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η_c-Nuclear Bound States

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The heavy-quarkonium nuclear bound states are recalculated by solving the Klein-Gordon equations. We find that the η_c(2980) can be bound in nuclei.

1. INTRODUCTION

Since the discovery of the J/ψ, the study of heavy-quark systems has been of continuous interest. Firstly, the non-relativistic approach can be applied for the study of heavy-quark systems. Secondly, the perturbative QCD can be used for the treatment of the production processes of the $(c\bar{c})$ due to its large momentum transfer. Besides, seven bound states have been found experimentally for the $(c\bar{c})$ system, which may provide valuable information for the test of QCD [1].

The η_c is the lowest bound state of the $(c\bar{c})$. Its spin is zero and its mass is 2980 MeV. According to the QCD theory, the η_c interacts with a nucleon through multiple gluon exchange. Similar to QED, this multi-gluon-exchange interaction is a kind of attractive Van de Waals force. However, unlike QED, this potential cannot have an inverse power law at large distances because of the absence of zero-mass gluonium states. Namely, there exists the screening effect.

On the other hand, because the $(c\bar{c})$ and nucleons have no quarks in common, the quark exchange potential should be negligible. Furthermore, since there is no Pauli blocking, the effective quarkonium-nucleon interaction will not have a short-range repulsion. Therefore, an effective nonrelativistic potential of a Yukawa form between the η_c and the nucleon was proposed by Brodsky et al. recently [2]. Since the spin of the gluon is 1, the interaction between the η_c and the nucleon is vector like. They related the vector-exchange contributions to the high-energy forward hadron-nucleon scattering amplitude and the vector part of the multiple-gluon-exchange potential at low energies and obtained

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the parameters of the effective η_c-N potential. By using the same Yukawa forma and considering the effect of the nuclear form factor the parameters of the η_c-nucleus potential were obtained.

Based on these potentials and by using the variational method Brodsky et al. studied the possibilities of forming η_c-nuclear bound states. It was found that the η_c-^{3}He bound state can be formed with binding energy of 19 MeV. However, the binding energy is found to be 140 MeV for the η_c-^{4}He system. Experimentally, a search for the nuclear bound charmonium is of fundamental significance for testing the QCD predictions. The experimental feasibility to measure the formation of the (η_c-^{3}He) bound state is proposed [3].

However, there are at least two points in the calculation of Brodsky et al. that need further studying. First, since the η_c-nucleus potential is written in the Yukawa form $V(r) = -\alpha e^{-\mu r}/r$, when r is small V is inversely proportional to r. It tends to negative infinite when r approaches to zero. For the ^{40}Ar nucleus they found that the η_c binding energy could be 1-2 GeV [4], which is unbelievable. From the experience of the Coulomb interactions between a charged particle and nuclei one knows that due to the finite size of the charge distribution in nuclei the Coulomb interactions between the charged particle and nuclei are finite at the origin, which is quite different from that between two point like charged particles. We expect that the potentials between the η_c and nuclei are different from those of the Yukawa form. On the other hand, because the η_c is a quite massive particle and the η_c-nuclear interactions are rather strong, the η_c wave function in the ground state of the η_c-nuclear system would be inside the nuclear core. Therefore, we expect that different behavior of the potentials at short distances will make big differences for the evaluation of the binding energies. Secondly, the variational method is an approximation to be justified for present cases. In order to have a quantitative understanding for the possibility to form the nuclear-bound charmonium, in this paper we present a different calculation, in which the Klein-Gordon equation with the η_c-nucleus optical potentials obtained by folding the η_c-nucleon interaction with the nuclear densities is solved numerically.

In Sec. 2, we outline the formulae used in our calculation. The calculated results and discussions are given in Sec. 3.

2. THE η_c-NUCLEUS POTENTIAL

According to [2], the effective η_c-nucleon interaction is taken to be the Yukawa form,

$$V_{\eta_c\text{-N}}(r) = -\alpha \frac{e^{-\mu r}}{r}, \tag{1}$$

where α is the strength parameter. After considering the partial screening effect [2] gives $\alpha = 0.4$ and $\mu = 0.6$ GeV. For the case without screening $\alpha = 0.6$. It is well known that in the case of the coulomb interaction between a charged particle and the nucleus, the coulomb potential is obtained by folding the point-like Coulomb interaction with the charge distribution density of the nucleus. Similarly, we assume that the η_c-nucleus potential can be obtained by the following folding,

$$V_{\eta_c\text{-A}}(r) = \int d^3r' \rho(r') V_{\eta_c\text{-N}}(\boldsymbol{r} - \boldsymbol{r}'), \tag{2}$$

where $\rho(r)$ is the nuclear density function normalized to the number of nucleons in the nucleus. From Eq. (1) we find

$$V_{\eta_c\text{-A}}(r) = -4\pi\alpha\Big[\frac{e^{-\mu r}}{r}\int_0^r r'^2 dr' \rho(r') I_0(\mu r') + I_0(\mu r)\int_r^\infty r' dr' \rho(r') e^{-\mu r'}\Big]. \tag{3}$$

Here $I_0(x)$ is the zero-order Bessel function with imaginary variable ix. We can easily prove that when $\mu \to 0$, Eq. (3) gives the form of the Coulomb potential commonly used.

Ref. [2] has also given the parameters of the Yukawa potential for different nuclei,

$$\mu^{-2} = \langle R_A^2 \rangle / 6,$$
$$\alpha = 3A\beta^2\mu^2/2\pi, \quad (4)$$

Here $\langle R_A^{\ 2}\rangle^{1/2}$ is the rms radius of the nucleus and $\beta = 1.85\ \mathrm{GeV}^{-1}$ the flavor-independent Pomeron-quark coupling constant [5]. A is the number of the nucleons in the nucleus.

It is clear that the resulting interactions between the η_c and nuclei quite different from that obtained by Eq. (1) with parameter Eq. (4). In Fig. 1, as examples, we show the potentials obtained from Eq. (3) for the η_c-^{3}He, η_c-^{4}He, η_c-^{12}C, and η_c-^{40}Ca systems with partial screening for the η_c-N interaction. The nuclear density functions are taken from the electron scattering experiments [6] and the finite size of nucleons has been corrected [7]. The dashed lines in Fig. 1 represent the η_c-A potentials from Eq. (1). Although our potentials are rather similar to that from [2] at large separation distances for very light nuclei, they deviate from each other at shorter distances.

The range of the potentials is essentially determined by the size of the nucleus. The potential contains a minimum which is located at a position not far from the origin. The depth of the potential changes with nuclei ranging roughly from 15.0 MeV for the light nuclei to 35.0 MeV for the heavy ones calculated. It is obvious that the binding energies of the η_c-nuclear states will be less than 35.0 MeV in general. Without taking into account the screening of the η_c-N interaction, the η_c-nucleus potentials are deeper. As a result, the binding energies will be larger.

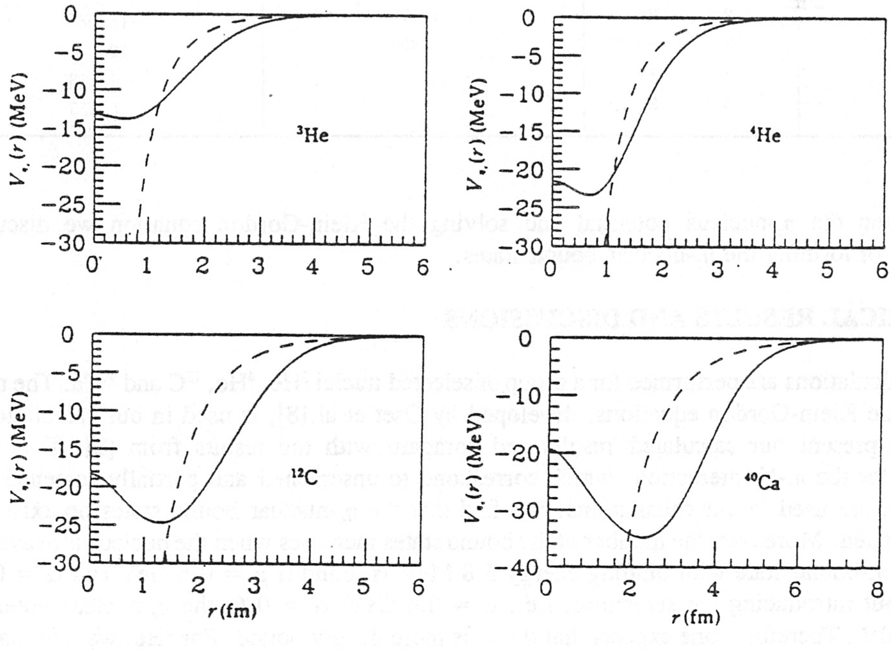

Fig. 1

The η_c-nucleus potentials for a selected group of nuclei. The solid lines are our calculations. The dashed lines represent the results obtained by using a Yukawa form with different parameters α and μ for different nuclei.

Table 1

Binding energies of the η_c-nuclear bound states.
(The numbers in parentheses are results from [1].)

Nuclei	n	l	E_{nl}(MeV) I ($\alpha=0.4, \mu=0.6$)	E_{nl}(MeV) II ($\alpha=0.6, \mu=0.6$)
^3He	1	0	-3.844(-3.0)	-5.741(-19.0)
			-	-
^4He	1	0	-9.171(−51.0)	-14.283(-143.0)
			-	-
^{12}C	1	0	-17.214	-29.216
	1	1	-9.267	-20.011
	1	2	-	-9.190
	2	0	-	-6.351
^{40}Ca	1	0	-29.366	-44.667
	1	1	-25.826	-40.921
	1	2	-20.680	-37.015
	1	3	-	-30.168
	2	0	-15.612	-28.482
	2	1	-10.370	-22.677
	2	2	-4.820	-15.988
	3	0	-2.954	-12.516
	3	1	-	-6.468
	3	2	-	-1.255

By using the η_c-nucleus potential and solving the Klein-Gordon equation we discuss the possibilities of forming the η_c-nuclear bound states.

3. NUMERICAL RESULTS AND DISCUSSIONS

The calculations are performed for a group of selected nuclei ^3He, ^4He, ^{12}C and ^{40}Ca. The method of solving the Klein-Gordon equations, developed by Oset et al.[8], is used in our calculations. In Table 1 we present our calculated results and compare with the results from [2]. Two sets of parameters for the η_c-N interaction, which correspond to unscreened and partially screened cases, respectively, are used in our calculations. We find that the η_c-nuclear bound states do exist for all nuclei calculated. Moreover, the number of the bound states increases when the nucleus is heavier. For ^3He, the 1s η_c-bound state with binding energy 3.8 MeV is found if $\mu = 0.6$ GeV and $\alpha = 0.4$ are used. Without introducing the screening, i.e., $\mu = 0.6$ GeV, $\alpha = 0.6$, the η_c-nuclear potential is deeper by 50%. Therefore, one expects that the η_c is more deeply bound. For ^3He, we find that E_b = 5.7 MeV for the 1s state. Compared with the results of [2], one can see that with partial screening our calculated binding energy is close to that from [2]. Without screening, our results is 13.7 MeV smaller than that found in [2].

For ^4He, our calculated result is quite different from that of [2]. We find one bound state with quantum numbers $n = 1$, $l = 0$. In the unscreened case, the binding energy is 9.17 MeV, which is very different from 51 MeV found in [2].

For heavier nuclei, more η_c-nuclear bound states are found. The discrepancy between our calculation and that in [2] comes mainly from the η_c-nucleus potential. Due to the finite size of the nucleus, the η_c-nucleus interaction generated by the η_c-N interaction becomes finite. We find that the depth of the η_c-nuclear potential is less than 25 MeV for light nuclei, which limits the η_c binding energies in nuclei.

From our calculations we can conclude that the η_c can be bound in nuclei. An experimental search for the η_c-nuclear bound state would provide valuable information for the purely gluonic potential.

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