High Energy Physics and Nuclear Physics

Volume 17, Number 3

Strong Υ Decays Via Three Gluons

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Although the Lund model can be used to qualitatively explain enhanced baryon rates in direct Υ decays, a quantitative explanation has not been given. With no additional parameters, we analyze Υ three-gluon decays in the framework of the "quark production rule" and the "quark combination rule", which were successfully used in explaining $e^+e^- \to q\bar{q} \to 2$ jets events, and obtain a good quantitative explanation.

1. INTRODUCTION

The study of gluon fragmentation mechanism is an important subject of the present particle physics. For this study, the Υ decay has several advantages over high energy continuum:

a) in the lowest order of quantum chromodynamics (QCD), the strong Υ decay proceeds only through a three-gluon intermediate state, so that there is no need to distinguish gluon jet from quark jet as $e^+e^- \to q\bar{q}g$ events;

b) either in e^+e^- or h-h reactions the cross section of the Υ production is much larger than that of gluon jet in the continuum;

c) in the strong Υ decay data, all the possible angular and energy distributions of three-gluon intermediate state are available, the uncertainties in the measurement of angles between jets and energies of gluons are excluded in this case;

d) Although the accuracy for the separation of gluon jet from quark jet and the determination of jet energies can be improved for high energy jets, the influences of parton cascade process become significant. In $\Upsilon \to 3g \to h's$ process, the energy of each gluon is so low that the final hadronic state can reflect gluon hadronization mechanism more directly.

Supported by the National Natural Science Foundation of China. Received on September 7, 1992.

It was found experimentally that the production rates of baryons (except Λ (1520)) on Υ are 2-3 times larger than the adjacent continuum data, while the production rates of mesons are only slightly enhanced [1,2]. Of all the phenomenological models, only the Lund model can be used to qualitatively explain this fact [3,4]. The Lund model suggests that in the color interaction, a color octet gluon is equivalent to a bicolor system, and a color charge of one gluon forms an attractive string with an anticolor charge of another gluon. In this way, the three-gluon state in the Υ decay is treated as a triangular closed string (as shown in Fig. 1) which is stretched and breaks into pieces the same way as that of a $q\bar{q}$ state. The large total multiplicity in three-gluon fragmentation is due to the "longer" triangular closed string compared with the straight q-$\bar{q}$ string in the adjacent continuum. For baryons, there is an even larger difference because in the closed string, there is no end region with lower baryon production probability. Consequently, the Lund model can explain the enhanced baryon rates in the Υ region qualitatively. However, further investigation indicated that the Lund model cannot explain quantitatively the recent ARGUS data on the particle yields and other features of Υ decays [5].

The Lund model succeeds in the description of $e^+e^- \rightarrow q\bar{q}$ events but fails in the quantitative explanation of the hadron yields in the Υ decay. There are two possibilities for this failure:

1) the color string picture of three-gluon intermediate state is not completely correct;

2) although the color string picture is essentially correct, the Lund model does not give a very good description for hadronization, especially for the baryon production.

In [6], we analyzed the color string structure of three-gluon intermediate state in the Υ decay from the color wave function in QCD, and came to the conclusion that the picture adopted by Lund model is approximately correct. In this paper we adopt the "quark production rule and combination rule" [7] which was built to describe hadronization of the $e^+e^- \rightarrow q\bar{q}$ event to replace Lund model in the calculation of the particle yields in Υ decays. In Sec. 2, the color string structure of three-gluon state in the Υ decay and the Dalitz plot are presented. In Sec. 3, we briefly introduce the "quark production rule and combination rule" and their application to the Υ decay. In Sec. 4, we simply describe the calculation methods of particle yields and list the results. Finally, in Sec. 5, an overall summary and some discussions are given.

2. COLOR STRING STRUCTURE AND DALITZ PLOT FOR THREE-GLUON DECAYS OF

In [6], we analyzed the color string structure of three-gluon middle state in the Υ decay in QCD from its color wave function and came to the conclusion that 97% of the configurations can from three color strings stretching three gluons and the remaining 3% can only form two color strings. In our calculation, the latter has a very small effect on the results, so the approximation that the three-gluon middle state can stretch three color strings is used as in Lund model. This allows us to study the impacts of altering the string fragmentation (hadronization) models on the results.

We denote the energies and momenta of three gluons in the rest frame of Υ by $E_i (i=1,2,3)$ and $p_i (i=1,2,3)$. In [4], the color string between two gluons i, j came from a field between two color charges which belong to these gluons respectively, i.e., a gluon was treated as a bicolor system, then energy E_{ij} and momentum p_{ij} of the string were give by

$$E_{ij} = \frac{1}{2}(E_i + E_j) = \frac{1}{2}(|p_i| + |p_j|) \tag{1}$$

$$p_{ij} = \frac{1}{2}(p_i + p_j) \tag{2}$$

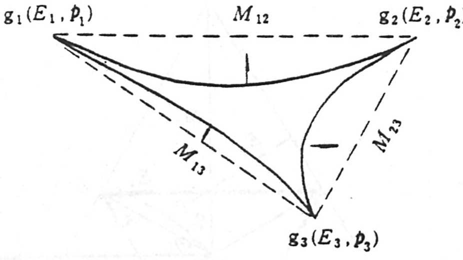

Fig. 1

The color string structure of $\Upsilon \to 3g$ in the Lund model: The three-gluon state is treated as a triangular string with the gluon in vertexes moving apart and trenching the string like force field. There are color strings among the three gluons, all of which are transversely excited. M_{ij} is the center-of-mass energy of the string.

In Eq. (1) massless gluon were assumed. Eq. (2) indicates that, in the rest frame of Υ, these three color strings are not at rest, but "transversely exited". The center-of-mass energy of each string, i.e., the invariant mass M_{ij} is the effective energy Q which produces quark pairs through QCD vacuum excitation (as shown in Fig. 1). Apparently, one has

$$M_{ij} = \sqrt{E_{ij}^2 - |p_{ij}|^2} = \sqrt{\frac{p_i p_j}{2}(1 - \cos\theta_k)} \tag{3}$$

where i, j and k take the commutation of 1, 2, 3. θ_k is the angle between the motion directions of g_i and g_j, i.e., the angle between p_i and p_j.

Four-momentum conservation gives out

$$\left.\begin{aligned} M_\Upsilon &= E_1 + E_2 + E_3 \\ O &= p_1 + p_2 + p_3 \end{aligned}\right\} \tag{4}$$

where $M\Upsilon$ is the rest mass of Υ. The energy of each gluon can be determined for a certain angular distribution of the three-gluon state.

The three-gluon decay of Υ can be described by the Dalitz plot. Because gluon is massless, the kinematically allowed region of the $\Upsilon \to 3g$ decay Dalitz plot is a triangle as shown in Fig. 2 [8]. Each point in the triangle (except the three vertexes) corresponds to a three-gluon state with definite gluon energies or angles. The probability of the Υ decay is given by

$$d\omega_{ba} \propto |m_{ba}|^2 ds \tag{5}$$

where m_{ba} is the decay amplitude and ds is the area element in triangle. Apparently, $|m_{ba}|^2$ is directly proportional to the density of event points in the Dalitz plot and determines the relative weight of three-gluon state with a certain angular distribution.

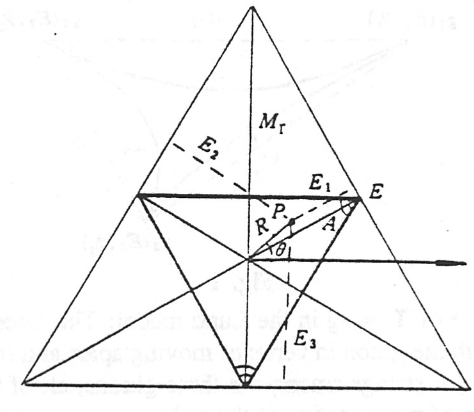

Fig. 2

Dalitz plot for the three-gluon decay of Υ. Physical area is ΔABC (except three vertexes A.B.C). Every point P can be expressed by $P(R, \theta)$. The distance from each point inside the triangle to the three sides of the triangle correspond to the energies of three gluons, E_1, E_2 and E_3, respectively. The cutoff taken during our calculation are three identical small triangles on three vertexes, where BE = 0.4 GeV.

A. Ore et al. calculated the transition probability of the orthopositronium three-photons process in QED [9]. Brodsky and Fritzsch verified that this calculation is also suitable for the $\Upsilon \mp 3g$ process [10], namely,

$$|m_{ba}|^2 \propto \left\{ \frac{[(M_\Upsilon/2) - E_1]^2}{E_2^2 E_3^2} + \frac{[(M_\Upsilon/2) - E_2]^2}{E_1^2 E_3^2} + \frac{[(M_\Upsilon/2) - E_3]^2}{E_1^2 E_2^2} \right\} \tag{6}$$

We can therefore use Eqs. (1)-(3) to calculate the invariant mass M_{ij} of three color strings in those models. In particular, when the energies of initial partons are not high enough (e.g., in the Υ decay where the energy E_i are lower than ½$M\Upsilon$ = 4.73 GeV), is there any parton shower evolution? How can Q_0 be determined? Many indistinct questions arise. The "quark production rule" avoids the questions mentioned above and determines the number of quark pairs before hadronization from energy conservation.

3. "QUARK PRODUCTION RULE" AND "QUARK COMBINATION RULE"

As mentioned in Sec. 1, the way to deal with the Υ decay in the Lund model is to apply Lund's q-$\bar{q}$ fragmentation model in the $e^+e^- \to q\bar{q} \to 2$ jet process to the colour string fragmentation in $\Upsilon \to 3g$. Thus, the latter fragmentation is entirely determined by the former without additional free parameters. In [6], we also proposed a q-$\bar{q}$ fragmentation model and unifiedly explained the experimental phenomena and basic rules in $e^+e^- \to qq\bar{q} \to 2$ jets. In comparison with the other phenomenological models including the Lund model, the striking advantage of this new model is that there are a few free parameters and that the baryon and meson productions can be unifiedly given. In the light q-$\bar{q}$ fragmentation of $\Upsilon \to 3g$, there is only one adjustable parameter, which can be determined by the $e^+e^- \to 2$ jets experiment. Let us briefly introduce the "quark production rule" and the "quark combination rule" which are used in this paper.

1) Quark Production Rule

According to the "quark production rule", the average number $\langle N_{ij}\rangle$ of quark pairs which are produced through vacuum excitation of a color string can be determined from the energy conservation as:

$$Q(=M_{ij}) = 2\langle N_{ij}\rangle m + \langle N_{ij}\rangle(2\langle N_{ij}\rangle - 1)V \tag{7}$$

where m is the average mass of these constituent quarks. Taking accepted u, d, s quark masses and the production ratio P_u:P_d:P_s = 1:1:0.3, we obtain m = 0.36 GeV. V is the average potential among $2\langle N_{ij}\rangle$ quarks and antiquarks just before hadronization. From Eq. (7) one has

$$\langle N_{ij}\rangle = (\alpha^2 + \beta M_{ij})^{\frac{1}{2}} - \alpha \tag{8}$$

where

$$\beta = (2V)^{-1} \qquad \alpha = \beta m - \frac{1}{4} \tag{9}$$

Equation (8) is called the "quark production rule". β (or V) is the only free parameter which was determined in [7] from the HRS data of the light quark jets in $e^+e^- \to 2$ jet events, and

$$\beta = 3.6\mathrm{GeV}^{-1}, \tag{10a}$$

Substituting λ and m into Eq. (9), we have

$$\alpha = 1.046 \tag{10b}$$

Using Eqs. (10) and (8) directly, we can calculate the number of quark pairs produced by three color strings in the direct Υ decay respectively. Apparently, the total number of quark pairs $\langle N\rangle$ before hadronization is the sum of the three parts $\langle N_{12}\rangle$, $\langle N_{23}\rangle$, $\langle N_{31}\rangle$,

$$\langle N\rangle = \sum_{ij=12,23,31} [(1.094 + 3.6M_{ij})^{\frac{1}{2}} - 1.046] \tag{11}$$

2) Quark Combination Rule

The transformation of quarks and antiquarks produced before hadronization into observable mesons and baryons is the essential problem that all the fragmentation models must answer. The Lund model can explain meson production directly, but its baryon production mechanism is rather sophisticated. Because the experiments indicated that the striking feature in the Υ decay is baryon enhancement, it is worthwhile to study the Υ decay by using the new hadronization model instead of the Lund string fragmentation.

In [11], we studied the rule by which N quark pairs combine into mesons and baryons stochastically. We found that the near correlation in rapidity is consistent with the fundamental requirement of QCD and uniquely determines the combination rule of quarks and antiquarks into mesons and baryons. According to the combination rule, the combination probability of N quark pairs into M mesons, B baryons, and B antibaryons is given by

$$X_{MB}(N) = \frac{2N(N!)^2(M+2B-1)!}{(2N)!M!(B!)^2} 3^{M-1}\delta_{N,M+3B} \tag{12}$$

The average number of mesons $M(N)$ and that of baryons $B(N)$ produced directly in the combination are given by:

$$\left.\begin{aligned} \overline{M}(N) &= \sum_M \sum_B M X_{MB}(N) \\ \overline{B}(N) &= \sum_M \sum_B B X_{MB}(N) \end{aligned}\right\} \tag{13}$$

Apparently, for $N < 3$, one has $M(N) = N$, $B(N) = 0$.

In experiment, the corresponding data are average value over events. If the distribution function of N is $P(N)$, then

$$\left.\begin{aligned} \langle M \rangle &= \sum_N P(N)\overline{M}(N) \\ \langle B \rangle &= \sum_N P(N)\overline{B}(N) \end{aligned}\right\} \tag{14}$$

If $P(N)$ is Poisson distribution, $\langle M \rangle$ and $\langle B \rangle$ depend only on $\langle N \rangle$ and can be calculated from Eqs. (12)-(14) directly without any adjustable parameters. The authors of [12] calculated $\langle M \rangle$ and $\langle B \rangle$ when $\langle N \rangle \le 10$,

$$\left.\begin{aligned} \langle M \rangle &= a\langle N \rangle + b \\ \langle B \rangle &= (1-a)\langle N \rangle/3 - b/3 \end{aligned}\right\} \tag{15}$$

where $a = 0.6577$, $b = 0.6848$. $\langle M \rangle$ and $\langle B \rangle$ in Eq. (15) correspond to mesons and baryons produced directly from $\langle N \rangle$ quark pairs via hadronization, respectively. In [7], as in other models, we considered $L = 0$, 1 mesons and $J^{PC} = 3^+/2$, $1^+/2$ ($L=0$) baryons only and verified that $L = 1$ mesons account for 25% of all mesons produced directly. In [13] one gave out the baryon spin suppression factor $3^+/2:1^+/2 = 0.36$. For hadrons in the same $SU_f(3)$ multiplet, the production weights satisfy the $SU_f(3)$ symmetry with a strangeness suppression factor $\lambda = 0.3$ for the strange quark production. Furthermore, using the unstable particle branching ratios given by the Particle Data Group [14], we can calculate hadron yields which can be compared with data.

The question is now how to apply the "quark combination rule" (which is suitable for the q-$\bar{q}$ straight string in the $e^+e^- \to 2$ jets event) to the closed triangular color string in the Υ decay. There may be two ways:

1) the number of $q\bar{q}$ pairs $\langle N_{ij} \rangle$ in Eq. (8) is substituted into Eq. (15) to calculate $\langle M \rangle$ and $\langle B \rangle$ for $\langle N_{12} \rangle$, $\langle N_{23} \rangle$, $\langle N_{31} \rangle$ separately, i.e., the q-$\bar{q}$ pairs produced by three strings combine into hadrons independently;

2) by using the total number $\langle N \rangle$ given by Eq. (11), the total $\langle M \rangle$ and $\langle B \rangle$ of the Υ decay can be directly calculated, i.e., all quark pairs produced by three strings combine into hadrons together.

The dynamical foundation of the "quark combination rule" and the essence of the requirement of the near correlation in rapidity revealed in [11] indicated that in a straight string, the smaller the difference in rapidity, the longer, the time they interact, so there is enough time for a $q\bar{q}$ pair to become color singlet then to form a meson and a $q\bar{q}$ pair to become 3 state, then to combine with another q to form a baryon. In the Υ three-gluon decay, although the rapidity cannot be defined, the $\Delta p = |p_i - p_j|$ of two partons (quark or antiquark) in the Υ rest system can still be used to reflect the length of interaction time. Quarks (or antiquarks) whose momenta are closet combine into hadrons at first, whatever they belong to the same $\langle N_{ij}\rangle$ or different $\langle N_{ij}\rangle$, i.e., the number of quark pairs $\langle N\rangle$ produced by three straight strings obtained in Eq. (11) must combine into hadrons together. Substituting $\langle N\rangle$ into Eq. (15), the hadron yields can be calculated. That is to say, quarks are produced according to three straight strings and combined into hadrons according to a closed triangular string.

4. CALCULATION AND RESULT OF FRAGMENTATION

On the basis of principles and methods presented above, we can calculate the average hadron yield $\langle h_i\rangle$ of various hadrons h_i for the fragmentation of three-gluon with a certain angular distribution. As stressed in Sec. 1, the contributions from three-gluon intermediate state with all possible angular distributions are available in the Υ decay data. Thus, $\langle h_i\rangle$ must be averaged over the kinematically allowed region in the Dalitz plot:

$$\langle\langle h_i\rangle\rangle = \int |m|^2\langle h_i\rangle ds \Big/ \int |m|^2 ds \tag{16}$$

The meanings of.ds and $|m|^2$ are the same as those in Sec. 2. The averages of $\langle N\rangle$, $\langle M\rangle$ and $\langle B\rangle$ over all physical points in the Dalitz plot are denoted as $\langle\langle N\rangle\rangle$, $\langle\langle M\rangle\rangle$ and $\langle\langle B\rangle\rangle$, respectively. A physical point P in the Dalitz plot is expressed by $P(R, \theta)$, shown in Fig. 2. The energies of three gluons are given by:

$$\begin{aligned} E_1 &= (M_\Upsilon/3) - R\cos(\theta - \pi/6) \\ E_2 &= (M_\Upsilon/3) + R\cos(\theta + \pi/6) \\ E_3 &= (M_\Upsilon/3) + R\sin\theta \end{aligned} \tag{17}$$

M_{ij} and $\langle N\rangle$ are calculated according to Eqs. (1)-(4), and $\langle N\rangle$ is also the function of R and θ. Averaging over all physical area, one obtains

$$\langle\langle N\rangle\rangle = \int \langle N\rangle |m|^2 ds \Big/ \int |m|^2 ds \tag{18}$$

In order to exclude the non-physical points (three vertices of the triangle in Fig. 2), we must take a cutoff shown in Fig. 2. Three small triangles of which the weight account for 0.027% of the remaining part are excluded from the physical area in the Dalitz plot. Thus, the cutoff does not affect the final result $\langle\langle N\rangle\rangle$, and we have $\langle\langle N\rangle\rangle = 6.27$. This indicates that the linear relations between $\langle N\rangle$ and $\langle M\rangle$, and $\langle N\rangle$ and $\langle B\rangle$ in Eq. (15) can be used to calculate $\langle\langle M\rangle\rangle$ and $\langle\langle B\rangle\rangle$, then one has

$$\begin{aligned} \langle\langle M\rangle\rangle &= a\langle\langle N\rangle\rangle + b \\ \langle\langle B\rangle\rangle &= (1-a)\langle N\rangle/3 - b/3 \end{aligned} \tag{19}$$

The particle yields $\langle\langle h_i\rangle\rangle$ from the Υ decay can then be calculated using the methods described above. In Table 1, our result and ARGUS data are listed. With no more additional parameters, the calculated baryon yields are in agreement with the ARGUS data within experiment errors. The baryon data except P and the meson data are adopted from [2]. π^0, η and ϕ the data predictions agree with the data, but $\pi^\pm$, $K^\pm$ theoretical results are lower than the corresponding data.

The baryon enhancement in Υ direct decays excludes strongly many present models. We believe that the direct methods for testing models quantitatively is to compare Υ decays data with predictions. Because in $e^+e^- \to q\bar{q}$ events, theoretical calculations depend significantly on the knowledge of charm

Table 1
Comparison of the calculated particle yields in direct Υ decays with the ARGUS data [1,2].

Particle	ARGUS Data	Our calculation
$p+\bar{p}$	0.507 ± 0.028	0.48
$\Lambda+\bar{\Lambda}$	$(2.28\pm0.03\pm0.21)\times10^{-1}$	2.08×10^{-1}
$\Xi^-+\bar{\Xi}^-$	$(2.06\pm0.17\pm0.23)\times10^{-2}$	2.23×10^{-2}
$\Sigma^0+\bar{\Sigma}^0$	$(5.64\pm1.69\pm1.13)\times10^{-2}$	5.67×10^{-2}
$\Sigma^-(1385)+\bar{\Sigma}^-(1385)$	$(1.42\pm0.17\pm0.20)\times10^{-2}$	1.78×10^{-3}
$\Sigma^+(1385)+\bar{\Sigma}^+(1385)$	$(1.68\pm0.29\pm0.23)\times10^{-2}$	1.78×10^{-3}
$\Xi^0(1530)+\bar{\Xi}^0(1530)$	$(4.78\pm1.14\pm0.62)\times10^{-3}$	5.87×10^{-3}
$\Omega^-+\bar{\Omega}^-$	$(1.83\pm0.62\pm0.32)\times10^{-3}$	1.76×10^{-3}
$\pi^++\pi^-$	7.55 ± 0.14	6.98
K^++K^-	$0.906\pm0.016\pm0.023$	0.61
π^0	$3.917\pm0.23\pm0.38$	3.73
η	$0.40\pm0.14\pm0.09$	0.28
ϕ	$0.0545\pm0.0022\pm0.0034$	0.046

and beauty baryon productions and decays which are very uncertain, the model predictions for continuum are not accurate. This has been demonstrated by Scheck, who found that in Lund model the r for all baryons decay are very sensitive even to the branching ratio of $\Lambda_c \rightarrow \Lambda + X$ [5] (r is the ratio of particle production rates in direct Υ decays compared with adjacent continuum). Therefore, although we have calculated hadron yields in adjacent continuum which agree with data, we will discuss it no further.

5. CONCLUSIONS

Decays can provide accurate information on the gluon fragmentation. The investigations have been carried out for ten years. Specifically, the ARGUS Collaboration recently provided highly accurate measurements. So far no model can quantitatively explain the fact that the Υ decay rate for baryons is 2-3 times larger than the continuum data.

In this paper, we extended the "quark production rule" and the "quark combination rule" to the three-gluon decay of Υ. It is assumed that quarks are produced according to three straight strings independently, and combined into hadrons according to a closed triangular string. With no more additional parameters, the calculated particle yields are in good agreement with the data. This indicates that the string picture of the three-gluon state used in the Lund model is reasonable, although the Lund model cannot describe baryon production rules very well.

The ARGUS Collaboration also carried out further experimental investigations on baryon production rules on the Υ resonance and the adjacent continuum, such as "strange suppression", "spin suppression," baryon-antibaryon flavor correlations, $\Lambda(1520)$ production, dual baryon (pp, $\Lambda\Lambda$) production rates and baryon fractional momentum spectrum which are not consistent with Lund model predictions. The first four problems have been naturally explained in our stochastic combination model [13,15], so the results in this paper do not appear to be accidental.

ACKNOWLEDGMENT

We are grateful to Liu Ximing, Chen Esheng and Wang Qun for their helpful discussion. One of the authors (Xie) thanks Prof. D. Wegener for the discussion and useful information.

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