

Dynamics of Heavy Ion Collisions in the Fermi Domain (III) Multifragmentation and Limiting Excitation Energy

Zuo Wei¹, Ge Lingxiao² and Zhang Fengshou²

¹(Department of Modern Physics, Lanzhou University, Lanzhou, Gansu, China)

²(Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, Gansu, China)

In the case of central collisions, the mechanism of fragmentation is studied within the framework of Boltzmann-Uehling-Uhlenbeck theory from 10 MeV to 100 MeV for $^{20}\text{Ne} + ^{20}\text{Ne}$ collision system. The exciting energy and the problem about whether complete thermalization be attained have also been discussed.

1. INTRODUCTION

The study of nuclear multifragmentation is one of the major subjects in intermediate energy heavy ion collisions (bombarding energies between 10 and 100 MeV/u) [1-6], which not only provides interesting insight in reaction mechanism itself from medium to high energy, but also gives us unique fingerprints of the nuclear equation of state (EOS) at high excitation energies. In this energy region, both the mean field and the collisions between the nucleons including Pauli blocking play an important role in collision dynamics. In comparison to the cases at low and high energies, the heavy ion collision at intermediate energy has displayed many new properties. One of the most remarkable properties is the transition from incomplete fusion (ICF) to nuclear multifragmentation.

As we know, at bombarding energies below about 10 MeV/u, in central collisions, the projectile and target can merge in a single system if they are not too heavy. This process is called complete

Supported by the National Natural Science Foundation of China. Received on April 24, 1992.

© 1994 by Allerton Press, Inc. Authorization to photocopy individual items for internal or personal use, or the internal or personal use of specific clients, is granted by Allerton Press, Inc. for libraries and other users registered with the Copyright Clearance Center (CCC) Transactional Reporting Service, provided that the base fee of \$50.00 per copy is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923. An annual license may be obtained only directly from Allerton Press, Inc. 150 5th Avenue, New York, NY 10011.

fusion (CF). As the bombarding energy increases, the fusion becomes incomplete because only part of the projectile fuses with part of the target. Many experiments now indicate that incomplete fusion measured by detecting the normal deexcitation products (evaporation residues and/or fission) decreases with increase in bombarding energy, and disappears completely when the excitation energy of the residual nucleus reaches a limiting value (about 3-6 MeV/u, depending on the system). Beyond this limiting excitation energy, the nucleus becomes unstable and explodes into several pieces (multifragmentation). A number of models, such as simple schematic model, statistical model, microscopic dynamical model and so on, have been used to explore the above experimental phenomenon [1-7], but a complete and systematic comprehension is still lacking. There are many open questions to be settled. For example, is the mechanism responsible for nuclear multifragmentation a normal evaporation process or is it a simultaneously dynamical process? Under which conditions do nuclei break up? How are the excitation energy limits predicted? This work presents a qualitative insight in the transition from ICF to multifragmentation and giving semi-quantitative estimates of the explosion threshold as well as the limiting excitation energy in the framework of BUU simulations for $^{20}\text{Ne} + ^{20}\text{Ne}$ central collision.

2. THE MODEL

The BUU equation is a transport equation for the one-body distribution function $f(\mathbf{r}, \mathbf{p}, t)$, which reads [2]

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{r}} f - \nabla_{\mathbf{r}} U(\mathbf{r}) \cdot \nabla_{\mathbf{p}} f = I_{\text{coll}}[f]. \quad (1)$$

The mean-field potential $U(\mathbf{r})$ used here is only density-dependent

$$U(\mathbf{r}) = A\rho + B\rho^\lambda. \quad (2)$$

In our calculation, we take $A = -124$ MeV, $B = 70.5$ MeV and $\lambda = 2$ which reproduce nuclear matter saturation properties and give an incompressibility coefficient of $K = 380$ MeV. The right-hand side of Eq. (1) is the collision integral including the Pauli blocking, i.e.,

$$I_{\text{coll}}[f] = - \int \frac{d^3 p_2 d^3 p'_1 d^3 p'_2}{(2\pi)^9} \sigma v_{12} [f f_2 (1 - f'_1)(1 - f'_2) - f'_1 f'_2 (1 - f)(1 - f_2)] \times (2\pi)^3 \delta^{(3)}(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}'_1 - \mathbf{p}'_2), \quad (3)$$

where σ denotes the nucleon-nucleon cross section [8] and v_{12} is the relative velocity between two colliding nuclei. The simulation of Eq. (1) is based on the test particle method and particle-in-cell technique [8]. In present paper, 100 test particles per nucleon and a cell size of 1 fm are taken. Evolving the test particles by Newtonian mechanics is equivalent to solving the Vlasov equation, while the collision integral is treated in stochastic way, allowing test particles to undergo collisions with a probability proportional to the Pauli corrected cross section.

In order to calculate the excitation energy, we separate the total energy into two parts according to [1]

$$E_{\text{tot}} = E_{\text{int}} + E_{\text{coll}}, \quad (4)$$

where the kinetic collective energy is calculated from the collective current

$$E_{\text{coll}} = \frac{1}{2} m \int \frac{j^2}{\rho} d^3 r, \quad (5)$$

$$j = \int \frac{p}{m} f(r, p, t) d^3 p. \quad (6)$$

The intrinsic energy E_{int} may be splitted into two components,

$$E_{\text{int}} = E_{\text{int}}^0 + E^*, \quad (7)$$

where E^* is the excitation energy, and the cold limit of the intrinsic energy E_{int}^0 is defined as

$$E_{\text{int}}^0 = E_{\text{int}}(\rho, T = 0), \quad (8)$$

which can be calculated according to the parametrization of the mean-field potential in Eq. (1) [1,2],

$$E_{\text{int}}^0 = \int \rho E_{\text{int}}^0 dr, \quad (9)$$

$$\rho E_{\text{int}}^0 = \frac{3}{5} \frac{\hbar^2}{2m} \left(\frac{3\pi^2}{2} \right)^{2/3} \rho^{5/3} + \frac{A}{2} \left(\frac{\rho}{\rho_0} \right) \rho + \frac{B}{1+\lambda} \left(\frac{\rho}{\rho_0} \right)^\lambda \rho. \quad (10)$$

In addition, the quadrupole momentum, which is one of the criteria for thermalization, is defined as

$$\hat{Q}_{zz} = 2p_z^2 - p_x^2 - p_y^2, \quad (11)$$

$$\langle \hat{Q}_{zz} \rangle = \int \hat{Q}_{zz} f(r, p, t) d^3 r d^3 p. \quad (12)$$

3. CALCULATION RESULTS

To display the ICF-multifragmentation transition, the time evolution of the average density in the overlap zone for $^{20}\text{Ne} + ^{20}\text{Ne}$ central collisions at various bombarding energies is shown in Fig. 1.

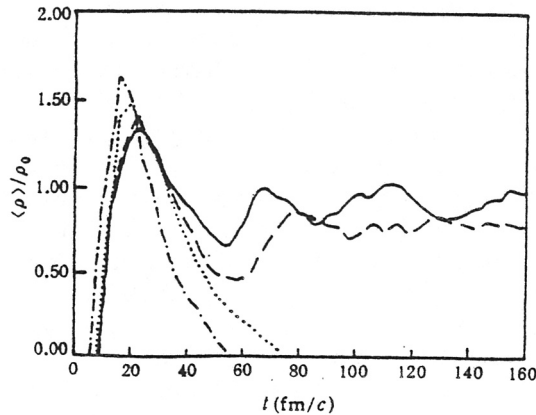


Fig. 1

The time evolution of the overlap density for $^{20}\text{Ne} + ^{20}\text{Ne}$ central collisions at various bombarding energies.

— 30 MeV/u; -- 50 MeV/u; ... 60 MeV/u; -.- 100 MeV/u

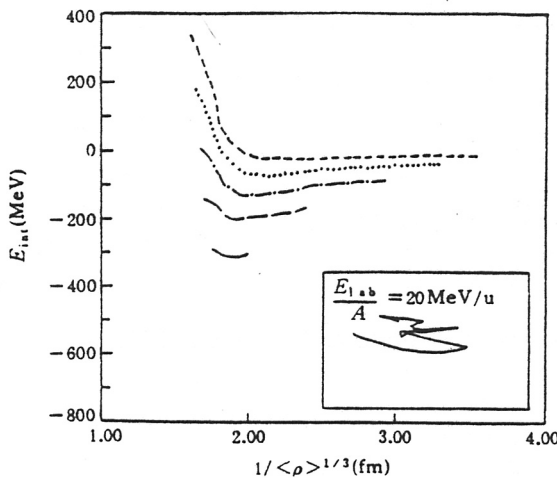


Fig. 2

The dynamic EOS of the $^{20}\text{Ne} + ^{20}\text{Ne}$ collision at various bombarding energies.

— 20 MeV/u; -- 40 MeV/u; -.- 60 MeV/u;
... 80 MeV/u; --- 100 MeV/u

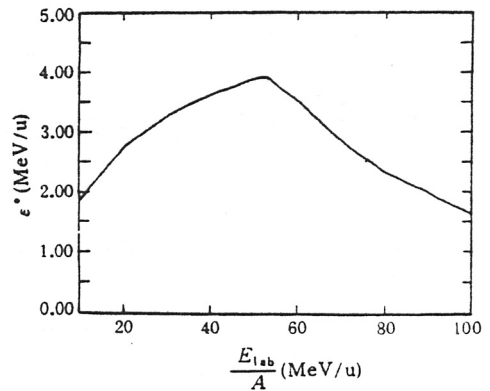


Fig. 3

The excitation energy per nucleon versus the bombarding energy per nucleon.

During the early stage of the collision the projectile and the target merge into a compound system which is compressed rapidly and then expands. It can be seen that the larger the bombarding energy, the larger the compression and the rapider the expansion process. At low energies ($E_{\text{lab}}/A \leq 50$ MeV/u) the compression-expansion sequence leads to a long-living correlative oscillation around a density value less than ρ_0 ($\sim 0.9\rho_0$ for $E_{\text{lab}}/A = 30$ MeV/u and $0.75\rho_0$ for $E_{\text{lab}}/A = 50$ MeV/u). This denotes a typical characteristic of ICF. But at higher energies ($E_{\text{lab}}/A \geq 60$ MeV/u) the compound system is unable to recontract after the expansion and indeed breads up. Thus, Fig. 1 indicates the progressive disappearance of the ICF process and the onset of nuclear multifragmentation at energies around 50-60 MeV/u for the $^{20}\text{Ne} + ^{20}\text{Ne}$ collision system. It can also be seen that the higher the bombarding energy, the rapider the multifragmentation. A more interesting way to look at the above results is to plot the intrinsic energy E_{int} as a function of $1/\langle\rho\rangle^{1/3}$ which represents the size of the compound system. This is called dynamic equation of state. The dynamic EOS of the $^{20}\text{Ne} + ^{20}\text{Ne}$ collision is shown in Fig. 2 where only that piece is given from maximum compression towards where an unambiguous relation $\langle\rho\rangle$ -time is possible. Again ICF and a long-living collective oscillation show up at lower energies ($E_{\text{lab}}/A < 60$ MeV/u). It turns out that the oscillation is in the vicinity of the equilibrium state and the nuclei formed by the collision process is bounded as a whole (One may check the whole dynamic trajectory for $E_{\text{lab}}/A = 20$ MeV/u sketched with a larger scale in the corner of Fig. 2 for a more special feeling). While at higher energies ($E_{\text{lab}}/A \geq 60$ MeV/u) the compound system can no longer be bounded in a potential well due to the large collective kinetic energy. From the time evolution of E_{int} one can also check that beyond 60 MeV/u the collective energy remains larger than the intrinsic energy E_{int} at any time during the expansion. In other words the system has enough kinetic energy to cross the potential barrier, which implies that the system becomes unstable and will explode.

The explosion threshold and limiting excitation energy are very important quantities to describe nuclear multifragmentation. The former marks the onset of multifragmentation and the latter indicates the dynamic limitation to the energy deposition in the compound system. To estimate these quantities, the excitation energy per nucleon as a function of bombarding energy per nucleon is given in Fig. 3.

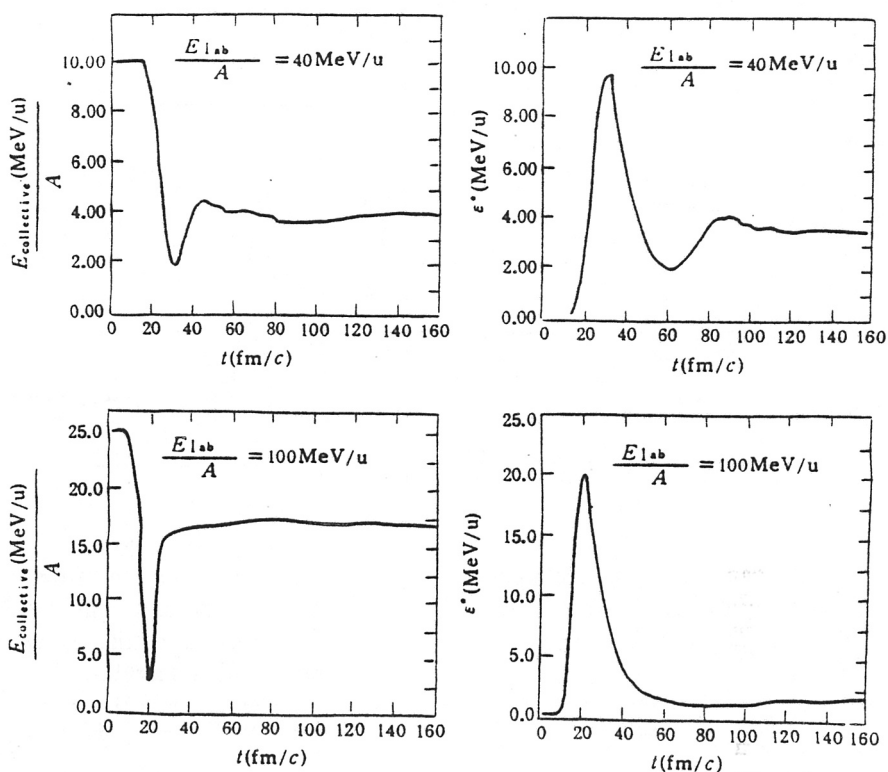


Fig. 4

The time evolution of the collective energy (first row) and the excitation energy (second row) per nucleon for $^{20}\text{Ne} + ^{20}\text{Ne}$ collisions at bombarding energies of 40 MeV/u and 100 MeV/u.

It turns out that below 52 MeV/u, the excitation energy increases with increasing bombarding energy, and reaches the maximum $\sim 3.9 \text{ MeV/u}$ at $E_{\text{lab}}/A \sim 52 \text{ MeV/u}$, then starts to decrease as the bombarding energy increases. This means that the explosion threshold is $\sim 52 \text{ MeV/u}$ and the limiting excitation energy is $\sim 3.9 \text{ MeV/u}$. Beyond the explosion threshold ($E_{\text{lab}}/A \geq 52 \text{ MeV/u}$) the decrease of the excitation energy may be due to two mechanisms: collective kinetic energy during the expansion and the fast particle emission. To illustrate this point more clearly, we give in Fig. 4 the time evolution of the collective energy (the first row) and excitation energy (the second row) for two bombarding energies (40 MeV/u and 100 MeV/u). It is seen that at 40 MeV/u only about 40% of the incident energy is transferred into the collective energy during the expansion process, while in the case of $E_{\text{lab}}/A = 100 \text{ MeV/u}$, almost 70% of the bombarding energy becomes the collective energy of fragments and emitted particles.

Finally, we give a brief discussion about thermalization. To this end, two quantities have been calculated. In Fig. 5 is plotted the time evolution of the quadrupole momenta at various bombarding energies. One can see that the larger the bombarding energy, the more rapid the thermalization process (which is mainly due to two-body collisions). This reflects that as the bombarding energy increases, the two-body collisions become more and more important. At low bombarding energies (below 60 MeV/u), the two-body collisions lead the compound system to a complete thermalization. The oscillation of $\langle Q_{33} \rangle$ around zero reflects the collective effect. Above 60-70 MeV/u bombarding

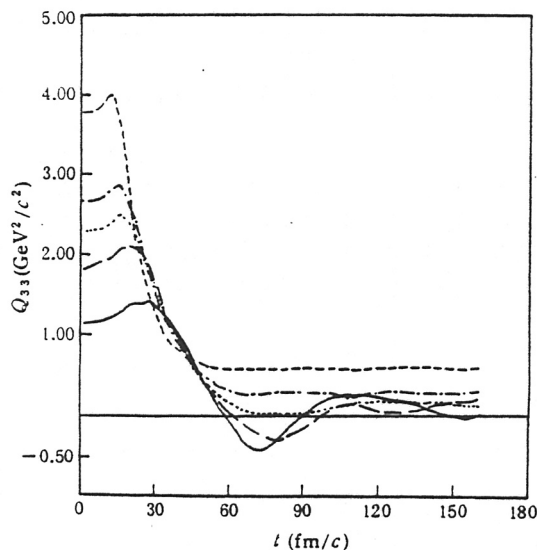


Fig. 5

The time evolution of the quadrupole momentum for $^{20}\text{Ne} + ^{20}\text{Ne}$ central collisions at various bombarding energies.

— 30 MeV/u; -- 50 MeV/u; ... 60 MeV/u;
-.- 70 MeV/u; --- 100 MeV/u

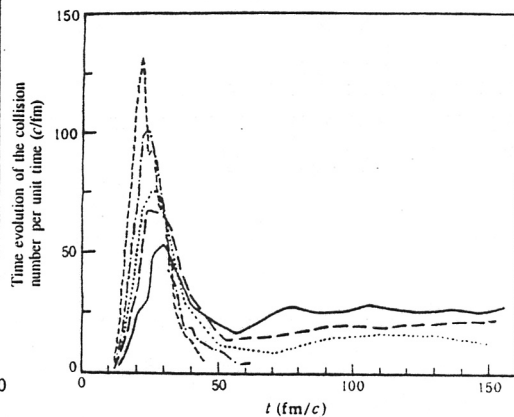


Fig. 6

The time evolution of the collision number per unit time for $^{20}\text{Ne} + ^{20}\text{Ne}$ central collisions at various bombarding energies.

— 30 MeV/u; -- 50 MeV/u; ... 60 MeV/u;
-.- 70 MeV/u; --- 100 MeV/u

energies, the complete thermalization of the whole system can no longer be arrived after compress. This may indicate that the compound system has started to explode before a full thermalization. In addition, the similar conclusions extracted from $\langle Q_{33} \rangle$ can also be obtained from the time evolution of the collision number per unit time (Fig. 6). It is seen that the larger the bombarding energy, the larger the peak value of the collision number per unit time and the faster it decreases. In low energy range (30-60 MeV/u), the collision number decreases to a certain value and then oscillates around this value, indicating the appearance of the thermal equilibrium. While in the cases of higher energies ($E_{\text{lab}} > 60$ MeV/u), the collision number is rapidly reduced to zero, denoting that the complete thermalization is not established. In the vicinity of about 60 MeV/u bombarding energy, both Figs. 5 and 6 show the competition between the statistical and dynamical mechanisms.

4. DISCUSSION AND OUTLOOK

From the above calculation and analysis the following conclusions can be drawn for the $^{20}\text{Ne} + ^{20}\text{Ne}$ central collision.

1) With the increase of the bombarding energy, there exists a limiting excitation energy in heavy ion collisions. As the excitation energy ε^* of the collision system reaches this limiting value, a transition of the reaction mechanisms from ICF to the multifragmentation will occur. The estimated explosion threshold and limiting excitation energy are ~ 52 MeV/u and ~ 3.9 MeV/u for the $^{20}\text{Ne} + ^{20}\text{Ne}$ symmetric system, respectively.

2) In the transition region from ICF to the multifragmentation, the statistical process and the dynamic process interweave and compete with each other and with the increase of the bombarding energy the dynamic process becomes more and more dominant. Nuclear multifragmentation process

occurs within a very short time. This implies that the dynamic mechanism plays a significant role in nuclear multifragmentations.

3) Beyond the threshold energy, the thermalization of the whole collision system becomes increasingly more incomplete.

Further refined and systematic investigation of nuclear multifragmentations demand a model which is able to treat on the same footing all the different phases (expression, expansion and multifragmentation) of the reaction. In other words, it should be a dynamic approach including in some way the many body correlations which are responsible for clusterization. One candidate is the QMD model, while another is to improve the BUU model by including the fluctuation of the mean field in it. Both directions are in currently being used.

REFERENCES

- [1] E. Suraud et al., *Nucl. Phys.*, **A495**(1989)73c.
- [2] G. F. Bertsch and S. das Gupta, *Phys. Rep.*, **160c**(1988)189.
- [3] S. Leray et al., *Nucl. Phys.*, **A495**(1989)283.
- [4] Li Zhuxia et al., *Phys. Rev.*, **C44**(1991)824.
- [5] J. Randrup, *Nucl. Phys.*, **A495**(1989)245c.
- [6] J. Desbois et al., *Z. Phys.*, **A328**(1987)101; C. Ngo et al., *Nucl. Phys.*, **A495**(1989)267.
- [7] K. Krishan et al., *Nucl. Phys.*, **A495**(1989)65c.
- [8] Zhang Fengshou and Ge Lingxiao, *High Energy Phys. and Nucl. Phys.* (in Chinese), **14**(1990)1045; Ge Lingxiao et al., *Proc. of the International Summer School on Heavy Ion Reaction Theory*, Lanzhou, China, 1988, Shen Wenqing, Liu Jianye and Ge Lingxiao (Editors), World Scientific, Singapore, 1989, p. 283.