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Vertex Correction at Finite Temperature and Decoupling Phase Transition in 2+1-Dimensional Chiral Gross-Neveu Model

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With the aid of imaginary-time temperature field theory and fermion mass spectrum derived from Ward-Takahashi identities with composite fields, the vertex correction among fermions and σ meson is calculated beyond the leading order in $1/N$ expansion in 2+1 dimensional chiral Gross-Neveu model. The behavior of the vertex function with temperature and decoupling phase transition are discussed. The critical temperature of decoupling phase transition arises when the fermion mass and the π meson mass at zero temperature increase; the vertex correction resulting from thermal fluctuation will influence the fermion dynamical mass and cannot be ignored at finite temperature.

1. INTRODUCTION

At high temperature and high density, QCD vacuum undergoes structural changes, and there are confinement transition and chiral phase transition [1]. These properties have been supported by lattice Monte Carlo simulations. Since chiral phase transition and bound state spectra in QCD are still unsolved, it usually utilizes a model approach to discuss the chiral phase transition and the bound state spectra. From the strong-coupling expansion in lattice QCD, we can see that the four-fermion interaction can be regarded as the effective interaction between quarks [2]. From the fact that the four-

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fermion interactions are renormalizable in 2+1 dimensions [3] and chiral four-fermion models poses many properties of low-energy physics, which results from chiral symmetry, based one our previous work [4], we study the vertex correction decoupling phase transition at finite temperature in 2+1-dimensional chiral Gross-Neveu model.

In chiral Gross-Neveu model, when fermions have a current mass, the chiral symmetry will be broken explicitly, which is called explicit breaking in the following. In addition, the attractive interaction between fermions will make fermion pairs condense in the vacuum and then make chiral symmetry dynamically broken, which is called dynamical breaking below. Although in the explicit breaking case chiral symmetry can not be restored, thermal effects will excite fermion pair and make fermion pair condensate to melt gradually, which makes dynamical breaking restore gradually. The thermal effects will also influence the four-fermion coupling (equivalent to a meson-fermion coupling). In the mean field approximation or at the leading order in $1/N$ expansion, the latter effect cannot be included. As the meson-fermion coupling plays an important role in the dynamical breading, we use imaginary-time temperature field theory to discuss the correction of the meson-fermion coupling at finite temperature and decoupling transition.

In Sec. 2, with the aid of the chiral Ward-Takahashi identities at finite temperature and in terms of the properties that the current mass can be taken as the external source of composite field, the corresponding chiral Ward-Takahashi identities and the mass spectra of the fermion and bound states are obtained at finite temperature. According to the Feynman rules in the $1/N$ expansion, the vertex correction between fermions and σ meson is evaluated beyond the leading order in Sec. 3. The critical temperature of decoupling transition is obtained and the influences of σ, π mesons and N to the decoupling temperature are discussed. Conclusions are drawn in Sec. 4.

2. THE CASE AT FINITE TEMPERATURE

The (2+1)-dimensional chiral Gross-Neveu model with the explicit breaking is described by[5]

$$\mathscr{L} = \mathscr{L}_0 - m_0\bar{\psi}\psi, \tag{2.1}$$

where m_0 is a fermion current mass, $\mathscr{L}_0$ is

$$\mathscr{L}_0 = i\bar{\psi}\partial\!\!\!/\psi + \frac{g^2}{2N}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2] \tag{2.2}$$

In order to describe the dynamical breaking, the composite external sources are introduced. At finite temperature, the quantum statistical partition function is

$$\begin{aligned} Z_\beta[\bar{\eta},\eta;K,K_5] &\equiv e^{-W_\beta[\bar{\eta},\eta;K,K_5]} \\ &= \int D\bar{\psi}D\psi \exp(-\int_0^\beta d\tau\int d^2x[\mathscr{L}_0 + \bar{\eta}\psi + \bar{\psi}\eta + (K - m_0)\bar{\psi}\psi + K_5\bar{\psi}i\gamma_5\psi]). \\ &\equiv e^{-W^s_\beta[\bar{\eta},\eta;K-m_0,K_5]} \end{aligned} \tag{2.3}$$

where the massless fermion field ψ^a $(a=1,2,\cdots,N)$ is an antiperiodic function on $R^2 \times [0,\beta]$, and W_β^s is the connected generating functional of the chiral symmetry. From Eq. (2.3), one can see that the explicit breaking term can be taken as an external source. Thus, there is the following relation between the generating functional with explicit breaking and the symmetry generating functional

$$W_\beta[\bar{\eta},\eta;K,K_5] = W^s_\beta[\bar{\eta},\eta;K - m_0,K_5]. \tag{2.4}$$

where a physically meaningless constant term has been omitted.

In terms of the chiral Ward-Takahashi identities for symmetric generating functional at finite temperature [6]

$$\int_0^\beta d\tau \int d^2x [\bar{\eta}(x) \frac{i}{2} \frac{\delta W_\beta^s[J]}{\delta \bar{\eta}(n)} + \frac{\delta W_\beta^s[J]}{\delta \eta(x)} \frac{i}{2} \eta(x)] = 0, \tag{2.5a}$$

$$\int_0^\beta d\tau \int d^2x [\bar{\eta}(x) \frac{i}{2} \gamma_5 \frac{\delta W_\beta^s[J]}{\delta \bar{\eta}(x)} - \frac{\delta W_\beta^s[J]}{\delta \eta(x)} \frac{i}{2} \gamma_5 \eta(x) + \frac{\delta W_\beta^s[J]}{\delta K_5(x)} K(x) - \frac{\delta W_\beta^s[J]}{\delta K(x)} K_5(x)] = 0. \tag{2.5b}$$

it is easy to get

$$\int_0^\beta d\tau \int d^2x [\bar{\eta}(x) \frac{i}{2} \frac{\delta W_\beta[J]}{\delta \bar{\eta}(x)} + \frac{\delta W_\beta[J]}{\delta \eta(x)} \frac{i}{2} \eta(x)] = 0, \tag{2.6a}$$

$$\int_0^\beta d\tau \int d^2x [\bar{\eta}(x) \frac{i}{2} \gamma_5 \frac{\delta W_\beta[J]}{\delta \bar{\eta}(x)} - \frac{\delta W_\beta[J]}{\delta \eta(x)} \frac{i}{2} \gamma_5 \eta(x) + \frac{\delta W_\beta[J]}{\delta K_5(x)} (K(x) - m_0) - \frac{\delta W_\beta[J]}{\delta K(x)} K_5(x)] = 0 \tag{2.6b}$$

which are chiral Ward-Takahashi identities for the generating functional with the explicit breaking.

In order to derive mass spectra, we express Ward-Takahashi identities as the form of the effective action

$$\int_0^\beta d\tau \int d^2x [\bar{\psi}_\beta(x) \frac{\delta \Gamma_\beta}{\delta \bar{\psi}_\beta(x)} + \frac{\delta \Gamma_\beta}{\delta \psi_\beta(x)} \psi_\beta(x)] = 0 \tag{2.7a}$$

$$\int_0^\beta d\tau \int d^2x [\bar{\psi}_\beta(x) \frac{i}{2} \gamma_5 \frac{\delta \Gamma_\beta}{\delta \bar{\psi}_\beta(x)} - \frac{\delta \Gamma_\beta}{\delta \psi_\beta(x)} \frac{i}{2} \gamma_5 \psi_\beta(x) + m_0 (\bar{\psi}_\beta(x) i\gamma_5 \psi_\beta(x) + G_5^\beta(x)) + \frac{\delta \Gamma_\beta}{\delta G_\beta(x)} G_5^\beta(x) - \frac{\delta \Gamma_\beta}{\delta G_5^\beta(s)} G_\beta(x)] = 0. \tag{2.7b}$$

where G_β, G_5 are defined as

$$\frac{\delta W_\beta[J]}{\delta K(x)} = \bar{\psi}_\beta(x) \psi_\beta(x) + G_\beta(x), \tag{2.8a}$$

$$\frac{\delta W_\beta[J]}{\delta K_5(x)} = \bar{\psi}_\beta(x) i\gamma_5 \psi_\beta(x) + G_5^\beta(x). \tag{2.8b}$$

Differentiating Eqs. (2.7) several times with respect to the fields $\bar{\psi}_\beta$, ψ_β, G_β and G_5^β, we can get Ward-Takahashi identities for the proper vertices at finite temperature.

The broken direction is chosen as

With the aid of the Ward-Takahashi identities for the two-point vertices, we can obtain the mass spectra of the fermions and bound states [6]

$$m_f = m_0 + \Gamma^{(3)}_{\psi_\beta, \bar{\psi}_\beta; \sigma_\beta}(p, -p; 0)\,|_{p^2=0} \cdot \langle\sigma\rangle_\beta, \tag{2.9a}$$

$$m_\pi^2 = -\, m_0 \frac{\langle\bar{\psi}\psi\rangle_\beta}{\langle\sigma\rangle_\beta^2}, \tag{2.9b}$$

$$m_\sigma^2 = m_\pi^2 + \Gamma^{(3)}_{\sigma_\beta, \pi_\beta; \sigma_\beta}(p, -p; 0)\,|_{p^2=0} \langle\sigma\rangle_\beta, \tag{2.9c}$$

where $p = (i\omega, p_1, p_2)$, and the σ, π mesons are composed of the composite fields $\bar{\psi}(x)\psi(x)$, $\bar{\psi}(x)i\gamma_5\psi(x)$. The meson fields are defined as

$$\sigma(x) = \frac{\langle\sigma\rangle_0}{\langle\bar{\psi}\psi\rangle_0}\bar{\psi}(x)\psi(x), \tag{2.10a}$$

$$\pi(x) = \frac{\langle\sigma\rangle_0}{\langle\bar{\psi}\psi\rangle_0}\bar{\psi}(x)i\gamma_5\psi(x), \tag{2.10b}$$

where the $\langle A\rangle_0$ denotes the vacuum expectation value at zero temperature.

In the $1/N$ expansion, the four-fermion couplings are renormalizable. In order to obtain physical results, it is necessary to introduce a fermion wavefunction renormalization constant Z_ψ, a coupling renormalization constant Z_{g^2}, a composite field renormalization constant $Z_{\bar{\psi}\psi}$ $(=Z_\sigma)$ and a current mass renormalization constant Z_m.

$$\psi_{ren}(x) = Z_\psi^{1/2}\psi(x), \tag{2.11a}$$

$$(\bar{\psi}(x)\psi(x))_{ren} = Z_{\bar{\psi}\psi}\bar{\psi}(x)\psi(x), \tag{2.11b}$$

$$(\bar{\psi}(x)i\gamma_5\psi(x))_{ren} = Z_{\bar{\psi}\psi}\bar{\psi}(x)i\gamma_5\psi(x), \tag{2.11c}$$

$$g_{ren}^2 = Z_{g^2}g^2, \tag{2.11d}$$

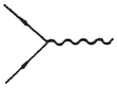
$$m_0^{ren} = Z_{m_0}m_0. \tag{2.11e}$$

In [9], the Feynman rules in the $1/N$ expansion are given below

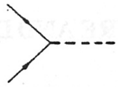
$$G(p) = \frac{i}{p - M_f}\delta_{ij}, \tag{2.12a}$$

$$D_\sigma(p) = \frac{2\pi}{iN}\frac{1}{2(M_f - M) + \dfrac{-p^2 + 4M_f^2}{\sqrt{-p^2}}\tan^{-1}\sqrt{-p^2}/2M_f}, \tag{2.12b}$$

$$D_\pi(p) = \frac{2\pi}{iN}\frac{1}{2(M_f - M) + \sqrt{-p^2}\tan^{-1}\sqrt{-p^2}/2M_f}, \tag{2.12c}$$



$$\gamma_5\,\delta_{ij}, \tag{2.12d}$$



$$- i\delta_{ij}. \qquad (2.12e)$$

where M_f and M satisfy

$$M_f = \frac{1}{2}[M + \sqrt{M^2 + m^2}], \qquad (2.13a)$$

$$\frac{1}{g^2} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{-p^2 + M^2}, \qquad (2.13b)$$

where m is a parameter describing the explicit breaking, which is related to the current mass m_0 by the relation*

$$m^2 = 4\pi m_0 Z_{m_0}/g^2 Z^*{}_{g^2}. \qquad (2.14)$$

We assume that the explicit breaking is very small, $m/M \ll 1$. As the fermion mass and σ meson are the order of M, the corrections resulted from m can be omitted in the mass spectra of the fermion and the σ meson. However, it is quite different for the π meson. In the low momentum limit, Eqs. (2.13a) and (2.12c) indicate that

$$D_\pi(p) \approx \frac{4\pi M}{N} \frac{i}{p^2 - m^2}. \qquad (2.15)$$

which shows that m is the π meson mass. In the low energy limit, $D_\pi^{-1}(p^2) \to \frac{N}{4\pi M} m_\pi^2$. Using the π meson mass Eq. (2.9b), one can get

$$m_\pi^2 = - \Gamma_\pi^{(2)\mathrm{ren}}(0) \frac{4\pi M}{N} = m^2, \qquad (2.16)$$

where $M \approx \langle\sigma\rangle_{\mathrm{ren}}$. From Eq. (2.16), one can see that the π mass spectrum derived from Ward-Takahashi identities is in agreement with that in the $1/N$ expansion.

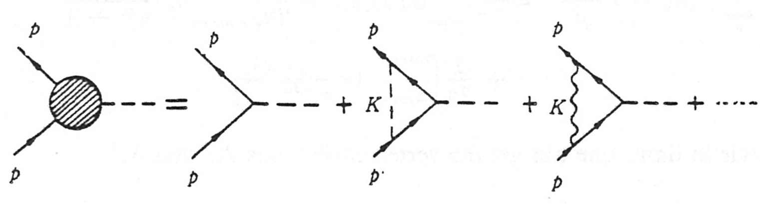


Fig. 1
The exact proper vertex $\Gamma^{(3)}$ beyond the leading order in the $1/N$ expansion.

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*The definitions of the renormalized quantities here are different from those in [7]. The relations are $Z_1 = Z_\psi$, $Z_2 = Z_\psi^{1/2} Z_{\psi\psi}^{1/2}$, $Z_3 = Z_{g^2}^{-1}$, and $Z_4 = Z_{m_0}^{-1}$.

3. VERTEX CORRECTION AT FINITE TEMPERATURE AND DECOUPLING TRANSITION

In terms of the relation between field theory and imaginary-time temperature field theory [8], the Feynman rules at finite temperature in the $1/N$ expansion can be easily obtained. In the $1/N$ expansion, the vertex function $\Gamma^{(3)}$ is shown in Fig. 1.

According to the Feynman rules, the vertex function beyond the leading order is

$$i\Gamma^{(3)}_{\psi_\beta,\bar{\psi}_\beta,\sigma_\beta}(p,-p;0)|_{p^2=0} = -i + i\Lambda_\sigma(p,-p;0)|_{p^2=0} + i\Lambda_\pi(p,-p;0)|_{p^2=0}. \quad (3.1)$$

where

$$i\Lambda_\sigma(p,-p;0)|_{p^2=0} = \frac{i}{\beta}\sum_n\int\frac{d^2k}{(2\pi)^2}(-i)\frac{i}{\gamma_0(\omega-\omega_n)-\gamma\cdot(p-k)-M} \cdot(-i)\frac{i}{\gamma_0(\omega-\omega_n)-\gamma\cdot(p-k)-M} \cdot(-i)\frac{2\pi}{iN}\frac{\sqrt{-\omega_n^2+k^2}}{(-\omega_n^2+k^2+4M^2)\tan^{-1}\sqrt{-\omega_n^2+k^2}/2M}. \quad (3.2)$$

$$i\Lambda_\pi(p,-p;0)|_{p^2=0} = \frac{i}{\beta}\sum_n\int\frac{d^2k}{(2\pi)^2}\gamma_5 \cdot\frac{i}{\gamma_0(\omega-\omega_n)-\gamma\cdot(p-k)-M}(-i) \cdot\frac{i}{\gamma_0(\omega-\omega_n)-\gamma\cdot(p-k)-M}\gamma_5\frac{2\pi}{iN} \cdot\frac{1}{\frac{m^2}{2M}+\sqrt{-\omega_n^2+k^2}\tan^{-1}\sqrt{-\omega_n^2+k^2}/2M}, \quad (3.3)$$

where $\omega_n = i\frac{2n}{\beta}\pi(n=0,\pm1,\cdots),\omega=i\frac{2m+1}{\beta}\pi$.

Using the identity

$$\frac{i}{\beta}\sum_n f(\omega_n=i\frac{2n\pi}{\beta}) = \frac{1}{2\pi}\int_{-i\infty}^{i\infty}dx f(x) + \frac{1}{2\pi}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}dx\frac{f(x)}{e^{\beta x}-1} + \frac{1}{2\pi}\int_{-i\infty-\epsilon}^{i\infty-\epsilon}dx\frac{f(x)}{e^{-\beta x}-1}. \quad (3.4)$$

in the non-relativistic limit, one can get the vertex corrections Λ_σ^β and Λ_π^β

$$i\Lambda_\sigma^\beta(p,-p;0)|_{p^2=0} = i\Lambda_\sigma^0(p,-p;0)|_{p^2=0} - \frac{2\pi}{N}i\int\frac{d^2K}{(2\pi)^2}\left\{\frac{8}{e^{\beta E_m}+1} \cdot\frac{1}{Em}\frac{2\ln3-1}{3^2M\ln^2 3} - \frac{\beta e^{\beta E_m}}{(e^{\beta E_m}+1)^2}\frac{2M}{E_m^2}\frac{1}{3\ln3} - \frac{1}{e^{\beta E_m}+1}\frac{1}{E_m^3}\frac{2M}{3\ln3}\right\} \quad (3.5)$$

$$= i\Lambda_\sigma^0 - i\frac{2}{N}\left[\frac{4}{9\beta M\ln^2 3}(2\ln3-1)\ln(1+e^{-\beta M}) - \frac{1}{3\ln3}\frac{1}{e^{\beta M}+1}\right],$$

$$i\Lambda_\pi^3(p, -p;0)|_{p^2=0} = i\Lambda_\pi^2 + i\frac{2\pi}{N}\int\frac{d^2k}{(2\pi)^2}\Big[\frac{1}{e^{\beta E_m}+1}\frac{1}{E_m}\frac{1}{M}\frac{8}{3\ln^2 3}$$
$$+\frac{1}{e^{\beta E_m}-1}\frac{1}{E_\pi}\frac{2}{M}+\frac{2M}{\ln 3}\Big(\frac{\beta e^{\beta E_m}}{(e^{\beta E_m}+1)^2}\frac{1}{E_m^2}+\frac{1}{e^{\beta E_m}+1}\frac{1}{E_m^3}\Big)\Big] \tag{3.6}$$
$$= i\Lambda_\pi^0 + i\frac{2}{N}\Big[\frac{1}{\beta M}\frac{4}{3\ln^2 3}\ln(1+e^{-\beta m}) - \frac{1}{\beta M}\mathrm{Ln}(1-e^{-\beta m_\pi})$$
$$+\frac{1}{\ln 3}\frac{1}{e^{\beta M}+1}\Big].$$

where $E_m^2 = k^2 + M^2$, $E_\pi^2 = k^2 + m_\pi^2$. Λ_σ^0, Λ_π^0 are the vertex corrections at zero temperature, and we have used the relations

$$\frac{i}{\tan^{-1}i/2} = \frac{2}{\ln 3}, \tag{3.7a}$$

$$\frac{ia}{\tan^{-1}ia/2M}|_{a\to 0} = 2M, \tag{3.7b}$$

$$\frac{i2M}{\tan^{-1}i} = 0. \tag{3.7c}$$

From the statistical properties of fermions and mesons, it is easy to find that the terms with Fermi-Dirac distribution function correspond to the fermionic thermal fluctuation, and the term with Bose-Einstein distribution function corresponds to the mesonic thermal fluctuation. Thus, both fermion and meson thermal fluctuations contribute to vertex corrections. This is different from [9].

Applying Eqs. (4.7) and (4.10), one finds that the effective coupling between fermions and σ meson beyond the leading order is

$$g_{\rm eff}(\beta) = g_{\rm eff}(0) + \frac{2}{N}\Big[-\frac{1}{\beta M}\ln(1-e^{-\beta m_\pi}) + \frac{1}{e^{\beta M}+1}\frac{4}{3\ln 3}$$
$$+\frac{8}{\beta M 9\ln^2 3}(2-\ln 3)\ln(1+e^{-\beta M})\Big]. \tag{3.8}$$

where $g_{\rm eff}(0)$ denotes the effective coupling at zero temperature.

In the $1/N$ expansion, the vertex function $\Gamma^{(3)}_{\psi,\bar\psi;\sigma}$ is

$$i\Gamma^{(3)}_{\psi,\bar\psi;\sigma}(p, -p;0)|_{p^2=0} = -i - i\int\frac{d^3k}{(2\pi)^3}\frac{i}{-\not k+\not p-M}(-i)\frac{i}{-\not k+\not p-M}(-i)$$
$$\cdot\frac{2\pi}{iN}\frac{\sqrt{-k^2}}{(-k^2+4M^2)\tan^{-1}\sqrt{-k^2}/2M}$$
$$+\int\frac{d^3k}{(2\pi)^3}\gamma_5\frac{i}{-\not k+\not p-M}(-i)\frac{i}{-\not k+\not p-M}\gamma_5\frac{2\pi}{iN} \tag{3.9}$$
$$\cdot\frac{1}{\frac{m^2}{2M}+\sqrt{-k^2}\tan^{-1}\sqrt{-k^2}/2M}.$$

In the non-relativistic limit ($p^2/2M^2 \ll 1$), it is easy to get the effective coupling

$$g_{\rm eff}(0) = -1 + \frac{0.18}{N}, \tag{3.10}$$

which shows that at zero temperature, the vertex corrections are very small.

From Eq. (3.8), one can see that the vertex correction at finite temperature results from two contributions: one is the correction at zero temperature, which comes from quantum fluctuation in the vacuum; the other is thermal fluctuation, which is classical thermal effects. As ultraviolet divergences result from quantum fluctuation and usually exist in the vertex correction at zero temperature, they can be removed by the renormalization procedure at zero temperature. Note that if $m_\pi = 0$, the thermal fluctuation of the π meson will cause an infrared divergence, which is difficult to handle. In this paper, a small explicit breaking is introduced to get rid of the infrared divergence at finite temperature.

From Eqs. (3.8) and (3.10), we get

$$g_{\rm eff}(\beta) = -1 + \frac{0.18}{N} + \frac{2}{N}\Big[-\frac{1}{\beta M}\ln(1 - e^{-\beta m_\pi}) + \frac{1}{e^{\beta M}+1}\frac{4}{3\ln 3} + \frac{1}{\beta M}\frac{8(4-\ln 3)}{9\ln^2 3}\ln(1 + e^{-\beta M})\Big]. \tag{3.11}$$

where M is the function of temperature. From Eq. (2.13b), it is determined by [6,10]

$$M - M_0 + \frac{2}{\beta}\ln(1 + e^{-\beta M}) = 0. \tag{3.12}$$

where M_0 denotes the dynamically generated mass at zero temperature, which is related to the fermion pair condensate. Usually, thermal excitations make the fermion pair condensate to melt gradually, so M decreases with temperature. The dependence of the effective coupling $g_{\rm eff}(\beta)$ with temperature is shown in Figs. 2 and 3 at different M_0 and m_π.

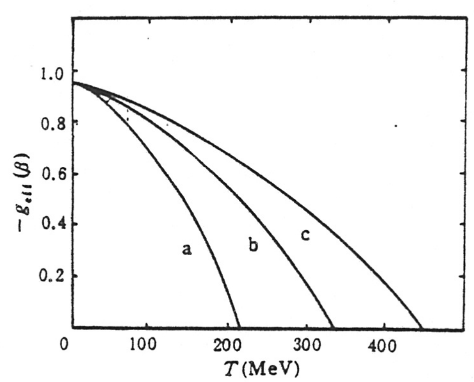

Fig. 2

The behaviors of the effective coupling $g_{\rm eff}(\beta)$ with temperature and fermion mass where the π meson mass m_π and N are fixed, $m_\pi = 10$ MeV, $N=4$, and the curves a, b, c correspond to a) $M_0 = 450$ MeV, b) $M_0 = 750$ MeV, c) $M_0 = 1050$ MeV respectively.

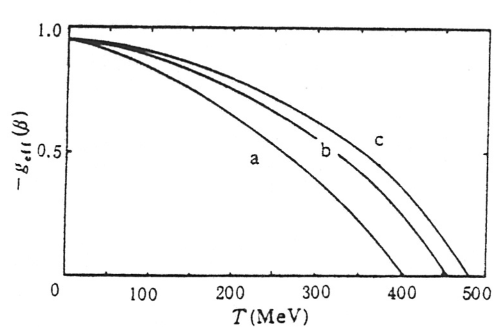

Fig. 3

The behaviors of the effective coupling $g_{\rm eff}(\beta)$ with temperature and π meson mass, where the fermion mass and N are fixed, $M_0 = 1000$ MeV, $N=4$, and the curves a, b, c correspond to a) $m_\pi = 10$ MeV, b) $m_\pi = 30$ MeV, c) $m_\pi = 50$ MeV respectively.

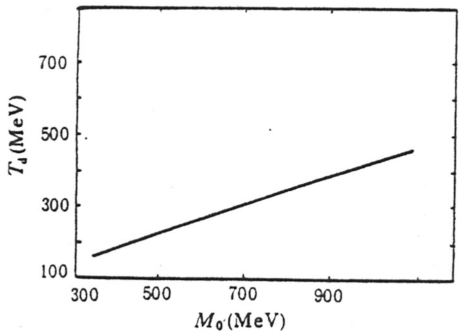

Fig. 4
The behaviors of the decoupling temperature T_d and the melting temperature T_d with the fermion mass M_0, where m_π=5 MeV, N=5.

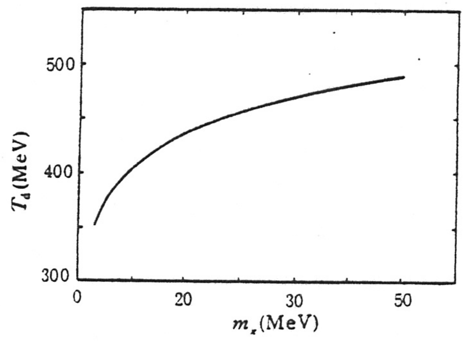

Fig. 5
The behaviors of the decoupling temperature T_d with the π meson mass m_π, where M_0=1000 MeV, N=4.

In Figs. 2 and 3 and Eq. (3.11), we can see that with temperature increasing, $-g_{eff}(\beta)$ decreases gradually and the attractive interaction between fermions becomes weaker. When the temperature exceeds a critical temperature $T_d(g_{eff}(\beta_d)=0)$, the attractive interaction will be screened by the thermal fluctuation, and becomes inclusive interaction. This makes it impossible for fermion pair to condense in the vacuum. Then, the dynamical breaking resulting from fermion pair condensate will be restored.

The critical temperature of the decoupling transition is determined by Eqs. (3.11) and (3.12). The behaviors of the decoupling transition temperature T_d with M_0 and m_π are shown in Figs. 4 and 5. In Figs. 4 and 5, we can see that the decoupling temperature arises when M_0 and m_π increase.

From Eq. (2.9a), the fermion mass M_f, M_f', corresponding to the cases at the leading order and beyond the leading order, can be expressed as

$$M_f - m_0^{ren} = -\frac{g_{ren}^2}{N}\langle\bar{\psi}\psi\rangle_\beta. \tag{3.13}$$

$$M'_f - m_0^{ren} = g_{eff}(\beta)\frac{g_{ren}^2}{N}\langle\bar{\psi}\psi\rangle_\beta. \tag{3.14}$$

The behaviors of $M_f - m_0^{ren}$ and $M_f - m_0^{ren}$ with temperature are shown in Fig. 7. It turns out that

1) when $T \leq T_d$, M_f is much larger than the π mass m_π, which shows that it is reasonable to take the current mass as a small one at finite temperature;

2) the vertex corrections resulting from thermal fluctuation will influence the fermion dynamical mass and cannot be ignored at finite temperature.

It should be pointed out that the dynamical breaking restoration discussed here is different from that in the mean field approximation or at the leading order in the $1/N$ expansion. They have different physical mechanisms. In the latter, thermal fluctuation excites the fermion pair condensate and makes it melting. The critical temperature T_c is determined by the fermion pair condensate $\langle\bar{\psi}\psi\rangle_\beta = 0$, although the dynamical breading restoration discussed here results from the decoupling affect, where not only the melting effect of the fermion pair condensate (which represents the M dependence on temperature), but also the screening effect (which results from the vertex corrections by thermal fluctuation), are considered together. Based only on dynamical analysis, we find that when temperature exceeds the decoupling temperature, dynamical breaking will be restored.

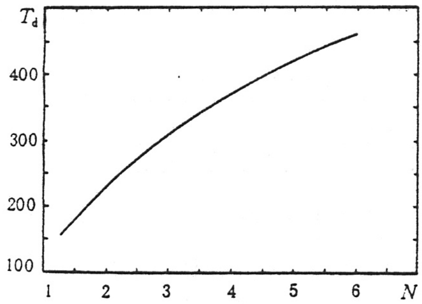

Fig. 6
The behaviors of the decoupling temperature T_d with N, where $M_0=1000$ MeV, $m_\pi=5$ MeV.

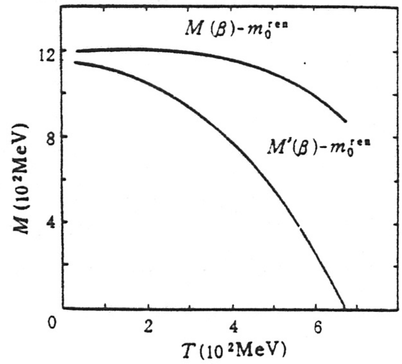

Fig. 7
The behaviors of M_f-m^{ren} and M_f-m^{ren} with temperature, where $N=4$, $m_\pi=50$ MeV and $M_0=600$ MeV.

Note that although we have taken some approximations in calculating the vertex corrections the above results still hold qualitatively.

4. CONCLUSIONS

In this paper, we use chiral Ward-Takahashi identities to derive the mass spectra and calculate the vertex correction among fermions and the σ meson beyond the leading order in the $1/N$ expansion. The dependence of the critical temperature of the decoupling transition with the fermion mass and π meson mass and the fermion mass correction at $1/N$ order are discussed. The critical temperature of the decoupling transition rises when the fermion mass and the π meson mass at zero temperature increase; at finite temperature, the vertex correction resulting from thermal fluctuation will influence the fermion dynamical mass and cannot be ignored.

REFERENCES

[1] E. V. Shuryak, *Phys. Rep.*, **115**(1984)151.

[2] D. Ebert and H. Reihardt, *Nucl. Phys.*, **B271**(1986)188; A. Dhar, R. Shanker and S. R. Wadia, *Phys. Rev.*, **D31**(1986)3256.

[3] K. Shizuya, *Phys. Rev.*, **D21**(1980)2327; B. Rosenstein, B. J. Warr and S. H. Park, *Phys. Rev. Lett.*, **62**(1989)1433.

[4] Shen Kun and Qiu Zhongping, *High Energy Phys. and Nucl. Phys*, (in Chinese), **16**(1992)29; Shen Kun and Qiu Zhongping, *Journal of Physics. G.*, **18**(1992)745.

[5] D. J. Gross and A. Neveu, *Phys. Rev.*, **D10**(1974)3235.

[6] Shen Kun and Qiu Zhongping, *High Energy Phys. and Nucl. Phys*. (in Chinese), **17**(1993)208.

[7] B. Rosenstein, B. J. Warr and S. H. Park, *Phys. Lett.*, **B219**(1989)469; B. Rosenstein and B. J. Warr, *Phys. Lett.*, **B218**(1989)465.

[8] C. W. Bernard, *Phys. Rev.*, **D9**(1974)3312; L. Dolan and R. Jackiw, *Phys. Rev.*, **D9**(1974)3320.

[9] Zheng Guotong and Su Rukeng, *High Energy Phys. and Nucl. Phys*. (in Chinese), **15**(1991)689.

[10] B. Rosenstein, B. J. Warr and S. H. Park, *Phys. Rev.*, **D39**(1989)3088.