$ = 91 GeV are P_c = 0.170 and P_b = 0.220, which are consistent with the measured results in [5]: $P_c = 0.148 \pm 0.044^{+0.043}_{-0.038}$, and $P_b = 0.220 \pm 0.016 \pm 0.024$.

3. AVERAGE NUMBERS OF THE LIGHT QUARKS PRODUCED BY VACUUM EXCITATION

According to the "tunneling effect" the probability for a primary quark pair $q_i\bar{q}_i$ produce a new quark pair $q_\nu\bar{q}_\nu$ through vacuum excitation can be expressed as [6]

$$N_\nu \propto e^{-km_\nu}, \tag{5}$$

where k is a constant related to the strong interaction among the quarks. Putting the values of masses of five kinds of quarks into Eq. (5) and Letting N_s/N_u be equal to the strange suppression factor $\lambda = 0.3$, we can determine the value of k, and then obtain

$$N_u:N_d:N_s:N_c:N_b = 1:1:0.3:10^{-13}:10^{-134}. \tag{6}$$

It is thus obvious that the heavy flavor quark pair $c\bar{c}$ and $b\bar{b}$ can be completely ignored, and the newly produced quark pairs are all light quark pairs $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$, with ratios $1:1:\lambda$. These conclusions obtained by "tunneling effect" have been verified and it is a good evidence that the calculated P_c and P_b are consistent with the measured ones when assuming the quarks $c(\bar{c})$ and $b(\bar{b})$ in charmed and bottom particles to be primary quarks.

In order to calculate the average number $\langle N\rangle$ of the light quark pairs produced by a primary quark pair $q_i\bar{q}_i$ through the vacuum excitation, we introduce the effective energy Q which is used for $q_i\bar{q}_i$ to produce new quark pairs from the vacuum. The experiments indicate that for the an event in which the primary quark pair is the light quark pair, one cannot judge which quark in the final-state hadrons is the primary quark by measuring the hadrons' energies. It is believed that the primary light quark pair $q_i\bar{q}_i$ gives almost all its energy $\sqrt{s}$ to produce new quark pairs. Thus we can write the effective energy $Q_l \approx \sqrt{s}$ for the light quark jets. However, for the heavy quark jets, many experiments indicate that two leading hadrons containing heavy quarks carry most of the energy $\sqrt{s}$ of the primary quark pair $c\bar{c}$ or $b\bar{b}$. This means that the effective energy Q_c or Q_b is only a small part of $\sqrt{s}$. The measured average fractional momenta of the charmed and bottom mesons are independent of $\sqrt{s}$, and their values are $\langle x\rangle_c = 0.58$ and $\langle x\rangle_b = 0.75$. According to the assumption that heavy quarks almost do not vary their momenta when they get the light quarks from the vacuum and combine with them to from heavy particles, Peterson *et al.* have deduced the momentum distribution function of heavy particle by the quantum mechanics method [7]:

$$\frac{d\sigma}{dx} \sim \frac{1}{x\left(1-\frac{1}{x}-\frac{\varepsilon}{1-x}\right)^2}, \tag{7}$$

where ϵ is the parameter related to the mass of heavy quark. A lot of experiments show the Eq. (7) is consistent with the data if a suitable value of parameter ϵ is taken. Based on Peterson's formula (7), the fact that $\langle x\rangle_c$ and $\langle x\rangle_b$ are independent of $\sqrt{s}$ is naturally explained. The definition of the fractional energy Z and momentum x of a particle are

$$Z \equiv E/E_{max} = 2E/\sqrt{s}, \quad x \equiv p/p_{max} = P\Big/\left[\left(\frac{\sqrt{s}}{z}\right)^2 - M^2\right]^{1/2}. \tag{8}$$

where E, P and M are the energy, momentum and mass of the particle, respectively; E_{max} and P_{max} are

Table 1
The weights of the directly produced mesons and baryons.

h_i	J_i	α_i	$(2J_i+1)\lambda^{\alpha_i}$	$C(h_i)$	h_i	J_i	β_i	α_i	$\beta_i\lambda^{\alpha_i}$	$C(h_i)$
$\pi^+\pi^-\pi^0$	0	0	1	0.048	$n\ p$	1/2	1	0	1	0.203
$K^0\bar{K}^0K^+K^-$	0	1	0.3	0.014	Λ	1/2	1	1	0.3	0.061
η	0	4/3	0.2	0.097	$\Sigma^-\Sigma^0\Sigma^+$	1/2	1	1	0.3	0.061
η'	0	2/3	0.45	0.022	$\Xi^-\Xi^0$	1/2	1	2	0.09	0.018
$\rho^+\rho^-\rho^0$	1	0	3	0.145	$\Delta^-\Delta^0\Delta^+\Delta^{++}$	3/2	0.36	0	0.36	0.069
$K^{*0}\bar{K}^{*0}K^{*+}K^{*-}$	1	1	0.9	0.043	$\Sigma^{*-}\Sigma^{*0}\Sigma^{*+}$	3/2	0.36	1	0.108	0.021
ω	1	0	3	0.145	$\Xi^{*-}\Xi^{*0}$	3/2	0.36	2	0.032	0.006
ϕ	1	2	0.27	0.013	Ω^-	3/2	0.36	3	0..010	0.002

maximum of E and P, respectively. From Eq. (8) and $E^2 = P^2 + M^2$, we get the relation between Z and x:

$$Z = [x^2 + 4M^2(1 - x^2)/s]^{1/2}. \tag{9}$$

Let the average values of Z and x be $\langle Z\rangle$ and $\langle x\rangle$ and assume that the relation between $\langle Z\rangle$ and $\langle x\rangle$ and that between Z and x are the same. Then we can write

$$\langle Z\rangle = [\langle x\rangle^2 + 4M^2(1 - \langle x\rangle^2)/s]^{1/2}. \tag{10}$$

Thus the effective energy Q can be written as

$$Q = \sqrt{s}\,(1 - \langle Z\rangle) = \sqrt{s}\,\{1 - [\langle x\rangle^2 + 4M^2(1 - \langle x\rangle^2)/s]^{1/2}\}. \tag{11}$$

For the c-jet events, taking $M = 2$ GeV and $\langle x\rangle = 0.58$, we get

$$Q_c = \sqrt{s}\,[1 - (0.336 + 10.624/s)^{1/2}]. \tag{12}$$

For the b-jet events, taking $M = 5.27$ GeV and $\langle x\rangle = 0.75$, we get

$$Q_b = \sqrt{s}\,[1 - (0.563 + 48.658/s)^{1/2}]. \tag{13}$$

In [1], assuming the interaction among the light quarks produced by the effective energy Q to be unsaturated and using the energy conservation formula

$$Q = 2\langle N\rangle\overline{m} + \langle N\rangle(2\langle N\rangle - 1)\overline{V}, \tag{14}$$

here $\langle N\rangle$ and m are the average number and mass of the light quarks respectively, the authors have deduced the formula of $\langle N\rangle$ as following

$$\langle N\rangle = (\alpha^2 + \beta Q)^{1/2} - \alpha, \tag{15}$$

where

$$\beta = 1/(2\bar{V}), \quad \alpha = \beta\bar{m} - 1/4, \tag{16}$$

with $\bar{m} = (0.34 \times 0.45\lambda)/(2 + \lambda) = 0.36$ GeV. In [1], $\bar{V}$ is select to be 0.11 GeV. Substituting $\bar{m} =$ 0.36 GeV and $\bar{V} = 0.11$ GeV into Eq. (16) gives $\beta = 4.7\ \mathrm{GeV}^{-1}$ and $\alpha = 1.44$. From Eqs. (15), (12), (13) and $Q_1 = \sqrt{s}$, we obtain the average numbers of the light quarks in the three following kinds of jets:

$$\langle N \rangle_1 = (1.44 + 4.7\sqrt{s})^{1/2} - 1.44, \tag{17}$$

for the light quark jets;

$$\langle N \rangle_c = \{1.44 + 4.7[1 - (0.336 + 10.624/s)^{1/2}]\}^{1/2} - 1.44, \tag{18}$$

for c-jets and

$$\langle N_b \rangle = \{1.44 + 4.7[1 - (0.563 + 48.658/s)^{1/2}]\}^{1/2} - 1.44, \tag{19}$$

for b-jets.

It should be noticed that in $<N>_c$ or $<N>_b$ the light quarks in two leading particles are not taken into account.

4. THE CALCULATION FORMULAE OF THE HADRON YIELDS

Assume N light quark pairs combining into M mesons, B baryons and $\bar{B}$ antibaryons. Because of the baryon number conservation in the strong interaction process, the number of antibaryons is equal to that of the baryons, i.e., $B = \bar{B}$. The total numbers of quarks before and after their combination into hadrons are the same. It follows that

$$2N = 2M + 6B. \tag{20}$$

Using the nearest rapidity correction in the combination of the quarks into hadrons and the simulation method of the computer, [8] gives the following equations:

$$\begin{array}{lll} B(N) = \bar{B}(N) = 0.114N - 0.243, & M(N) = 0.658N + 0.685, & N \geqslant 3 \\ B(N) = \bar{B}(N) = 0, & M(N) = N, & N < 3 \end{array} \tag{21}$$

Since M and B calculated by Eq. (21) do not strictly satisfy Eq. (20), we substitute 0.729 for 0.685 in Eq. (21). It is regarded the N follows a Poisson distribution, i.e.,

$$P(N) = \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!}, \tag{22}$$

where $<N>$ is the average value of N. For a given $<N>$, the average values of B, $\bar{B}$ and M can be obtained by the following formulas:

Table 2
$F(h_i)$ and $G(h_i)$ of various hadrons.

hi	π^+	π^-	π^0	K^+	K^-	K^0_S	K^0_L	η	$'\eta$	ρ^+	ρ^-
$F(h_i)$	0.595	0.595	0.615	0.064	0.064	0.062	0.062	0.024	0.022	0.145	0.145
hi	ρ^0	K^{*+}	K^{*-}	K^{*0}	$\bar{K}^{*0}$	ω	ϕ				
$F(h_i)$	0.151	0.043	0.043	0.043	0.043	0.145	0.013				
hi	n	p	Λ	Σ^+	Σ^0	Σ^-	Ξ^-	Ξ^0	Δ^{++}	Δ^+	Δ^0
$G(h_i)$	0.498	0.502	0.221	0.060	0.060	0.060	0.023	0.023	0.069	0.069	0.069
hi	Δ^-	Σ^{*+}	Σ^{*0}	Σ^{*0}	Ξ^{*0}	Ξ^{*-}	Ω	π^+	π^-	π^0	K^-
$G(h_i)$	0.069	0.021	0.021	0.021	0.006	0.006	0.002	0.146	0.342	0.250	0.001

$$\langle B\rangle = \langle \bar{B}\rangle = \sum_{N=3} P(N)B(N) = \sum_{N=3} \frac{(0.114N - 0.243)\langle N\rangle^N e^{-\langle N\rangle}}{N!},$$

$$\langle M\rangle = \sum_{N} P(N)M(N) = \sum_{N=1}^{2} \frac{\langle N\rangle^N e^{-\langle N\rangle}}{(N-1)!} + \sum_{N=3} \frac{(0.658N + 0.729)\langle N\rangle^N e^{-\langle N\rangle}}{N!}. \quad (23)$$

For $<N> \; > 5$, Eq. (23) can be approximately expressed as

$$\langle B\rangle = \langle \bar{B}\rangle = 0.114\langle N\rangle - 0.243,$$
$$\langle M\rangle = 0.658\langle N\rangle + 0.729. \quad (24)$$

Using Eqs. (17)-(23) we can get the average numbers of the mesons, baryons and antibaryons in the light quark jets, i.e., $<M>_l$, $<B>_l$ and $<\bar{B}>_l(<\bar{B}>_l = <B>_l)$, and that in the heavy quark jets, i.e., $<M>_c$, $<M>_b$, $<B>_c = <\bar{B}>_c$ and $<B>_b = <\bar{B}>_b$, in where the leading particles containing heavy quark c($\bar{c}$) or b($\bar{b}$) are not taken into account.

According to the $SU(6)$ symmetry with strange suppression $\lambda(\lambda = 0.3)$, in $<M>_u(u = 1,c,b)$ mesons the relative weight $C(M_i)$ of the directly produced M_i meson with α_i strange quarks and spin J_i is proportional to $(2J_i + 1)\lambda^{\alpha_i}$. In Table 1 are listed the relative weights $C(M_i)$ for various directly produced mesons. For the directly produced baryons, we ought to consider the spin suppression in addition to the strange suppression. In $<B>_k(k = 1,c,b)$ baryons the relative weight $C(B_i)$ of the baryons B_i with α_i strange quarks and spin J_i is proportional to $\beta_i\lambda^{\alpha_i}$, here $\beta_i = 1$ (for $(1/2)^+$ baryon) or 0.36 (for $(3/2)^+$ baryon). The relative weights $C(B_i)$ of various directly produced baryons are also listed in Table 1.

The measured hadron yields contain the contributions from the decays of other short-lived hadrons. In order to compare the calculated hadron yields with the measured ones, we must take the contributions from the decays of other short-lived hadrons into account. The yields of all mesons (0^- and 1^- mesons), except π mesons, are the contributions only from the decays of mesons (for K mesons there is a small contribution from the decay of Ω baryon, which is negligible. These contributions are from the directly produced mesons, not from the successive decays. Let $F(M_i)$ be the average number of mesons M_i produced by one of the $<M>_u$ $(u = 1,s,b)$ directly produced mesons, which include the directly produced meson M_i and the indirectly produced mesons M_i from decays of other short-lived mesons. We can write

$$F(M_i) = C(M_i) + \sum_j C(M_j)B(M_j \to M_i), \quad M_i \neq \pi^+, \pi^-, \pi^0, \tag{25}$$

where $B(M_j \to M_i)$ is the number of mesons M_i produced by decays of meson M_j. For example, the decay $\eta' \to \eta$ have two ways: $\eta' \to \eta\pi^+\pi^-$ and $\eta' \to \eta\pi^0\pi^0$, and their branch ratios are 0.441 and 0.205, respectively, so we have $B(\eta' \to \eta) = 0.441 + 0.205 = 0.646$. But the decay $\eta' \to \eta\pi^0$ has only one way: $\eta' \to \eta\pi^0\pi^0$, so we have $B(\eta \to \pi^0) = 2 \times 0.025 = 0.410$. In the yields of π meson, there are the contributions not only from meson decays, but also from baryons decays. In the meson decay mode, there are contribution to pions not only from decays of the directly produced meson, but also from successive decays.

The average number of mesons $\pi^{0,\pm}$ produced by a directly produced meson can be written as

$$F(\pi^{0,\pm}) = C(\pi^{0,\pm}) + \sum_{M_k \neq \pi^{0,\pm}} F(M_k)B(M_k \to \pi^{0,\pm}). \tag{26}$$

In the baryon decays which give π mesons there are both direct decays and the indirect decays. Let $G(\pi^{0,\pm})$ be the average number of $\pi^{0,\pm}$ given by a directly produced baryon and $D(\pi^{0,\pm})$ be the average number of $\pi^{0,\pm}$ given by a directly produced antibaryon. Obviously we have $D(\pi^\pm) = G(\pi^+)$, $D(\pi^0) = G(\pi^0)$. To calculate $G(\pi^{0,\pm})$, we should first calculate the average number of baryons given by a directly produced baryon.

For the yields of baryon $B_i \neq \Lambda$, p, n, there are only the contributions from the direct decays of other directly produced baryons. So we obtain

$$G(B_i) = C(B_i) + \sum_{B_j \neq B_i} C(B_j)B(B_j \to B_i), \quad B_i \neq \Lambda, \mathrm{p}, \mathrm{n}. \tag{27}$$

For the yields of baryons Λ, p, n and meson π, there are the contributions from the direct and indirect decays of directly produced baryons. So we get

$$G(\Lambda) = C(\Lambda) + \sum_{B_k \neq \Lambda, \mathrm{p}, \mathrm{n}} G(B_k)B(B_k \to \Lambda). \tag{28}$$

$$G(\mathrm{p}) = C(\mathrm{p}) + \sum_{B_k \neq \mathrm{p}, \mathrm{n}} G(B_k)B(B_k \to \mathrm{p}),$$

$$G(\mathrm{n}) = C(\mathrm{n}) + \sum_{B_k \neq \mathrm{p}, \mathrm{n}} G(B_k)B(B_k \to \mathrm{n}),$$

$$G(\pi^{0,\pm}) = \sum_k G(B_k)B(B_k \to \pi^{0,\pm}). \tag{29}$$

The average number $D(\bar{B}_i)$ of antibaryon $\bar{B}_i$ produced by a directly produced antibaryon is equal to $G(B_i)$, i.e., $D(B_i) = G(B_i)$. Table 2 gives $F(M_i)$, $G(B_i)$ and $G(\pi^{0,\pm})$ of various mesons and baryons.

For the hadron yields in the heavy quark jets, there are the contributions from the decays of the leading particles containing a primary heavy quark or antiquark. Let $K_c(h_i)$ and $K_b(h_i)$ be the number of hadrons h_i produced by the two charmed particles and two bottom particles, respectively. For the light quark jets, $K_l(h_i) = 0$. Various hadron yields in three kinds of jets are given as follows:

$$\langle M_i \rangle_\mu = \langle M \rangle_\mu F(M_i) + K_\mu(M_i), \quad M_i \neq \pi^+, \pi^-, \pi^0,$$

$$\langle\pi^{0,\pm}\rangle_\mu = \langle M\rangle_\mu F(\pi^{0,\pm}) + 2G(\pi^{0,\pm})\langle B\rangle_\mu + K_\mu(\pi^{0,\pm}),$$
$$\langle B_i\rangle_\mu = \langle \bar{B}_i\rangle_\mu = \langle B_i\rangle_\mu G(B_i) + K_\mu(B_i),$$
$$\mu = 1,c,b. \tag{30}$$

The yields of hadron $h_i(h_i = M_i,B_i)$ in e^+e^- annihilation can be written as

$$\langle h_i\rangle = \sum_{\mu=1,c,b} \langle h_i\rangle_\mu P_\mu, \tag{31}$$

where P_c and P_b are the probabilities for c-jets and b-jets to emerge in $e^+e^- \to h$ and $P_l = P_u + P_d + P_s$ is the probability for the light quark jets to emerge in $e^+e^- \to h$.

Since the properties about the decay of heavy particles are not known enough up to now, $K_c(h_i)$ and $K_b(h_i)$ are unknown, which creates difficulties in the calculation of $<h_i>$.

5. YIELDS OF LONG-LIVED CHARGED PARTICLES

In [9] the measured values for three kinds of charged particles in e^+e^- annihilation at $\sqrt{s} = 29$ GeV are

$$\langle\pi^+ + \pi^-\rangle_c = 10.8 \pm 0.9,\ \langle K^+ + K^-\rangle_c = 2.0 \pm 0.5, \langle p + \bar{p}\rangle_c = 0.6 \pm 0.5,$$
$$\langle\pi^+ + \pi^-\rangle_b = 13.8 \pm 1.0,\ \langle K^+ + K^-\rangle_b = 1.7 \pm 0.6, \langle p + \bar{p}\rangle_b = 0.7 \pm 0.6.$$

Substituting these data into Eq. (30) one has

$$K_c(\pi^+ + \pi^-) = 4.7 \pm 0.9, K_c(K^+ + K^-) = 1.4 \pm 0.5, K_c(p + \bar{p}) = 0.1 \pm 0.5,$$
$$K_b(\pi^+ + \pi^-) = 9.5 \pm 1.0, K_b(K^+ + K^-) = 1.3 \pm 0.6, K_b(p + \bar{p}) = 0.4 \pm 0.6.$$

From these results we can determine the values of these parameters except $K_c(p + \bar{p})$ and $K_b(p + \bar{p})$ (the errors of $K_c(p + \bar{p})$ and $K_b(p + \bar{p})$ are too large to determine the values of them): $K_c(\pi^+ + \pi^-) = 4.7$, $K_c(K^+ + K^-) = 1.4$, $K_b(\pi^+ + \pi^-) = 9.5$, and $K_b(K^+ + K^-) = 1.3$.

To determine the values of $K_c(p + \bar{p})$ and $K_b(p + \bar{p})$ we use another method. In c-jets the leading quark $c(\bar{c})$ is able to combine into a charmed meson $M_c(\bar{M}_c)$ or a baryon $B_c(\bar{B}_c)$ with a quark $\bar{q}_u(q_u)$ or with two quarks $q_u q_\nu(\bar{q}_u\bar{q}_\nu)$ from the $N = <N>_c + 1$ light quark pairs. In the second paper of [2], we calculated the ratio of the probability for quark $c(\bar{c})$ to combine into a charmed meson to that into a charmed baryon with N light quark pairs, and the result is

$$f_c = (N-1)/(3N-1) = \langle N\rangle_c/(3\langle N\rangle_c + 2), \tag{32}$$

From Eq. (32), we find that the probability for a leading quark $c(\bar{c})$ to become a charmed baryon is $f_c/(1 + f_c) = <N>_c/(4<N>_c + 2)$. Because of the baryon number conservation in baryon decays, after $c(\bar{c})$ become a charmed baryon, the charmed baryon will eventually become a proton $p(\bar{p})$ or a neutron $n(\bar{n})$ through its successive decays, whatever its decay ways may be. It is demonstrated that the probabilities for a charmed baryon eventually to become a $p(\bar{p})$ and an $n(\bar{n})$ are approximately the same. So the contributions from $B_c(\bar{B}_c)$ to the yield $<p>(<\bar{p}>)$ are $0.5<N>_c/(4<N>_c + 2)$ and that from B_c and $\bar{B}_c$ to the yields $<p + \bar{p}>$ are

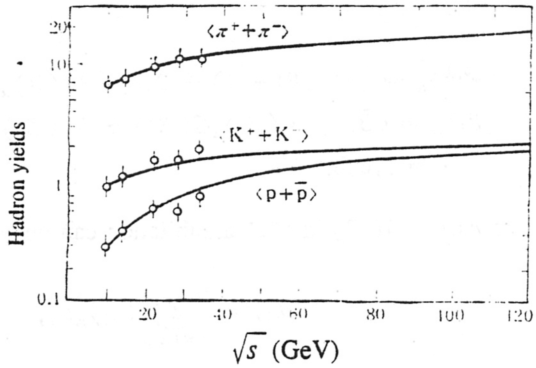

Fig. 2
The hadrons yields of the long-lived charged particles, $<\pi^+ + \pi^->$, $<k^+ + k^->$ and $<p + \bar{p}>$. The data for $\sqrt{s} = 10$, 29 GeV are taken from [10], the other from [11].

$$K_c(p+\bar{p}) = \langle N\rangle_c/(4\langle N\rangle_c + 2). \tag{33}$$

Making a similar analysis for the leading quark $b(\bar{b})$ gives

$$K_b(p+\bar{p}) = \langle N\rangle_b/(4\langle N\rangle_b + 2). \tag{34}$$

Since $<N>_c$ and $<N>_b$ depend on the energy $\sqrt{s}$, $K_c(p + \bar{p})$ and $K_b(p + \bar{p})$ also depend on $\sqrt{s}$. But this dependence is not strong. For example, for $\sqrt{s} = 20$, 50, 90 GeV, $K_c(p + \bar{p}) = 0.220$, 0.236, 0.240, $K_b(p + \bar{p}) = 0.210$, 0.232, 0.237, and for $\sqrt{s} \to \infty$, $K_c(p + \bar{p}) = K_b(p + \bar{p}) = 0.25$. $K_c(p + \bar{p})$ and $K_b(p + \bar{p})$ can be approximately regarded as constant (~ 0.23). Having known K_c and K_b, we can calculate the yields for three kinds of long-lived charged particles with the following formulas

$$\begin{aligned}\langle\pi^+ + \pi^-\rangle &= \sum_{\mu=l,c,b} \{[F(\pi^+) + F(\pi^-)]\langle M\rangle_\mu + 2[G(\pi^+) \\ &\quad + G(\pi^-)]\langle B\rangle_\mu + K_\mu(\pi^+ + \pi^-)\}P_\mu, \\ \langle K^+ + K^-\rangle &= \sum_{\mu=l,c,b} \{[F(K^+) + F(K^-)]\langle M\rangle_\mu + K_\mu(K^+ + K^-)\}P_\mu, \\ \langle p+\bar{p}\rangle &= \sum_{\mu=l,c,b} [2G(p)\langle B\rangle_\mu + K_\mu(p+\bar{p})]P_\mu,\end{aligned} \tag{35}$$

where $K_l(p + p) = 0$. The average number of charged particles in e^+e^-annihilation is

$$\langle n_{ch}\rangle = \langle\pi^+ + \pi^-\rangle + \langle K^+ + K^-\rangle + \langle p+\bar{p}\rangle. \tag{36}$$

6. DISCUSSION AND CONCLUSIONS

According to Eq. (35), we have calculated $<\pi^++\pi^->$, $<k^++k^->$ and $<p + \bar{p}>$, and compared them with the corresponding measured values in Fig. 2, where it is obvious that the

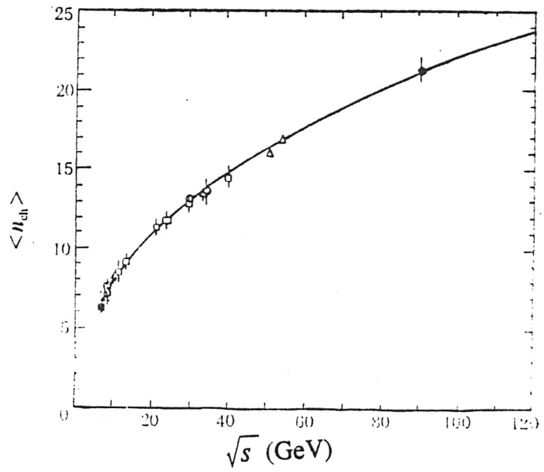

Fig. 3
The average charged multiplicities $<n_{ch}>$. The solid line represents the calculated results. The data are taken from [12]. • : ALEPH; Δ : TOPAZ; ○ : TASSO; O : JADE; ■ : LENA; ▲ : CLEQ.

calculated values are basically consistent with the available data. Figure 3 shows that the calculated curve of $<n_{ch}>$ is well consistent with the data in $\sqrt{s}$ = 10-90 GeV.

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