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Color Screening Effect in $b\bar{b}$ System

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The spectra, leptonic decay widths and E1 transitions of the $b\bar{b}$ system are calculated with a confinement potential in which the color screening effect is included. Meanwhile, the effect of relativistic correction on E1 transition is also evaluated.

Key words: color screening, bound, leptonic decay, E1 transition.

1. INTRODUCTION

As is well known, the heavy quarkonium spectrum can successfully be explained by the non-relativistic quark model. However in various models [1] in which the confinement potentials were adopted in different forms, the resultant leptonic decay width of $\psi(4160)$ was much smaller than the experimental value. The calculated leptonic decay width of highly excited states in the $c\bar{c}$ and $b\bar{b}$ systems, for instance $\psi(4415)$ and $\gamma(11020)$, were also apart from the empirical values.

Recently, E. Laermann [2] re-performed a lattice gauge calculation in which the virtual fermion loop was considered. The result indicated that when separation between quarks became larger, the confinement potential showed a much weaker behavior than the linear one. This character represents the sea quark effect and in called color screening effect.

We studied the $c\bar{c}$ system by phenomenologically employing an error-function type confinement potential where the color screening effect exists and showed that $\psi(4160)$ could be regarded as the 4^3S_1 state instead of 2^3D_1 state, while $\psi(4415)$ could be explained as the $5S$ state [3]. As a consequence, the experimental leptonic decay widths could be well reproduced.

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Table 1
S state spectrum of $b\bar{b}$ system (GeV).

	Exp. [7]	Present work	Barchielli	M-B	GRR	C-K	Cornell
1^3S_1	9.460	9.460	9.460	9.460	9.460	9.460	9.460
2^3S_1	10.023	10.017	10.000	10.021	10.013	10.023	10.050
3^3S_1	10.355	10.360	10.339	10.355	10.355	10.352	10.400
4^3S_1	10.580	10.629	10.616	10.620	—	10.611	10.670
5^3S_1	10.860	10.853	10.859	—	—	—	10.920
6^3S_1	11.020	11.033	11.079	—	—	—	11.140
$ss \times 10^{-4}$		21.13	30.45				69.35

Therefore, it is necessary to study the $b\bar{b}$ system in terms of the mentioned confinement potential and to reveal the color screening effect on the $b\bar{b}$ system.

2. THEORY AND CALCULATION

As shown in [3], the Hamiltonian of the system is chosen to include the one-gluon exchange potential where the relativistic correction is considered and a pure scalar coupling confinement potential.

$$H = 2m + B_0 + H_0 + H_1, \tag{1}$$

$$H_0 = -\frac{\nabla^2}{m} - \frac{4}{3}\frac{\alpha_s}{r} + V^{\rm Conf}(r), \tag{2}$$

$$V^{\rm Conf}(r) = V\mathrm{erf}(\mu_0 r), \tag{3}$$

$$H_1 = \frac{32\pi}{9m^2}\mathbf{S}_1 \cdot \mathbf{S}_2 \alpha_s \delta^3(\mathbf{r}) + \frac{4}{3m^2 r}\alpha_s S_{12} + \frac{1}{2m^2 r}\left(\frac{4\alpha_s}{r^2}\cdot\frac{\mathrm{d}V}{\mathrm{d}r}\right)^{\rm Conf}\mathbf{S}\cdot\mathbf{L}$$
$$+\frac{4\alpha_s}{3m^2}\delta^3(\mathbf{r}) + \frac{2\alpha_s}{m^2}\left[\frac{1}{r}\mathbf{p}_1\cdot\mathbf{p}_2 + \frac{1}{r^3}(\mathbf{r}\cdot\mathbf{p}_1)\cdot\mathbf{p}_2\right] - \frac{p^4}{4m^3}, \tag{4}$$

where B_0 is the zero-point energy and α_s stands for the strong coupling constant. The last term in H_1 is the relativistic correction of the kinetic energy, and the rest of the terms in H_1 are the relativistic corrections of the one-gluon exchange potential.

The Schrödinger equation $H_0\psi_0 = E_0\psi_0$ is solved as the zero-th order approximation, and then the relativistic corrections to energies are calculated in terms of H_1 as the first order perturbation. The parameters in the calculation are chosen as

$$\alpha_s = 0.323,\quad m_q = 5.13\,\mathrm{GeV},\quad \mu_0^{-1} = 1.94\,\mathrm{fm},\quad V = 2.06\,\mathrm{GeV},\quad B_0 = -0.833\,\mathrm{GeV}, \tag{5}$$

so that the overall deviation between the calculated values and experimental data can be minimized. Comparing these parameter values with those used in the $c\bar{c}$ system, it is found that in the $b\bar{b}$ system, α_s and μ_0^{-1} are smaller and larger than those in the $c\bar{c}$ system, respectively. This character is reasonable.

Table 2

P state properties of $b\bar{b}$ system (GeV).

	Exp. [7]	Present work	Barchielli	M-B	GRR	C-K	Cornell
1^3P_0	9.860	9.850	9.860	9.867	9.863	9.854	9.870
1^3P_1	9.890	9.877	9.891	9.916	9.893	9.886	9.903
1^3P_2	9.915	9.893	9.913	9.938	9.910	9.906	9.917
1^3P_{cog}	9.901	9.883	9.898	9.923	9.899	9.894	9.907
R_{1P}	0.670	0.667	0.710	0.449	0.577	0.625	0.424
2^3P_0	10.225	10.221	10.225	10.221	10.232	10.223	10.225
1^3P_1	10.255	10.243	10.253	10.261	10.252	10.246	10.251
1^3P_2	10.270	10.257	10.273	10.280	10.266	10.261	10.266
2^3P_{cog}	10.261	10.248	10.264	10.267	10.258	10.252	10.256
R_{2P}	0.750	0.682	0.720	0.475	0.700	0.652	0.577
$\Delta_{21}(1P)$(MeV)	25	16	22	22	17	20	14
$\Delta_{10}(1P)$(MeV)	30	27	31	49	30	32	33
$\Delta_{21}(2P)$(MeV)	15	15	20	19	14	15	15
$\Delta_{21}(1P)$(MeV)	20	22	28	40	20	23	26

2.1. Calculation of Energy Spectrum

The S state spectrum of the $b\bar{b}$ system resulted from Eq.(1) and parameters in the formulas of Eq.(5) is tabulated in Table 1. For comparison, some results from references are also put in the same table.

In this table, ss values represent the root-mean-square deviations between the theoretical values and experimental data. In comparison with the others, the result in this work is the best. The results of ^3P_J states are tabulated in Table 2, where

$$\Delta_{ij}(nP) = M(n^3P_i) - M(n^3P_j), \tag{6}$$

and

$$R_{nP} = [M(n^3P_2) - M(n^3P_1)]/[M(n^3P_1) - M(n^3P_0)]. \tag{7}$$

From this table, it is seen that the deviations between the theoretical values and experimental data in either this work or the others are in the same degree except our R_{1P} which is even closer to the empirical value. By analyzing the obtained spectrum, it is also revealed that the energy levels of the $b\bar{b}$ highly excited states are improved by the color screening effect. The reason is that at the larger separation r, the strength of the confinement potential defined in Eq.(3) is weaker than that of the linear one so that the energies of excited states are reduced. Moreover, it is shown that the relativistic correction can be ignored in the spectrum calculation.

2.2. e^+e^- Decay of n^3S_1 States

The leptonic decay width of n^3S_1 states with the correction from the strong interaction can be written as

$$\Gamma_{e^+e^-}(n^3S_1) = \frac{16\pi\alpha^2}{M_n^2} e_q^2 |\psi_{n^3S_1}(0)|^2 \left(1 - \frac{16}{3\pi}\alpha_s\right), \tag{8}$$

Table 3

Leptonic decay width in $b\bar{b}$ system (keV).

	1^3S_1	2^3S_1	3^3S_1	4^3S_1	5^3S_1	6^3S_1
Γ(Present work)	1.237	0.553	0.388	0.307	0.240	0.127
Γ(Barchielli)	1.07	0.44	0.31	0.26	0.23	0.19
Exp. [7]	1.34	0.60	0.44	0.24	0.31	0.13

Table 4

Ratio of leptonic decay widths ($\Gamma(n^3S_1)/\Gamma(\Upsilon(9460))$).

n	Present work	Barchielli	Cornell	Richardson	Exp. [7]
2	0.447	0.411	0.36	o.42	0.45
3	0.314	0.289	0.25	0.30	0.33
4	0.249	0.243	0.20	0.27	0.18
5	0.194	0.215	0.19	0.22	0.23
6	0.103	0.178	0.16	0.18	0.097

where the expression in the bracket represents the correction of the strong interaction, e_q and α are the charge of quark and the electromagnetic coupling constant, respectively, M_n is the mass of the n^3S_1 state meson and $\psi_{n^3S_1}(0)$ denotes the value of its wave function at the origin. The calculated decay widths by using Eq.(8) are tabulated in Table 3. In order to delete the theoretical uncertainty and to make comparison more easy, the ratios of the resultant decay widths of the S states and the decay width of Υ_{1^3S1}. It is found that the leptonic decay widths of the highly excited states can evidently be improved by the color screening effect. Namely, the values of the theoretical ratio of $\Upsilon_{6S}^{\ast}(11020)$ and $\Upsilon_{1S}(9460)$ in references were 0.16~0.18 which were larger than the empirical value of 0.097. In contrast with those, our value of 0.103 is much better.

The effect of the color screening can be deduced from the character of the confinement potential where the color screening effect exists. Because the confinement potential with the color screening effect becomes flat at larger r, it not only reduces the energies of highly excited states but also affects their wave functions, i.e., the values of the wave functions at origin would be reduced due to the wider distributions of the wave functions. For instance, the root-mean-square radius is 1.25 fm in the Cornell potential case and 1.39 fm in our error-function type potential case, then in our case, the distribution of the wave function becomes wider and the value of the wave function at origin becomes smaller. As a consequence, the leptonic decay widths can be reduced.

3. PROBABILITIES OF E1 TRANSITIONS

Finally, the E1 transition of the $b\bar{b}$ system is studied. In order to examine the effect of the relativistic correction of an E1 transition, two types of studies are carried out. Firstly, in the non-relativistic approximation,

$$\Gamma(E1) = \frac{4}{27} e_q^2 \alpha |I_1|^2 (2J_f + 1) K^3, \tag{9}$$

Table 5

E1 transition width in $b\bar{b}$ system (keV).

Process	Grotch [4]	Present work	Exp. [7]
$2^3S_1 \to 1^3P_0 + \gamma$	0.807	1.31(1.11)	1.8±0.9
$2^3S_1 \to 1^3P_1 + \gamma$	1.34	2.08(1.81)	2.9±.10
$2^3S_1 \to 1^3P_2 + \gamma$	1.46	2.11(1.85)	2.8±1.0
$3^3S_1 \to 2^3P_0 + \gamma$	1.30	1.35(1.31)	1.5±0.3
$3^3S_1 \to 2^3P_1 + \gamma$	2.35	2.41(2.27)	2.8±0.5
$3^3S_1 \to 2^3P_2 + \gamma$	2.71	2.59(2.39)	2.7±0.4
$3^3S_1 \to 1^3P_0 + \gamma$	0.0004	0.0385(0.0288)	—
$3^3S_1 \to 1^3P_1 + \gamma$	0.0475	0.102(0.0685)	—
$3^3S_1 \to 1^3P_2 + \gamma$	0.383	0.148(0.0933)	—
$1^3P_0 \to 1^3S_1 + \gamma$	24.8	23.6(17.9)	<22.9
$1^3P_1 \to 1^3S_1 + \gamma$	30.1	29.7(22.2)	22.4±5.1
$1^3P_2 \to 1^3S_1 + \gamma$	33.3	34.4(26.3)	29.2±5.6
$2^3P_0 \to 1^3S_1 + \gamma$	12.0	8.64(7.76)	<10
$2^3P_1 \to 1^3S_1 + \gamma$	15.2	9.29(8.37)	4.2±1.8
$2^3P_2 \to 1^3S_1 + \gamma$	16.3	9.77(8.84)	19±7
$2^3P_0 \to 2^3S_1 + \gamma$	4.86	10.33(8.75)	13±10
$2^3P_1 \to 2^3S_1 + \gamma$	6.99	13.39(11.45)	14±6
$2^3P_2 \to 2^3S_1 + \gamma$	10.7	16.00(13.84)	17±9

Table 6

Results of products of branching ratios.

Narain [6]	Present work	Grotch [4]
$(1.7\pm0.4\pm0.6)\times 10^{-3}$	$2.71\times 10^{-3}(2.1\times 10^{-3})$	3.7×10^{-3}

with K to be the energy of the radiated photon, and

$$I_1 = \int R_f(r)R_i(r)r^3 \mathrm{d}r, \tag{10}$$

where R_f and R_i are the radial wave functions of the final and initial states, respectively, with respect to H_0 and J_f stands for the total angular momentum of the final state.

Secondly, consider the first order correction of the wave function due to the relativistic correction term H_1 and correction of the finite size of quark, and ignore the contribution from the abnormal magnetic moment of quark. The E1 transition formula can be written as

$$\Gamma(E1) = \frac{4}{27} K_0^2 K e_q^2 \alpha (2J_f + 1) I_1^2 \left[1 - \frac{KI_{3J}}{10m_q I_1}\right], \tag{11}$$

with $K_0 = m_i - m_f$, and

$$I_{3J} = \int r^4 \frac{\mathrm{d}R_{3S_1}}{\mathrm{d}r} R_{n^3P_J} \mathrm{d}r, \tag{12}$$

The results of using Eq.(9) are listed in the third column of Table 5, and the values obtained from Eq.(11) are tabulated in the brackets in the same column.

From this table, one sees that for processed $2^3S_1 \to 1^3P_J + \gamma$, $3^3S_1 \to 1^3P_J + \gamma$, $1^3P_J \to 1^3S_1 + \gamma$ and $2^3P_J \to 2^3S_1 + \gamma$, the calculated transition widths in the present calculation and Grotch's paper [4] have distinct difference. Because of the large experimental error, it is hard to determine which one is closer to the empirical value for most of the processes. But for $2^3S_1 \to 1^3P_J + \gamma$, our result coincides with the empirical one but the others' results [4,5] were smaller than the experimental data.

For $3^3S_1 \to 1^3P_J + \gamma$, there is no experimental data available, and different predictions from various models were reported. Recently, M. Narain *et al.* [6] showed the product relation of branching ratios $B(\Upsilon'' \to 1^3P_J + \gamma)$ and $B(1^3P_J \to 1^3S_1 + \gamma)$

$$\sum_J B(\gamma'' \to 1^3P_J + \gamma)B(1^3P_J \to 1^3S_1 + \gamma) = (1.7 \pm 0.4 \pm 0.6) \times 10^{-3}, \qquad (13)$$

which gave us a criterion to judge the theoretical results. Our results are tabulated in Table 6. In this table, the values in brackets include the relativistic corrections from Eq.(11). It is seen that the relativistic correction for wave function and the quark finite size correction are very important in the $3^3S_1 \to 1^3P_J + \gamma$ process. In fact, due to the large contribution from I_{3J}/I_{1J}, the finite size correction is important in this process but not in the other processes. As a consequence, the E1 transition width becomes about four times larger. Meanwhile, the relativistic correction for wave function can reduce the transition width, and especially in the above mentioned process, it can reduce the width to 1/5. From the above analysis, one concludes that the relativistic correction for wave function and the correction from the finite quark size give evident contributions to the E1 transition in the $b\bar{b}$ system and therefore cannot be ignored. Although the calculated E1 transitions are only between lower energy levels, the color screening effect still plays a certain role.

3. CONCLUSION

This investigation shows that the color screening effect not only affects the properties of highly excited states to a large degree, but also play a certain role in the states of lesser excitement. It reduces the energies of the highly excited states and widens the distribution of wave function so that the value of the wave function at the origin decreases. As a result, it improves the energy levels of highly excited states and the leptonic decay widths. Therefore, the color screening effect is important in the $b\bar{b}$ system.

Moreover, although the relativistic correction cannot improve the energy spectrum, it can affect E1 transition evidently, so that it should be taken into consideration.

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