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# Inclusive $(\pi,\eta)$ Reactions on Nuclei

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**By assuming that the $\eta$-N interaction is dominated by the $N^*(1535)$ resonance, the inclusive $(\pi,\eta)$ reaction is studied in the framework of the DWBA model, and the cross sections are calculated for the inclusive $^{12}C(\pi,\eta)X$ reaction. A good agreement between our theoretical result and the experimental data is achieved, and it is found that the interaction between the $N^*$ and N is attractive. The Pauli blocking effect is not obvious for the $\eta$ production process.**

**Key words: inclusive $(\pi,\eta)$ reaction, $\eta$-nucleus interaction, double differential cross section.**

## 1. INTRODUCTION

Since the first experimental observation of the $(\pi,\eta)$ reactions performed at LAMPF [1,2], the $\eta$-nucleus interaction has always been one of the most interesting subjects in medium energy physics. As we know, the $\eta$ meson, $\pi$ meson and K meson belong to the same $SU(3)$ octet, but their quark structures are different in the quark model. The study of the $\eta$-nucleus interaction can improve our understanding of the global picture of meson-nucleus interactions. Great progress has been made in the study of $\pi$-nucleus interactions and the K-nucleus interactions have been extensively studied both theoretically and experimentally. On the contrary, little is known for the $\eta$-nucleus interactions. The main reason is that the lifetime of the $\eta$ meson is so short that its interactions with nucleons and nuclei can not be investigated directly through scattering experiments. Based on the quark model, some

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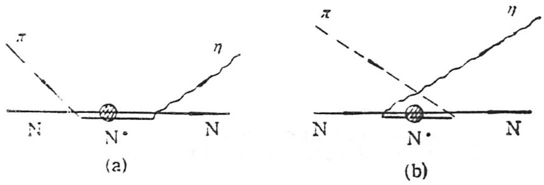


**Fig. 1**
Feynman diagrams contributing to the reaction $\pi N \rightarrow \eta N$.

characters of the $\eta$-nucleon interaction are predicted [3]. By taking into account the contributions of the $N^*(1440)$, $N^*(1520)$ and $N^*(1535)$ resonances, R. S. Rhalerao and L. C. Liu [4] studied the $\eta$-nucleon interaction and analyzed qualitatively the possibility of the formation of the $\eta$-nucleon bound states. However, the $\eta NN^*$ coupling constant was determined by a fit to the $\pi$-N scattering phase shifts, which may cause large uncertainty due to the complexity of the pion-nucleon interaction in this energy region. From the Review of Particle Properties [5], one can see that in the region of $\sqrt{s} \leq 1600$ MeV, the $\eta$ meson appears to have a significantly stronger coupling to the $N^*(1535)$ than any other baryons. Therefore, H. C. Chiang, E. Oset and L. C. Liu [6] proposed a $N^*$ resonance model for the $\eta N$ interaction near threshold. By using this model and taking into account the many-body corrections for the $N^*$ self-energy in nuclear matter, a $\eta$-nucleus optical potential at $q = 0$ is obtained and the possibility of the $\eta$ nuclear bound state formation is studied.

Since there exist no $\eta$ beams, the information on the $\eta$-N and the $\eta$-nucleus interactions comes mainly from the $\eta$ productions. By using the $N^*(1535)$ model, H. C. Chiang and W. W. Wang [7] studied the $\pi N \rightarrow \eta N$ reaction and discussed some factors which might effect the $\eta NN^*$ coupling constant. A comparison of their results with experimental data showed that the correct trend of the cross sections for the $\pi$-induced $\eta$ productions near threshold could be predicted by using the $N^*$ model.

The threshold energy of the $\pi$-induced $\eta$ production on nuclei is about 600 MeV, which is lower than that on nucleons. The near-threshold $\pi N \rightarrow \eta N$ reactions have relatively large cross sections. Experimental studies of the $\eta$ productions on nuclei have made good progress. However, the experimental data of the pionic $\eta$ productions are mainly the inclusive $(\pi,\eta)$ reactions currently. In this paper, we will study these inclusive reactions by using the $N^*(1535)$ model.

In Section 2 we present our model formulae and the numerical results and discussions will be given in Section 3.

## 2. BASIC FORMULAS OF THE THEORY

Let us consider the inclusive reaction as shown below:

$$\pi + A \rightarrow \eta + X, \tag{1}$$

a $\pi$ meson with momentum $k$ collides with a nucleus and produces an $\eta$ meson. In experiments, the direction and energy of the finial state $\eta$ meson is observed, while the rest of the system remains unobserved. In fact, this process is rather complicated. To make our calculation feasible, we assume that the elementary dominant process in the inclusive $A(\pi,\eta)X$ reaction is a resonant $N^*(1535)$ excitation. One of nucleons in the nucleus is excited by the $\pi$ to form the $N^*(1535)$ and then the $N^*$ may decay into an $\eta$ and an unbound nucleon. With these approximations and by using the Fermi gas model for the nucleus, we calculate the inclusive cross section in the framework of the DWBA model.

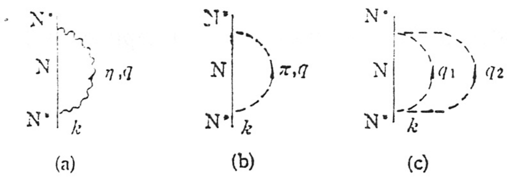

**Fig. 2**
Mechanisms contributing to the width of the $N^*$. (a) $N^* \rightarrow N\eta$; (b) $N^* \rightarrow N\pi$; (c) $N^* \rightarrow N\pi\pi$.

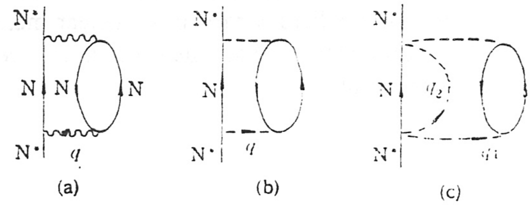

**Fig. 3**
Many-body diagrams in the $N^*$ self-energy. (a) $N^* \rightarrow$ N ph driven by $\eta$ exchange; (b) $N^* \rightarrow$ N ph driven by $\pi$ exchange; (c) $N^* \rightarrow$ N ph $\pi$ from $N^* \rightarrow N\pi\pi$ with one pion exciting a ph.

Following the Bjorken and Drell convention [8], the double differential cross section for the inclusive reaction can be written as

$$\frac{d\sigma}{d\Omega dE_\eta} = 2\int d^3 r \int \frac{d^3 p_{N_1}}{(2\pi)^3} \cdot n(p_{N_1}) \cdot \frac{1}{|V_{rel}|} \cdot \frac{|p_\eta|}{(4\pi)^2} \cdot \frac{1}{E_\pi} \cdot \frac{M_{N_1}}{E_{N_1}} \cdot \frac{M_{N_2}}{E_{N_2}}$$
$$\cdot \bar{\Sigma}\Sigma |T|^2 \cdot \delta(E_\pi + E_{N_1} - E_\eta - E_{N_2} - \varepsilon) \cdot \theta(|p_{N_2}| - k_F), \tag{2}$$

where $N_1$ and $N_2$ are the initial and final nucleons, $p$ and $E$ represent the momentum and energy, respectively, and $E = \sqrt{p^2 + m^2}$. $\varepsilon$ is the binding energy of the nucleon, $\bar{\Sigma}$ represents the average over the spin projections of the initial nucleon and the sum of the spin projections of the final nucleon.

$n(p_N)$ represents the occupation number of initial nucleons in the Fermi sea

$$n(p_{N_1}) = \begin{cases} 1 & |p_{N_1}| \leqslant k_F \\ 0 & |p_{N_1}| > k_F, \end{cases} \tag{3}$$

Because of the Pauli blocking effect, the $\Theta$ function restricts the final nucleon momentum to be above the Fermi momentum $k_F$,

$$\theta(|p_{N_2}| - k_F) = \begin{cases} 1 & |p_{N_2}| > k_F, \\ 0 & |p_{N_2}| \leqslant k_F, \end{cases} \tag{4}$$

In the Fermi gas model, there is a relationship between $k_F$ and the nuclear matter density $\rho$,

$$k_F^3 = 3\pi^2\rho/2. \tag{5}$$

In the framework of the DWBA model, the $T$-matrix can be expressed as

$$T^{\pi\eta}(p, k_\pi, k_\eta) = t^{\pi\eta}(p, k_\pi, k_\eta) + \int \frac{d^3 k'}{(2\pi)^3} t^{\pi'\eta}(p, k', k_\eta) g(k') T^{\pi\pi'}(k', k_\pi), \tag{6}$$

where $p$, $k_\pi$ and $k_\eta$ are the momenta of the nucleon, $\pi$ meson and $\eta$ meson, respectively.

The first term in Eq.(6) corresponds to $\pi$-induced $\eta$ production without distortion of the incident $\pi$, the second term includes the effects of initial-state $\pi$-nucleus interaction. The $g(k')$ is the propagator of the intermediate $\pi$ meson in nuclear matter.

In order to evaluate the $t$-matrix $t^{\pi\eta}$, we consider the Feynaman diagrams as depicted in Fig. 1. $t^{\pi\eta}$ is given by

$$\begin{aligned} -\mathrm{i}t^{\pi\eta} = & (-\mathrm{i})g_{\pi NN^*}\frac{1}{\sqrt{s}-M_{N^*}-\Sigma_{N^*}(s)+\mathrm{Re}\Sigma_N}(-\mathrm{i})g_{\eta NN^*} \\ & +(-\mathrm{i})g_{\pi NN^*}\frac{1}{\sqrt{s}-M_{N^*}-\Sigma_{N^*}(s)+\mathrm{Re}\Sigma_N-E_\pi(\boldsymbol{k}_\pi)-E_\eta(\boldsymbol{k}_\eta)} \\ & \cdot(-\mathrm{i})g_{\eta NN^*}, \end{aligned} \tag{7}$$

where $\Sigma_{N^*}$ is the self-energy of the $N^*$ in nuclear matter, $\mathrm{Re}\Sigma_N$ the real part of the self-energy of the nucleon in nuclear matter. $\sqrt{s}$ is the total energy of the system in the C.M. frame and $M_{N^*}$ is the $N^*$ mass. The first term in Eq.(7) is the contribution of Feynman Fig. 1(a), the second term is the contribution of its cross Fig. 1(b).

The $N^*$ self-energy $\Sigma_{N^*} = \mathrm{Re}\Sigma_{N^*} + \mathrm{i}\mathrm{Im}\Sigma_{N^*}$. The detail of calculating the imaginary part of the $N^*$ self-energy has been shown in [6]. Here we review the main features of such calculations. In addition to the decay channels as those depicted in Fig. 2, inside the nucleus new decay channels for the $N^*$ are possible, as those depicted in Fig. 3. These decay channels will contribute to the imaginary part of the $N^*$ decay width. By using the Feynman rules [8,9] and the Cutkosky rules [8] ($\Sigma \rightarrow 2\mathrm{i}\mathrm{Im}\Sigma$, $G(p) \rightarrow 2\mathrm{i}\Theta(p^0)\mathrm{Im}G(p)$, $D(q) \rightarrow 2\mathrm{i}\Theta(q^0)\mathrm{Im}D(q)$, here $G(p)$ and $D(q)$ are the Fermion and Boson propagators, respectively), the calculation of $\mathrm{Im}\Sigma$ is straightforward.

Since there is no experimental information on the free $NN^*$ interaction which would lead to a microscopic basis for evaluating $\mathrm{Re}\Sigma_{N^*}$, following [6] we assumed that

$$\begin{aligned} &\mathrm{Re}\Sigma_{N^*} = (\rho/\rho_0)V_{N^*}, \\ &V_{N^*} = -50, 0, \quad 50\ \mathrm{MeV}, \end{aligned} \tag{8}$$

where $V_{N^*}$ is the strength for the $N^*$ potential, $\rho$ the nuclear density.

In the meantime, we define

$$\mathrm{Re}\Sigma_N = (\rho/\rho_0)V_N,\ V_N = -50\mathrm{MeV}. \tag{9}$$

For the decay of the $N^*$ in nuclear matter we have to care that the final nucleon momentum is above the Fermi momentum $k_F$, which would introduce corrections to the $N^*$ decay width associated with the $N^*$ decay channels as those depicted in Figs. 2 and 3. We take the nucleon propagator in nuclear matter to have the form

$$\begin{aligned} &G^0(k, E) = \frac{1}{E - E_k + \mathrm{i}\varepsilon} + \mathrm{i}2\pi n(\boldsymbol{k})\delta(E - E_k), \\ &n(\boldsymbol{k}) = \begin{cases} 1 & |\boldsymbol{k}| \leqslant k_F \\ 0 & |\boldsymbol{k}| > k_F, \end{cases} \end{aligned} \tag{10}$$

with

$$T^{\pi\pi'} = T^{os} F(k,k';E), \tag{11}$$

$k_F$ is the Fermi momentum of nucleons. The first term in Eq.(10) is simply the free nucleon propagator and the second term is a genuine Pauli blocking correction. By using the Feynman rules and Cutkosky rules, the corrections introduced by the Pauli Blocking effect can be easily obtained.

In our calculations, we assumed that the initial $\pi$-N scattering amplitude $T^{\pi\pi'}$ has the form [10,11]

$$F(k,k';\ E) = \frac{v(k)v(k')}{v^2(k^0)};$$

$$v(k) = (1 + \alpha k^2)^{-2}; \alpha = 0.224\mathrm{fm}^2. \tag{12}$$

where $T^{os}$ is the full on-shell $\pi$-N scattering amplitude, and the factor $F(k,k';E)$ represents the off-shell extension and can be written in the form

$$g(\boldsymbol{k}') = \frac{1}{\boldsymbol{k}^{0 2} - \boldsymbol{k}'^2 + W + i\varepsilon}, \tag{13}$$

The on-shell scattering amplitude $T^{os}$ can be obtained from a fit to the experiential data of the $\pi$N scattering phase shifts [12-17].

The propagator of the intermediate $\pi$ meson can be written in the form [10,12]

$$\rho(r) = \rho_0(1 + \alpha(r/a)^2)\exp(-(r/a)^2), \tag{14}$$

where $k^0$ is the on-shell momentum of the $\pi$ meson. $W$ is a complex number that characterizes the mean free path of the pion in the nucleus, and is usually related to the first-order $\pi$-nucleus optical potential [10-12]. When a $\pi$ meson collides with a nuclear target, it can absorbed by nucleons, which is called the $\pi$ true absorption. Therefore, we should take into account the $\pi$ absorption correction in the $W$ term as well.

## 3. RESULTS AND DISCUSSIONS

In this paper, we calculated the inclusive $(\pi,\eta)$ cross section on the $^{12}C$ target. In our calculations, we choose the nuclear density function of $^{12}C$ as the H.O. function [18],

$$g_{\pi NN^*} = 0.664,\ g_{\eta NN^*} = 2.03,\ g_{\pi\pi NN^*} = 9.067 m_\pi^{-1}. \tag{15}$$

with $\alpha = 1.067$, $a = 1.687$ and $\rho_0 = 0.132\ \mathrm{fm}^{-3}$.

By taking $M_{N^*} = 1535$ MeV, the total $N^*$ decay width $\Gamma = 150$ MeV, the branching ratios of $N^* \to N\pi$, $N\eta$, $N\pi\pi$ to be 40%, 50% and 10%, respectively, we obtain [1,8]

$$F_i(q) = \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q^2},\ (i = \pi, \eta), \tag{16}$$

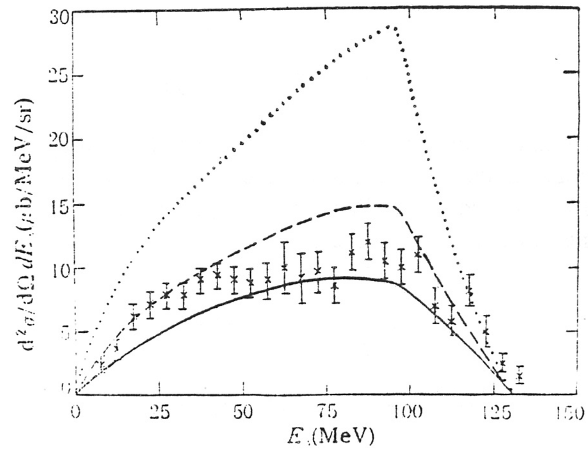


**Fig. 4**

$(\pi^+,\eta)$ inclusive cross sections on $^{12}C$ target at $p_\pi^{lab} = 680$ Mev/$c$. Solid curve is for $V_{N^*} = -50$ MeV, Dashed curve for $V_{N^*} = 0$ MeV and Dotted curve for $V_{N^*} = 50$ MeV.

While calculating the contributions to the $N^*$ self-energy associated with Fig. 3, we choose the form factor of the vertex to be the form

$$\Lambda_\eta = 1.5\text{GeV}, \quad \Lambda_\pi = 1.3\text{GeV}. \tag{17}$$

with $\Lambda_\eta = 1.5$ GeV, $\Lambda_\pi = 1.3$ GeV.

The inclusive $(\pi,\eta)$ cross section on the $^{12}C$ target versus the $\eta$ kinetic energy is shown in Fig. 4, for $\pi$ laboratory momentum 680 MeV/$c$, the experimental data is from [1]. The data are averaged over the angles from $0° \leq \Theta_\eta \leq 30°$, so we have averaged our theoretical calculation over the same angle region.

As shown in Fig. 4, our theoretical curves are in reasonably good agreement with experimental data when $V_{N^*} = -50$ MeV and $V_{N^*} = 0$ MeV. The experimental data fall in the region between the solid curve with $V_{N^*} = -50$ MeV and the dashed curve with $V_{N^*} = 0$ MeV, that is to say that the $NN^*$ interaction is attractive and the strength is between 50 MeV and 0 MeV.

In our calculations, it is found that although the Pauli blocking effect is very important for $\pi$ productions, it is very small for $(\pi,\eta)$ productions. This is due to the fact that the momenta transferred to the nucleons in the $\pi$-induced $\eta$ production are much larger. We also not that because the pions always lose energies after the $\pi N$ scattering, the scattered pions have little probability to produce the $\eta$ meson near threshold. We find that the contributions of the scattered pions can be neglected for the near-threshold $\Theta$ productions.

In summary, we have studied the inclusive $(\pi,\eta)$ reactions on nuclei in DWBA, and calculated the double differential cross sections for the inclusive $^{12}C(\pi,\eta)X$ reaction. A good agreement between our theoretical result and the experimental data is achieved, and it is found that the $NN^*$ interaction is attractive. The Pauli blocking effect is small for the $(\pi,\eta)$ reactions near-threshold. A comparison of our theoretical results with experimental data shows that the $N^*(1535)$ resonance plays an essential role in the near-threshold $\eta$ productions.

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