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Transverse Motion Correlation of Collective Flow in Relativistic Heavy Ion Collisions

Liu Yiming, Zhang Weigang, Huo Lei, Wang Shan, D. Keane[1], S. Y. Chu[2] and S. Y. Fung[2]

(Department of Physics, Harbin Institute of Technology, Harbin, Heilongjiang, China)
[1](Department of Physics, Kent State University, Kent, Ohio, U.S.A.)
[2](Department of Physics, University of California, Riverside, California, U.S.A.)

With 4π data for 1.2 A GeV Ar+BaI_2 and 2.1 A GeV Ne+NaF collisions at the Bevalac streamer chamber, the transverse motion correlations arising from collective flow are studied, which include azimuthal correlations of particle distribution and correlations of transverse momentum magnitude. The azimuthal correlations of particle distribution, the correlations of transverse momentum magnitude and transverse motion in collisions of Ar+BaI_2 are stronger than the corresponding ones in collisions of Ne+NaF respectively. Comparing with the correlations of transverse momentum magnitude, the azimuthal correlations of particle distribution dominate correlations of transverse motion in two experimental data samples.

Key words: collective flow, transverse motion correlations, azimuthal correlations of particle distribution, correlations of transverse momentum magnitude.

1. INTRODUCTION

The collective sideward flow produced in relativistic heavy ion collisions provides valuable

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information for the study of the nuclear equation of state. Many methods, such as ellipsoid tensor [1,2], transverse momentum analysis [3,4,5], the method proposed by Beckmann *et al.* [6,7,8] and azimuthal distribution function analysis [9,10], based on the momentum distribution of particles in the final state for collective flow analyses have been raised. Recently, Wang *et al.* proposed an idea of collective correlations for particles to analyze the collective flow and established a method of particle-pair correlation function [11,12]. Jiang *et al.* extended this method, proposed the concept of high-order collective-flow correlations, and first quantitatively measured "collectivity" of collective flow [13]. The strength and collectivity, describing the collective flow from two complementary aspects, are closely related to anisotropic transverse motion [13] which involved both the azimuthal distribution and the distribution of transverse momentum magnitude of particles. For instance, the characteristic of collective flow would be dominated by anisotropy in the distribution of transverse momentum magnitude if the azimuthal distribution of particles were isotropic in an event, which is a possible limiting case. The azimuthal correlations exist among the final particles [11,12,13] in relativistic nucleus-nucleus collisions with a non-zero impact parameter. Do the correlations of transverse momentum magnitude exist among the particles? If there are this kind of correlations, how do we study this correlations quantitatively? Investigation of these questions is an interesting project in the field of collective flow analyses.

The experimental data samples for this investigation come from the two Bevalac streamer chamber 4π experiments, in which the systems 1.2 A GeV Ar on BaI_2 and 2.1 A GeV Ne on NaF were studied. A description in more detail can be found in [14,15]. There are 786 events with multiplicity $M \geq 30$ for the case of Ar on BaI_2 and 2707 events with multiplicity $M \geq 13$ for Ne on NaF. Assuming a simple geometrical picture, the impact parameters are between 0-6.0 fm for Ar on BaI_2 and 0-5.0 fm for Ne on NaF.

In this paper, we first examine the azimuthal correlations of particle distribution for the two experimental data. Then the correlations of transverse momentum magnitude of particles are studied and compared for these two data samples. And the idea of the azimuthal particle-pair correlation function is extended to define the correlation function of particle transverse motion, the properties of correlations of particle transverse motion are compared for these two data. Finally, the conclusions are given in Section 5.

2. AZIMUTHAL CORRELATIONS OF PARTICLE DISTRIBUTION

For an event of relativistic nucleus-nucleus collisions with a non-zero impact parameter, the azimuthal distribution in an interval of rapidity (y) is anisotropic, and the distribution function of the azimuth ϕ can be described by an expression of the form [9]

$$\frac{d\sigma}{d\phi} = A(1 + \lambda \cos \phi). \tag{1}$$

In the present study, we restrict our analyses for particle transverse motion to the rapidity region $y_{lab} \geq 0.75\ y_{beam}$ [9,10,11,12]. Wang *et al.* in [11,12] pointed out that the azimuthal probability distributions for two particles are independent with each other and satisfy Eq.(1) when we study the effect of azimuthal correlations arising from collective flow in relativistic heavy ion collisions. From Eq.(1) the probability of observing two particles with azimuthal angles ϕ_1 and ϕ_2 is

$$\frac{d^2\sigma}{d\phi_1 d\phi_2} = A^2(1 + \lambda \cos \phi_1)(1 + \lambda \cos \phi_2), \tag{2}$$

then the distribution probability of ψ, which is the angle between the transverse momenta of the two correlated particles, has the form [11,12]

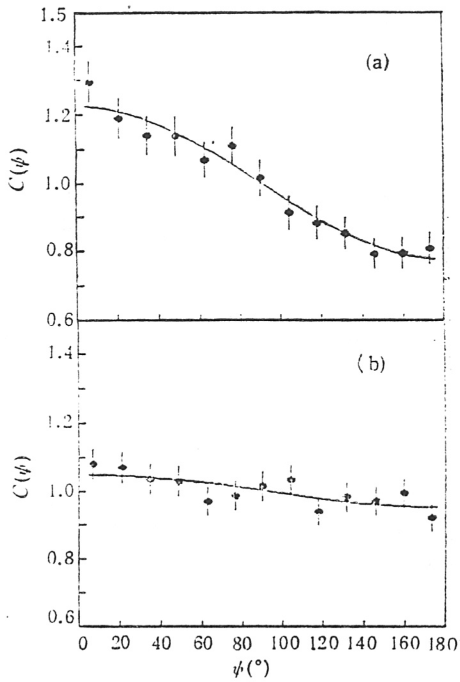

Fig. 1
Azimuthal correlation function $C(\psi)$ of particles with rapidity $y_{lab} \geq 0.75\ y_{beam}$. (a) is for Ar+$BaI_2$ collisions; (b) is for Ne+NaF collisions.

$$P(\psi) = A^2(1 + 0.5\lambda^2 \cos\psi). \tag{3}$$

Adapting the approach of interferometry analysis [16], we define the azimuthal particle-pair correlation function as

$$C(\psi) = \frac{P(\psi)}{PM(\psi)}, \tag{4}$$

where $PM(\psi)$ is the distribution probability of particle pairs with ψ for Monte Carlo events. Two particles in correlated pair are selected from the same collision event, and two particle in uncorrelated pairs are selected from the Monte Carlo event. Monte Carlo events are generated [3,11,12] by randomly mixing particles from different events within the same multiplicity range such that the azimuthal distribution of the particles should be isotropic. The azimuthal correlation function $C(\psi)$ is the ratio of the number of correlated particle pairs to uncorrelated particle pairs within the same bin of the azimuthal difference ψ.

Unlike other methods of collective flow analyses [1-10], the azimuthal correlation function proposed by Wang *et al.* [11,12] does not require any knowledge of the reaction plane. A direct consequence of this fact is the circumvention of the need for any event-by-event estimation of the reaction plane and the associated corrections for the dispersion of this plane about the true reaction plane. In addition, other systematic uncertainties associated with detector acceptance, efficiency, etc. can be minimized. The azimuthal correlation function provides a simple and powerful probe for heavy

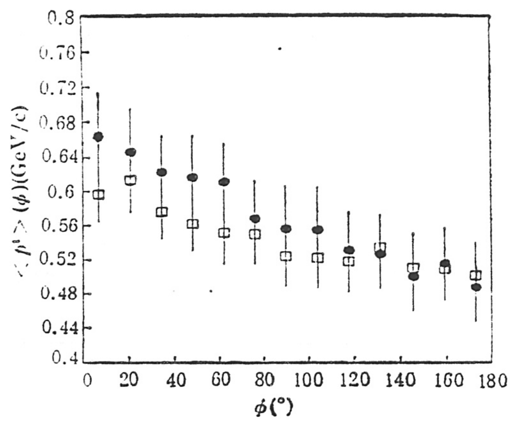

Fig. 2
Mean transverse momentum magnitude $<p_t>(\phi)$ of particles with azimuthal angle ϕ and rapidity $y_{lab} \geq 0.75\ y_{beam}$ in two experimental samples. The symbol □ is for Ne+NaF collisions; the symbol ● is for Ar+BaI_2 collisions.

ion collisions dynamics [17].

The solid circles in Fig. 1 show the calculated values of $C(\psi)$ for the two experimental samples. In order to eliminate the influence of the Coulomb interaction and the effect of quantum statistics for identical particles, we use the technique, shown in [11], which excludes the particle pairs with low relative momentum $|p_1 - p_2| \leq 50$ MeV/c. The particle pairs with relative momentum in this region make up 6% for Ar+BaI_2 collisions and 4% for Ne+NaF collisions of the total pair population. The uncorrelated particle pairs selected from Monte Carlo events, with ten times the statistics of the correlated pairs, make up the background. The solid lines in Fig. 1 indicate the fit to the data of $C(\psi)$ for the two experimental samples using Eq.(3). The fitted values of λ are $\lambda = 0.67 \pm 0.03$, $\chi^2/NDF = 5/13$, for Ar+BaI_2 collisions; $\lambda = 0.31 \pm 0.05$, $\chi^2/NDF = 7/13$, for Ne+NaF collisions.

3. CORRELATIONS OF TRANSVERSE MOMENTUM MAGNITUDE

The distribution of transverse motion is determined by the two factors: the azimuthal angle and the magnitude of transverse momentum for the final particles. Is the distribution of the magnitude of transverse momentum anisotropic? We examine the mean transverse momentum magnitude $<p^t>(\phi)$ for the particles within the same bin of the azimuth ϕ, the angle between particle transverse momentum and the reaction plane for the event and $0 \leq \phi \leq \pi$. The estimated reaction plane for an event is determined by a vector $\boldsymbol{Q}_\nu$ and the beam direction [3,4,5].

$$\boldsymbol{Q}_\nu = \sum_{\mu \neq \nu} \omega_\mu \boldsymbol{p}^t_\mu , \tag{5}$$

where $\omega_\mu = \pm 1$ for baryon with rapidity $y_{lab} - y_{cm} \gtrless \pm\delta$, otherwise $\omega_\mu = 0$. The condition $\mu \neq \nu$ removes the effect of "self correlation", $\boldsymbol{p}^t_\mu$ is the transverse momentum for the μ-th particle in the event, y_{cm} is the center-mass rapidity of the system. Figure 2 shows the calculated results of $<p^t>(\phi)$, which indicates an anisotropy of distribution in the entire region of the ϕ and the existence of the correlations of transverse momentum magnitude among the particles.

In order to quantitatively describe these correlations, we construct a variable $W_{i,\psi}$ for the i-th particle pair with the azimuthal difference ψ for an event:

$$W_{i,\psi} = \frac{p_a^t \cdot p_b^t}{\langle p^t \rangle^2}, \tag{6}$$

where $<p^t> \sum_{j=1}^{M} p_j^t/M$ is the mean transverse momentum of particle in the event, M is multiplicity of the event, and j is the index of particle, p_a^t and p_b^t are the transverse momentum magnitudes of the two particles in the pair. If the transverse momentum magnitudes of two particles in the pair are both equal to the mean transverse momentum of the particle in the event, then the variable $W_{i,\psi} = 1$ represents that there is a particle pair with the azimuthal difference ψ. If any transverse momentum magnitude of the particle in the pair deviates from the mean transverse momentum $<p^t>$ of the particle in the event, then the variable $W_{i,\psi}$ represents the weighted number of the particle pair with the azimuthal difference ψ including the contribution from correlations of the transverse momentum magnitude of the particle.

Assuming that the number of particle pair with the azimuthal difference ψ between two particle transverse momenta is $n(\psi)$ in the event, the mean value of variable $W_{i,\psi}$ can be written as

$$\langle W_{i,\psi} \rangle(\psi) = \frac{\sum_{i=1}^{n(\psi)} W_{i,\psi}}{n(\psi)}, \tag{7}$$

where the sum is for the $n(\psi)$ particle pairs with the azimuthal difference ψ. $<W_{i,\psi}>(\psi)$ excludes variation of the number for particle pairs with the azimuthal difference ψ arising from azimuthal correlations and reflects correlations of transverse momentum magnitude.

Adapting the approach of the azimuthal correlation function, we define the correlation function of transverse momentum magnitude as

$$CW(\psi) = \frac{\langle W_{i,\psi} \rangle (\psi)}{\langle WM_{i,\psi} \rangle (\psi)}, \tag{8}$$

where $<WM_{i,\psi}>(\psi)$ is the calculated result for Monte Carlo events by Eq.(7). The solid circles in Fig. 3 show the data of $CW(\psi)$ for the two experimental samples. The ratio of the number of correlated particle pairs in experimental events to the uncorrelated pairs in Monte Carlo events, and the way to eliminate the influence of the Coulomb interaction and the effect of quantum statistics for identical particles are the same as that in the analyses of the azimuthal correlations in Section 2.

It can be seen from the distribution of $CW(\psi)$ in Fig. 3 that $<W_{i,\psi}>(\psi)$ may be assumed to have a form

$$\langle W_{i,\psi} \rangle (\psi) = B^2(1 + 0.5\lambda_\rho^2 \cos\psi). \tag{9}$$

Using Eq.(9) to fit $CW(\psi)$ for the two experimental samples, the fitted results are:

$\lambda\rho = 0.28 \pm 0.04$, $\chi^2/NDF = 6/13$, for Ar + BaI_2 collisions;

$\lambda\rho = 0.28 \pm 0.04$, $\chi^2/NDF = 6/13$, for Ar + BaI_2 collisions.

The solid curves in Fig. 3 show the fits to the data of $CW(\psi)$. The values of χ^2/NDF show the high confidence level for using Eq.(9) as a fitted functional form. Since $\lambda_p > 0$, there are correlations of the transverse momentum magnitude among the particles. The correlations of the transverse momentum magnitude in Ar+BaI_2 collisions are stronger than in Ne+NaF collisions.

4. CORRELATIONS OF TRANSVERSE MOTION

The transverse motion of particles includes not only the azimuthal correlations of particle distribution but also the correlations of transverse momentum magnitude. To analyze collective flow through the transverse motion of particles should consider both the azimuthal correlations and the correlations of transverse momentum magnitude.

In the study of azimuthal correlations of particle distribution we count the number of particle pairs with the azimuthal difference ψ using the assignment expression $N(\psi) = N(\psi) + 1$. In the analyses of transverse motion including the azimuthal correlations of particle distribution and the correlations of transverse momentum magnitude, we count the weighted number of particle pairs with the azimuthal difference ψ including the effect of correlations of transverse momentum magnitude. The new assignment expression is

$$N(\psi) = N(\psi) + W_{i,\psi}. \tag{10}$$

We define the correlation function of transverse motion for particle-pair as

$$CT(\psi) = \frac{PW(\psi)}{PWM(\psi)}, \tag{11}$$

where $PW(\psi)$ and $PWM(\psi)$ are the weighted numbers of particle pairs with the azimuthal difference ψ calculated by Eq.(10) for experimental and Monte Carlo events, respectively. The correlation function of transverse motion is the ratio of the weighted number of correlated particle pairs to uncorrelated particle pairs within the same bin of azimuthal difference ψ taking account of correlations of transverse momentum magnitude.

The solid circles in Fig. 4 show the data of $CT(\psi)$ for the two experimental samples. The ratio of the number of correlated pairs in the experimental events to the uncorrelated pairs in the Monte Carlo events, the way to eliminate the factors affecting the assumption of the independence of the azimuthal probability distributions for two final particles are the same as that in the above two sections.

It can be seen from Eq.(7) that there is the following relation between $PW(\psi)$, the probability distribution of the weighted number of particle-pairs taking correlations of transverse momentum magnitude into account, and $P(\psi)$, the probability distribution of particle-pairs with ψ taking only the azimuthal correlations into account.

$$PW(\psi) = \xi\langle W_{i,\psi}\rangle(\psi)\cdot P(\psi), \tag{12}$$

where ξ is a normalization constant. From Eq.(3) and Eq.(9), Eq.(12) can be written as

$$PW(\psi) = \xi A^2B^2(1 + 0.5\lambda_p^2\cos\psi)(1 + 0.5\lambda^2\cos\psi).$$

Neglecting the term of $\lambda^2\lambda_p^2\cos^2\psi$, then

$$\begin{aligned} PW(\psi) &\simeq \kappa^2[1 + 0.5(\lambda_p^2 + \lambda^2)\cos\psi] \\ &= \kappa^2(1 + 0.5\lambda_t^2\cos\psi), \end{aligned} \tag{13}$$

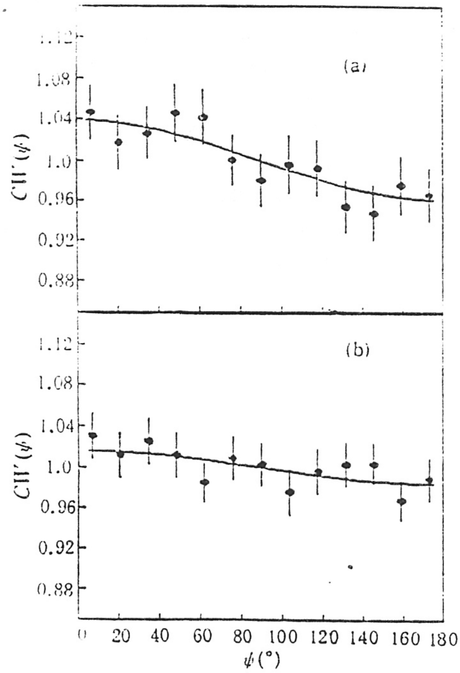

Fig. 3
Correlation function $CW(\psi)$ of transverse momentum magnitude with rapidity $y_{lab} \geq 0.75\, y_{beam}$. (a) is for Ar+ BaI_2 collisions; (b) is for Ne+NaF collisions.

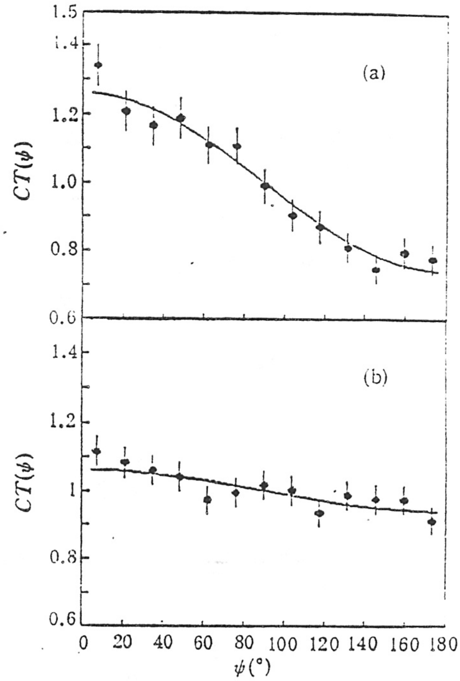

Fig. 4
Correlation function $CT(\psi)$ of transverse motion with rapidity $y_{lab} \geq 0.75\, y_{beam}$. (a) is for Ar+$BaI_2$ collisions; (b) is for Ne+NaF collisions.

where $\kappa^2 = \xi A^2 B^2$, $\lambda_{t^2} = \lambda_\rho^2 + \lambda^2$. Eq.(13) includes both informations from the azimuthal correlations of the particle distribution and from the correlations of the transverse momentum magnitude. The factor λ_t reflects a degree of anisotropy for the distribution of the transverse motion of the particles.

The fitted results to the data of $CT(\psi)$ for the two experimental samples using Eq.(13) are:

$$\lambda\rho = 0.72 \pm 0.03,\ \chi^2/NDF = 7/13, \text{ for Ar + BaI}_2 \text{ collisions;}$$

$$\lambda\rho = 0.36 \pm 0.05,\ \chi^2/NDF = 6/13, \text{ for Ne + NaF collisions.}$$

The solid lines in Fig. 4 are the fitted curves of $CT(\psi)$.

Comparing with the correlations of the transverse momentum magnitude, the azimuthal correlations of the particle distribution dominate the correlations of the transverse motion in the two experimental data samples.

5. CONCLUSIONS

The transverse motion of particles is related to both azimuthal correlations of particle distribution and correlations of transverse momentum magnitude in relativistic heavy ion collisions. The degrees of strength for the azimuthal correlations of particle distribution, the correlations of transverse momentum magnitude and the correlations of transverse motion can be described by the parameters of λ, λ_p and λ_t, respectively. It has been shown in the analyses of 1.2 A GeV Ar+BaI_2 and 2.1 A GeV Ne+NaF Bevalac streamer chamber experimental data that the three kinds of correlations in the former data are stronger than the corresponding ones in the latter, respectively. Comparing with the correlations of momentum magnitude, the azimuthal correlations of particle distribution dominate the correlations of transverse motion in the two experimental data samples. The azimuthal correlation function, the correlation function of transverse momentum magnitude and the correlation function of transverse motion describe the collective flow from different aspects. Unlike the traditional analyses of collective flow, the advantage of using these correlation functions is the circumvention of the need for the corrections for the dispersion of the estimated reaction plane about the true reaction plane and the elimination of the associated systematic uncertainties. The directness and precision of these three kinds of correlation function in the future high statistical experiments for the premier quantitative probes of relativistic heavy ion reaction dynamics play an important role.

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