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The Structures of Color String for $e^+e^- \to q\bar{q}g$ and $Y \to 3g$

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In the LUND model, the explanations of $e^+e^- \to q\bar{q}g \to$ 3jets and $Y \to 3g \to$ h's are based on the application of the LUND string fragmentation model to the color string structures of $q\bar{q}g$ and 3g systems. In this paper, starting from the color wave functions of the $q\bar{q}g$ and 3g systems, these color string structure are directly studied by using QCD. It is shown that the reasonableness and accuracy of the LUND string picture are revealed.

1. INTRODUCTION

How do quarks and gluons fragment into various hadrons? Are quark and gluon fragmentation two independent questions or different aspects of the same question? These are still fundamental problems in present particle physics and can only be understood gradually by comparing different models with experiments. The $e^+e^- \to q\bar{q} \to$ 2jet process is mostly suitable for the study of quark fragmentation, while the $e^+e^- \to q\bar{q}g$ process is a proper one for the studies of the difference and connection between the quark and gluon fragmentations. Because the Y strong decay can only proceed via the 3g intermediate state, it has many advantages in the study of gluon fragmentation over the high energy multi-jet events, such as:

A) there is no need to distinguish gluon jet from quark jet and to measure the angle between jets as in the $e^+e^- \to q\bar{q}g$ events;

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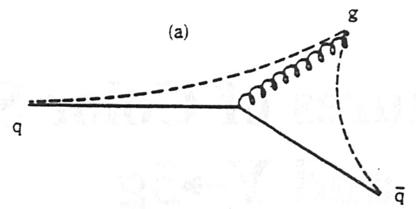

Fig. 1(a)

The color attractive string stretched between g and q (or $\bar{q}$) in the $q\bar{q}g$ system. The LUND model assumes that both of them are fragmented independently as a $e^+e^- \to q\bar{q}$ color singlet string. This is just a close approximation with the accuracy of 90%, in fact.

B) the energy of each gluon in $\Upsilon \to 3g$ is so low that the contribution from parton cascade processes can be neglected. Hence, the hadrons in the final state carry more directly the gluon hadronization information. The $\Upsilon \to 3g \to h$'s process is assumed to be an idea process to study the gluon fragmentation.

In QCD, the fundamental difference between the quark and gluon is that the quark is a color triplet state, whereas the gluon is a color octet state. This variance is differently treated in independent fragmentation [1] (IF) and string fragmentation [2] (SF) models. In the IF model, the fragmentation of partons is assumed to be independent of each other. The average hadron multiplicity from the gluon fragmentation, $\langle n\rangle_g$, is then much larger than that from the quark jet, $\langle n\rangle_q$, at the same energy. At the non-asymptotic energy, their ratio can be given by [3]

$$\langle n\rangle_g/\langle n\rangle_q = (9/4)[1 - 0.27\sqrt{\alpha_s} - 0.07\alpha_s], \qquad (1)$$

where α_s is the ordinary running coupling constant at that energy. In this case, both gluon and quark produce jets with symmetrical particle density. Then, in the SF model, much more attention is paid to the essential connection between quarks and gluon. Since $3 \otimes \bar{3} = 8 \oplus 1$, a color octet gluon can be regarded as a bicolor system, of which the color interaction is equivalent to that of the $q\bar{q}$ color octet system. In string language, the gluon in 3-jet events is introduced naturally as a kink carrying energy and momentum on the string stretched between a q and a $\bar{q}$ end. Thus the gluon has two pieces of string attached to it, whereas the q or $\bar{q}$ only has one. In particular, there is no string linked directly

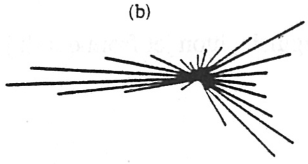

Fig. 1(b)

In fragmentation along the two strings shown in Fig. 1(a), "string effect" appears in the final particle density distribution.

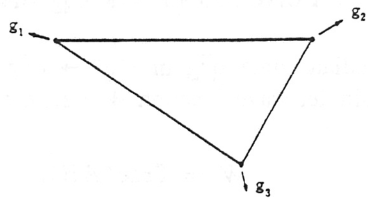

Fig. 2

In the $\Upsilon \rightarrow ggg$ decay, the LUND model assumes that three gluons always form a closed triangle string connected by three attractive strings. Our QCD demonstration shows that it is a very close approximation with the accuracy of 97%.

between q and $\bar{q}$ (Fig.1a). When the string pieces exist between q (or $\bar{q}$) and g fragments, the transverse motion of the string tends to boost the particles produced away from the central region, i.e., to deplete the region between q and $\bar{q}$ particles. This so-called "string effect" has been shown by Azimov et al.[4] within the framework of perturbative QCD. In their perturbative calculations, the asymmetry of the particle populations between jets in 3-jet events arises from interference among soft gluons radiated from the q, $\bar{q}$ and g. This "string effect" (Fig.1b), contradicting the prediction of Eq.(1), has been verified in the studies at PEP, PETRA and LEP energies [5]. In 1985, the LUND group extended the above hypothesis to the $\Upsilon \rightarrow 3g$ decay. They assumed that three gluons are connected by three attractive strings which form a closed triangle (Fig.2). That picture qualitatively explained the fact that the baryon yields in the $\mp$ decay is significantly higher than that in the adjacent $e^+e^- \rightarrow q\bar{q}$ continuum region [6].

From the above discussion, it shows that the hypothesis that the gluon is a bicolor particle and each pair of color charges stretches a string is reasonable. However, it showed recently that no matter what kind of the adjustment for free parameters was done, the LUND model could not quantitatively reproduce the precisely measured ARGUS results on various particle yields and other features of the Υ decay [7]. This shows that the $\Upsilon \rightarrow 3g \rightarrow h$'s process cannot perfectly be described by the LUND model.

In the LUND model, the treatments for $e^+e^- \rightarrow q\bar{q}g \rightarrow$ 3jets and $\Upsilon \rightarrow 3g \rightarrow h$'s include two parts: The first one is the assumption of the color string structures for the $q\bar{q}g$ and 3g systems, and the second one is the application of the LUND string fragmentation model to the above-mentioned string structures. Can the $q\bar{q}g$ system always form two independent (i.e., color singlet) attractive strings? Can the ggg system always form three attractive strings, and why are they not independent? These questions should be answered by the LUND model. In this paper, we present an approach to these questions. Our analysis method can be conveniently extended to the other multiparton systems, such as $q\bar{q}gg$, $q\bar{q}Q\bar{Q}$ etc. In Sec. 2, we explicitly write out the color singlet wave functions of the $q\bar{q}g$ and ggg systems. According to the perturbative QCD approximation, in Sec. 3, we study the configurations between all color charges to determine which configuration has the attractive feature and can form a string. Then we study the color configurations and their color string structures of the $q\bar{q}g$ and ggg systems from their wave functions in Secs. 4 and 5. Finally, in Sec.6, we give a brief summary.

2. COLOR SINGLET WAVE FUNCTIONS FOR $q\bar{q}g$ AND ggg SYSTEMS

Obviously, the intermediate state $q\bar{q}g$ in $e^+e^- \to q\bar{q}g \to$ 3jets can only be the color singlet obtained from $8 \otimes 8$. The singlet wave function W, i.e., the scalar product of A and B, is

$$W = \mathrm{Trac}(AB). \tag{2}$$

It can be expressed by the color R_q, G_q and B_q of q, the anticolor $\bar{R}_{\bar{q}}$, $\bar{G}_{\bar{q}}$ of and $\bar{B}_{\bar{q}}$ of $\bar{q}$ and the bicolor $(B\bar{R})_g$ of g:

$$\begin{aligned} W = [&R_q\bar{B}_{\bar{q}}(B\bar{R})_g + R_q\bar{G}_{\bar{q}}(G\bar{R})_g + B_q\bar{R}_{\bar{q}}(R\bar{B})_g + B_q\bar{G}_{\bar{q}}(G\bar{B})_g + G_q\bar{R}_{\bar{q}}(R\bar{G})_g \\ &+ G_q\bar{B}_{\bar{q}}(B\bar{G})_g] + \frac{2}{3}[R_q\bar{R}_{\bar{q}}(R\bar{R})_g + B_q\bar{B}_{\bar{q}}(B\bar{B})_g + G_q\bar{G}_{\bar{q}}(G\bar{G})_g] - \frac{1}{3}[R_q\bar{R}_{\bar{q}}(B\bar{B})_g \\ &+ R_q\bar{R}_{\bar{q}}(G\bar{G})_g + B_q\bar{B}_{\bar{q}}(R\bar{R})_g + B_q\bar{B}_{\bar{q}}(G\bar{G})_g + G_q\bar{G}_{\bar{q}}(R\bar{R})_g + G_q\bar{G}_{\bar{q}}(B\bar{B})_g]. \end{aligned} \tag{3}$$

It is well known that the strong decay of the quarkonium state with $J^{PC} = 1^{--}$ such as Υ, J/ψ etc., can only proceed via the 3g intermediate state, e.g., Υ → 3g → h's. Apparently, the 3g intermediate state can only be color singlet. We denote three gluons by A, B and C, respectively. A, B and C are traceless irreducible tensors of 8 dimensions

$$A^i_j, B^i_j, C^i_j \quad \text{and} \quad A^i_i = B^i_i = C^i_i = 0. \tag{4}$$

There are two independent ways to form a color singlet state of the ABC system:

$$W_+ = \mathrm{Trac}(ABC - BAC). \tag{5}$$

and

$$W_- = \mathrm{Trac}(ABC + BAC). \tag{6}$$

The intermediate state 3g must have the negative charge conjugation parity. Under the $\hat{C}$ transformation, each gluon wave function (e.g., A) transforms like

$$\hat{C}A = -A^T. \tag{7}$$

Thus, W_+ has C_+ parity while $_-$ has the $_-$ parity, i.e., $CW_- = -W_-$ and $\hat{C}W_+ = +W_+$. So the wave function of 3g intermediate state can only be W_-.

Similarly, we denote each gluon in the color octet state constructed by bicolor as

$$\left.\begin{aligned} &g_1 = R\bar{G}, g_2 = R\bar{B}, g_3 = G\bar{B}, g_4 = G\bar{R}, g_5 = B\bar{R}, g_6 = B\bar{G}, \\ &g_7 = \frac{1}{\sqrt{2}}(R\bar{R} - G\bar{G}), g_8 = \frac{1}{\sqrt{6}}(R\bar{R} + G\bar{G} - 2B\bar{B}). \end{aligned}\right\} \tag{8}$$

The matrices of A, B and C are given by

$$\begin{pmatrix} \frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}} & g_1 & g_2 \\ g_4 & -\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}} & g_3 \\ g_5 & g_6 & -\frac{2g_8}{\sqrt{6}} \end{pmatrix} \tag{9}$$

Substituting Eq.(9) into Eq.(6), one can get the color wave function of the 3g intermediate state as

$$\begin{aligned} W_- = & \left(\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_A\left[\left(\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_B\left(\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_C + g_{1B}g_{4C} + g_{2B}g_{5C}\right] \\ & + g_{1A}\left[g_{4B}\left(\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_C + \left(-\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_B g_{4C} + g_{3B}g_{5C}\right] \\ & + g_{2A}\left[g_{5B}\left(\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_C + \left(-\frac{2g_8}{\sqrt{6}}\right)_B g_{5C} + g_{6B}g_{4C}\right] \\ & + g_{4A}\left[\left(\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right) g_{1C} + g_{1B}\left(-\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_C + g_{2B}g_{6C}\right] \\ & + \left(-\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_A\left[g_{4B}g_{1C} + \left(-\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_B\left(-\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_C + g_{3B}g_{6C}\right] \\ & + g_{3A}\left[g_{5B}g_{1C} + \left(-\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_C g_{6B} + g_{6C}\left(-\frac{2g_8}{\sqrt{6}}\right)_B\right] \\ & + g_{5A}\left[\left(\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_B g_{2C} + g_{1B}g_{3C} + g_{2B}\left(-\frac{2g_8}{\sqrt{6}}\right)_C\right] \\ & + g_{6A}\left[g_{4B}g_{2C} + \left(-\frac{g_7}{\sqrt{2}} + \frac{g_8}{\sqrt{6}}\right)_B g_{3C} + g_{3B}\left(-\frac{2g_8}{\sqrt{6}}\right)_C\right] \\ & + \left(-\frac{2g_8}{\sqrt{6}}\right)_A\left[g_{5B}g_{2C} + g_{6B}g_{3C} + \left(-\frac{2g_8}{\sqrt{6}}\right)_B\left(-\frac{2g_8}{\sqrt{6}}\right)_C\right] + (A \leftrightarrow B) \end{aligned} \tag{10}$$

In the same way, W_- can be expressed in terms of the bicolors of A, B and C by substituting Eq.(8) into Eq.(10). Due to its lengthy expression, it is omitted here.

3. ATTRACTIVE STRINGS BETWEEN VARIOUS COLOR CHARGES

As mentioned in Sec.1, the extension of the LUND SF model to the cases containing gluons, i.e., $q\bar{q}g$ or ggg, is based on the following understanding of QCD: the color interaction of the gluon bicolor is equivalent to that of the color octet of $q\bar{q}$; each color charge of gluon is similar to the single color charge of q or $\bar{q}$ and may interact independently with the color charges of other partons. When the interaction is attractive a string between color charges is established.

Table 1

The values of C^2 and $\sum_{K=1}^{8} F_C^K(1)F_C^K(2)$ in the different state of color configurations between two partons and the attractive (or repulsive) features of these states.

The state of color configurations	1	3, $\bar{3}$	6, $\bar{6}$	8
The value of C^2	0	4/3	1/3	3
$\sum_{K=1}^{8} F_C^K(1)F_C^K(2)$	−4/3	−2/3	1/3	1/6
Repulsive (R) or Attractive (A)	A	A	R	R

There are six kinds of color charges R, B, G, $\bar{R}$, $\bar{B}$ and $\bar{G}$ and $6 + (6 \times 5)/2 = 21$ combinations, such as R-R, R-G, R-$\bar{G}$, $\bar{R}$-$\bar{G}$, etc. Though we are not able to calculate their interaction strength quantitatively, the study of hadron spectra indicates that even for these typical non-perturbative QCD phenomena, the qualitative judgement about whether the force between color charges given by the approximation of perturbative QCD

$$V \propto \sum_{K=1}^{8} F_C^K(1)F_C^K(2) = \frac{1}{2}(C^2 - \frac{8}{3}) \tag{11}$$

is attractive or repulsive is correct [8]. In Eq.(11), $F_C^K = \frac{1}{2}\lambda^K$, λ^K, where λ^K is the Gell-Mann matrix of $SU_c(3)$; C is the eigenvalue of the Casimir operator for the $SU_c(3)$ eigenstate of a pair of color charges.

Table 1 shows the values of C^2 and $\sum_{K=1}^{8} F_C^K(1)\, F_C^K(2)$ in different states. Whether the force between two color charges of different partons is attractive or repulsive (i.e., whether string can be stretched or not) is determined by the configuration of two color charges in the wave function of the whole system (e.g., Eq.(3) or (10)).

1) For the nine color-anticolor combinations: R-$\bar{R}$, B-$\bar{B}$, G-$\bar{G}$ and R-$\bar{G}$, R-$\bar{B}$, B-$\bar{G}$, B-$\bar{R}$, G-$\bar{R}$, G-$\bar{B}$ as $3 \otimes 3 = 8 \oplus 1$, there is an attractive string in the color singlet combination

$$\frac{1}{\sqrt{3}}(R\bar{R} + B\bar{B} + G\bar{G}) \tag{12}$$

but not in the color octet combinations

$$R\bar{G}, R\bar{B}, B\bar{G}, B\bar{R}, G\bar{R}, G\bar{B}. \frac{1}{\sqrt{2}}(R\bar{R} - B\bar{B}), \frac{1}{\sqrt{6}}(R\bar{R} + B\bar{B} - 2G\bar{G}) \tag{13}$$

2) For twelve color-color and anticolor-anticolor combinations: *R-B*, *B-G*, *G-R*, $\bar{R}$-$\bar{B}$, $\bar{B}$-$\bar{G}$, $\bar{G}$-$\bar{R}$ and *R-R*, *B-B*, *G-G*, $\bar{R}$-$\bar{R}$, $\bar{B}$-$\bar{B}$, $\bar{G}$-$\bar{G}$, there is an attractive string in asymmetry states 3 or 3

$$\left.\begin{array}{l}\frac{1}{\sqrt{2}}(RB\text{-}BR),\frac{1}{\sqrt{2}}(BG\text{-}GB),\frac{1}{\sqrt{2}}(GR\text{-}RG) \\ \frac{1}{\sqrt{2}}(\overline{RB}-\overline{BR}),\frac{1}{\sqrt{2}}(\overline{BG}-\overline{GB}),\frac{1}{\sqrt{2}}(\overline{GR}-\overline{RG})\end{array}\quad\begin{array}{c}\bar{3}\\3\end{array}\right\} \tag{14}$$

respectively, but not in symmetry states 6 and $\bar{6}$

$$\left.\begin{array}{l}RR,BB,GG,\frac{1}{\sqrt{2}}(RB+BR),\frac{1}{\sqrt{2}}(RG+GB),\frac{1}{\sqrt{2}}(GB+BG) \\ \overline{RR},\overline{BB},\overline{GG},\frac{1}{\sqrt{2}}(\overline{RB}+\overline{BR}),\frac{1}{\sqrt{2}}(\overline{RG}+\overline{GB}),\frac{1}{\sqrt{2}}(\overline{GB}+\overline{BG})\end{array}\quad\begin{array}{c}6\\\bar{6}\end{array}\right\} \tag{15}$$

From above discussion, we conclude that the combinations of the color and its non-complementary anticolor (e.g., $\bar{R}$-$\bar{B}$) or two identical colors (or anticolors) (e.g., *R-R*, $\bar{B}$-$\bar{B}$) can never form strings because they can only form 8, 6 and $\bar{6}$ states; while for the rest combinations, whether they can stretch a string or not depends on their states in the colour wave function of the whole system.

4. THE COLOR STRING STRUCTURE FOR $e^+e^- \to q\bar{q}g$ SYSTEM

In the above sections, we see that the color configurations for the $q\bar{q}g$ and 3g systems are treated in the similar ways in our approach. However, the color singlet wave function of $q\bar{q}g$ is much simpler than that of the 3g system. Unlike $\Upsilon \to 3g$, the center-of-mass energies of the qg and qg systems in the $e^+e^- \to q\bar{q}g$ process can be so large that the α_S is significantly less than 1 (e.g., in the present LEP energy region). This makes the analysis of the force string based on the perturbative QCD in Sec.3 more reliable. Moreover, according to the recent study of the LUND group on the parton-hadron duality[9], the multiplicity and momentum distributions of final state hadrons are mainly determined by the color string structure of $q\bar{q}g$ before hadronization, so the color string structure of $q\bar{q}g$ may even directly be tested by using three jet events. For the color singlet of the $q\bar{q}g$ system, the interaction between g and q (or $\bar{q}$) may come from that between the color charge of q (or anticolor change of $\bar{q}$) and anticolor (or color) charge of g. According to this combination, we rewrite Eq.(3) as

$$\begin{aligned} W = {} & [(R_q\bar{R}_g + B_q\bar{B}_g + G_q\bar{G}_g)(\bar{R}_{\bar{q}}R_g + \bar{B}_{\bar{q}}B_g + \bar{G}_{\bar{q}}G_q)] - \frac{1}{3}[(R_q\bar{G}_g)(\bar{R}_{\bar{q}}G_g) \\ & + (R_q\bar{B}_g)(\bar{R}_{\bar{q}}B_g) + (B_q\bar{R}_g)(\bar{B}_{\bar{q}}R_g) + (B_q\bar{G}_g)(\bar{B}_{\bar{q}}G_g) + (G_q\bar{R}_g)(\bar{G}_{\bar{q}}R_g) \\ & + (G_q\bar{B}_g)(\bar{G}_{\bar{q}}B_g) + (R_q\bar{R}_g)(\bar{R}_{\bar{q}} - R_g) + (B_g\bar{B}_g)(\bar{B}_{\bar{q}}B_g) + (G_q\bar{G}_g)(\bar{G}_{\bar{q}}G_g)] \end{aligned} \tag{16}$$

Obviously, in the first term of *W*

$$W_1 = [(R_q\bar{R}_g + B_q\bar{B}_g + G_q\bar{G}_g)(\bar{R}_{\bar{q}}R_g + \bar{B}_{\bar{q}}B_g + \bar{G}_{\bar{q}}G_g)] \tag{17}$$

both q-g and q-g form color singlet strings.

On the other hand, the interaction between g and q (or $\bar{q}$) may also come from that between the color charge of q (or anticolor charge of $\bar{q}$) and the color (or anticolor) charge of g, i.e., $3 \otimes 3$ (or $\bar{3} \otimes \bar{3}$). According to this combination, we rewrite Eq.(3) as

$$\begin{aligned} W =& \frac{2}{3}\{(R_q R_g)(\bar{R}_{\bar{q}}\bar{R}_g) + (B_q B_g)(\bar{B}_{\bar{q}}\bar{B}_g) + (G_q G_g)(\bar{G}_{\bar{q}}\bar{G}_g) \\ & + [(R_q B_g)(\bar{B}_{\bar{q}}\bar{R}_g) + (B_q R_g)(\bar{R}_{\bar{q}}\bar{B}_g)] \\ & + [(R_q G_g)(\bar{G}_{\bar{q}}\bar{R}_g) + (G_q R_g)(\bar{R}_{\bar{q}}\bar{G}_g)] \\ & + [(B_g G_g)(\bar{G}_{\bar{q}}\bar{B}_g) + (G_q B_g)(\bar{B}_{\bar{q}}\bar{G}_g)]\} \\ & + \frac{1}{3}\{(R_q B_g - B_q R_g)(\bar{B}_{\bar{q}}\bar{R}_g - \bar{R}_{\bar{q}}\bar{B}_g) + (R_q G_g - G_q R_g) \\ & (\bar{G}_{\bar{q}}\bar{R}_g - \bar{R}_{\bar{q}}\bar{G}_g) + (G_q B_g - B_q G_g)(\bar{B}_{\bar{q}}\bar{G}_g - \bar{G}_q\bar{B}_g)\} \end{aligned} \tag{16a}$$

In the second term

$$\begin{aligned} W_2 =& \{(R_q B_g - B_q R_g)(\bar{B}_q\bar{R}_g - \bar{R}_q\bar{B}_g) + (R_q G_g - G_q R_g)(\bar{G}_q\bar{R}_g - \bar{R}_{\bar{q}}\bar{G}_g) \cdot \\ & + (G_q B_g - B_q G_g)(\bar{B}_{\bar{q}}\bar{G}_g - \bar{G}_{\bar{q}}\bar{B}_g)\} \end{aligned} \tag{18}$$

both g-q and g-$\bar{q}$ are in asymmetry states (3 or $\bar{3}$), and can form strings. We denote this normalized string configuration by $[3, \bar{3}]$.

Therefore, the above study shows that the color singlet wave function of the $q\bar{q}g$ system cannot completely form the configuration [1,1] or $[3, \bar{3}]$. After tedious recombination and rearrangement, we find that

$$W = \frac{2}{3}W_1 + \frac{1}{3}W_2. \tag{16b}$$

Simple calculation shows that the probability of the configuration $[3, \bar{3}]$ is only 10% in the qqg singlet.

Comparing the color string structure of the $q\bar{q}g$ system obtained directly from QCD with that in the LUND model, we conclude that:

a) in the LUND model, the assumption that two strings can always be stretched between q and g and $\bar{q}$ and g, respectively, is reasonable.

b) the assumption that both color strings come from the interaction between color and anticolor and are independent color singlet strings (as the same as the string in the $e^+e^- \to q\bar{q}$ system) is only an approximation where the string configuration [1,1], whose probability is 90%, is considered.

5. COLOR STRING STRUCTURE FOR Y → 3g SYSTEM

Similar to that in Sec.4, starting from Eq.(10), the color wave function of the 3g intermediate state in Y → 3g → h's, we study its color string structure. Since Eq.(10) is more complicated than Eq.(3), here we only give out the final results.

Substituting Eq.(8) into Eq.(10), then recombining and classifying terms, we get

$$W_{-} = W_1 + W_2 + W_3. \tag{19}$$

where W_1, W_2 and W_3 are defined as following:

$$\begin{aligned} W_1 = \frac{2}{9}[&(R_A\overline{R}_B + B_A\overline{B}_B + G_A\overline{G}_B)(R_B\overline{R}_C \\ &+ B_B\overline{B}_C + G_B\overline{G}_C)(R_C\overline{R}_A + B_C\overline{B}_A + G_C\overline{G}_A) \\ &+ \text{(the terms with each color changed into its complementary color)} \end{aligned} \tag{20}$$

where the subscripts A, B and C denote three gluons (e.g., R_A represents the red color charge carried by gluon A, etc.), respectively. Obviously, the color and anticolor of the three pairs of gluons A-B, B-C and A-C in W_1 can form color singlet strings. We symbolize this kind of color string configuration by [1,1,1].

$$W_2 = D + (A \leftrightarrow B \text{ in } D) + (B \leftrightarrow C \text{ in D}) \tag{21}$$

where

$$\begin{aligned} D = &(11/81)(R_A\overline{R}_B + B_A\overline{B}_B + G_A\overline{G}_B)[(R_BB_C - B_BR_C)(\overline{R}_C\overline{B}_A - \overline{B}_C\overline{R}_A) \\ &+ (B_BG_C - G_BB_C)(\overline{B}_C\overline{G}_A - \overline{G}_C\overline{B}_A) + (G_BR_C - R_BG_C)(\overline{G}_C\overline{R}_A - \overline{R}_C\overline{G}_A)] \\ &+ \text{(the terms with each color changed into its complementary color).} \end{aligned} \tag{22}$$

It is easy to see that there are three strings between gluons in W_2. One of them is always in color singlet, the other two are in 3 and $\overline{3}$, respectively. We represent this string configuration by $[1,3,\overline{3}]$.

$$W_3 = E + (A \leftrightarrow C \text{ in } E) + (B \leftrightarrow C \text{ in } E) \tag{23}$$

where

$$\begin{aligned} E = &(2/27)\{(R_A\overline{B}_B)(B_BG_C - G_BB_C)(\overline{R}_A\overline{G}_C - \overline{G}_A\overline{R}_C) \\ &+ (R_A\overline{G}_B)(G_BB_C - B_BG_C)(\overline{R}_A\overline{B}_C - \overline{B}_A\overline{R}_C) \\ &+ (G_A\overline{R}_B)(R_BB_C - B_BR_C)(\overline{G}_A\overline{B}_C - \overline{B}_A\overline{G}_C) \\ &+ (G_A\overline{B}_B)(B_BR_C - R_BB_C)(\overline{G}_A\overline{R}_C - \overline{R}_A\overline{G}_C) \\ &+ (B_A\overline{R}_B)(R_BG_C - G_BR_C)(\overline{B}_A\overline{G}_C - \overline{G}_A\overline{B}_C) \\ &+ (B_A\overline{G}_B)(G_BR_C - R_BG_C)(\overline{B}_A\overline{R}_C - \overline{R}_A\overline{B}_C) \\ &+ (1/6)(R_A\overline{R}_B + B_A\overline{B}_B - 2G_A\overline{G}_B)[(G_BR_C - R_BG_C)(\overline{R}_A\overline{G}_C - \overline{G}_A\overline{R}_C) \\ &+ (B_BG_C - G_BR_C)(\overline{G}_A\overline{B}_C - \overline{B}_A\overline{G}_C) - 2(B_BR_C - R_BB_C)(\overline{R}_A\overline{B}_C - \overline{B}_A\overline{R}_C)] \\ &+ (1/2)(R_A\overline{R}_B - B_A\overline{B}_B)[(B_BG_C - G_BR_C)(\overline{B}_A\overline{B}_C - \overline{B}_A\overline{G}_C) \\ &- (G_BR_C - R_BG_C)(\overline{R}_A\overline{G}_C - \overline{G}_A\overline{R}_C)]\} \\ &+ \text{(the terms with each color changed into its complementary color).} \end{aligned} \tag{24}$$

Comparing W_2 with W_3, we notice that there is a fundamental difference between these two terms. Although in both W_2 and W_3, two of the three pairs of gluons A-B, B-C, A-C can form 3 and 3 strings, respectively, the remained pair in W_2 is in color singlet and can form a string, whereas the left pair in W_3 is in color octet and cannot form a string. That is, three gluons A, B and C can only be connected with 3 and 3 color strings, but cannot form a closed triangular string assumed in the LUND model. We denote this structure by $[3,\bar{3},8]$. This case is not considered in the LUND model.

Using Eqs.(19)-(24) W_- can be expressed by $[1,1,1]$, $[1,3,\bar{3}]$ and $[3,\bar{3},8]$. The probabilities of these three kinds of string structures can be calculated and the results turn to be 65%, 32% and 3%, respectively. However, in the LUND model, the 3% $[3,\bar{3},8]$ is neglected. Because the probability of $[1,3,\bar{3}]$ is about 32%, the color field between two gluons may be in 3, $\bar{3}$ or 1. Thus, if the whole system is connected by three strings, it is surely in color singlet. This feature has been reflected by the LUND model.

6. CONCLUSIONS

In the LUND model, the explanations for $e^+e^- \to$ 3jets and the Y strong decay are based on the color string structures of the $q\bar{q}g$ and 3g systems shown in Figs.1 and 2 (see Sec.I for details). In this paper, starting directly from the color wave functions of the $q\bar{q}g$ and 3g systems, we study their color string structures in QCD. For the $e^+e^- \to q\bar{q}g$ process, 10% configurations form 3 and 3 strings of the $q\bar{q}g$ system, which cannot fragment independently, the rest 90% form two color singlet strings. In this case, the LUND model is just an approximation where 10% $[3,\bar{3}]$ configurations are neglected. For the $Y \to 3g$ process, 3% configurations of the 3g system form two 3 and $\bar{3}$ color strings and the remaining 97% form three color strings where each of them is a mixture of the 1, 3 and $\bar{3}$ color states and only the closed triangular string formed by three strings is color singlet. In this case, the LUND model is equivalent to the case where the 3% $[3,\bar{3},8]$ configurations are neglected.

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