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Quantization of Interacting Fields in QED

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We propose a new quantization scheme with which and by the equation of motion the quantization problem of both independent and dependent interacting fields in QED can be solved simultaneously. When the external gauge field $A_\mu^{ex}(x) \neq 0$ (i.e., the Fermion field $\psi(x)$ and the electromagnetic field $A_\mu(x)$ are independent of each other) our scheme gives the same result as the usual quantization approach. When the external gauge field is absent the usual quantization conditions fail since it is not compatible with equation of motion, meanwhile our scheme is still valid. These results are demonstrated with solvable QED in 1+1 dimension.

Key words: dependent interacting fields, quantization, QED.

1. INTRODUCTION

The usual quantization condition in QED is applied under condition that the fermion field $\psi(x)$ and gauge field $A_\mu(x) \neq 0$ are independent, the nonvanishing anticommutative and commutative relations are [4,5]

$$\{\psi(x,t),\psi^\dagger(y,t)\} = \delta^3(x-y), \ [A_\mu(x,t),\Pi_\nu(y,t)] = i\delta_{\mu\nu}\delta^3(x-y). \tag{1}$$

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where $\Pi_\mu(x) = \dot{A}_\mu(x)$. Since $A_\mu(x)$ ($\mu = 0, 1, 2, 3$) are not completely independent, the quantized Lorentz gauge condition is

$$\langle | \partial_\mu A^\mu(x) | \rangle = 0. \tag{2}$$

Inversely, the usual quantization condition (1) can be taken as criteria by which one can judge the correctness of the operator solution of quantized field equation. Does this still work when the interacting fields are dependent? This is an open question [1,2,6]. To answer this question is one of our purposes. This paper is organized as follows. In Section 2, we proposes a new quantization scheme of interacting fields. We use the quantization condition of free fields and the field equation to solve the quantization problem of interacting fields. This scheme does not concern whether $\psi(x)$ and $A_\mu(x)$ are independent or not. We prove in Section 3 that our scheme and the usual scheme are equivalent when the external gauge field exists, $A_\mu^{ex}(x) \neq 0$. We discuss the quantization problem of dependent interacting fields $\psi(x)$ and $A_\mu(x)$ when $A_\mu^{ex}(x) = 0$ in Section 4, and show that the usual quantization condition (1) are not compatible with the solutions of field equation, so condition (1) cannot serve as quantization condition for the dependent fields $\psi(x)$ and $A_\mu(x)$. Therefore, it is meaningless that condition (1) is a criteria whether an operator solution of a quantized field equation is correct or not. Section 5 is a concrete example of QED_2 as demonstration of Section 4. Section 6 is a short conclusion.

2. ANOTHER QUANTIZATION APPROACH OF QED

In QED, fields ψ and A_μ satisfy the following equations:

$$\begin{gathered}\{\gamma^\mu(\partial_\mu - ieA_\mu(x)) + m\}\psi(x) = 0, \\ \partial_\lambda\partial^\lambda A_\mu(x) = -j_\mu(x).\end{gathered} \tag{3}$$

where $j^\mu(x) = ie\bar{\psi}(x)\gamma^\mu\psi(x)$. For gauge field $A_\mu(x)$, the Lorentz condition $\partial_\lambda A^\lambda(x) = 0$ (A_μ as classical field) has been applied.

Eqs. (2) and (3) also can be viewed as equation of motion for the quantized fields, only the normal product notation (::) is neglected. The formal solution of Eq. (3) is

$$\begin{aligned}A_\mu(x) &= A_\mu^{ex}(x) - ie\int_{-\infty}^{+\infty} d^4y G^R(x-y)\bar{\psi}(y)\gamma_\mu\psi(y), \\ \psi(x) &= \psi^{Free}(x) + ie\int_{-\infty}^{+\infty} d^4y G_\psi^R(x-y)\gamma^\mu A_\mu^{ex}(y)\psi(y) \\ &+ e^2\int_{-\infty}^{+\infty} d^4y_1 d^4y_2 G_\psi^R(x-y_1)G^R(y_1-y_2)\gamma^\mu\bar{\psi}(y_2)\gamma_\mu\psi(y_2)\psi(y_1).\end{aligned} \tag{4}$$

where $A_\mu^{ex}(x)$ and $\psi^{Free}(x)$ satisfy the equations for free fields

$$\begin{gathered}\partial_\lambda\partial^\lambda A_\mu^{ex}(x) = 0, \\ (\gamma^\mu\partial_\mu + m)\psi^{Free}(x) = 0.\end{gathered} \tag{5}$$

$A_\mu^{ex}(x)$ should satisfy the Lorentz condition. $A_\mu^{ex}(x)$ represents the external potential, that is not stimulated by a charged particle. If $A_\mu^{ex}(x)$ is absent, $A_\mu^{ex}(x) = 0$, we can see from Eq.(4) that $\psi(x)$ and

A_μ are no longer independent of each other. $G^R(x-y)$ and $G^R_\psi(x-y)$ are the retarded Green functions satisfying the following equations

$$\partial_\lambda \partial^\lambda G^R(x-y) = \delta^4(x-y),$$
$$(\gamma^\mu \partial_\mu + m) G^R_\psi(x-y) = \delta^4(x-y). \tag{6}$$

From the structure of Eq. (4), we know that $\psi(x)$ and $A_\mu(x)$ can be expanded in the form.

$$A_\mu(x) = \sum_{n=0}^{\infty} e^n A_\mu^{(n)}(x), \quad \psi(x) = \sum_{n=0}^{\infty} e^n \psi^{(n)}(x). \tag{7}$$

and the $A_\mu^{(N)}(x)$ and $\psi^{(N)}(x)$ obey the recursion relation

$$\begin{aligned}
A_\mu^{(0)}(x) &= A_\mu^{ex}(x), \quad \psi^{(0)}(x) = \psi^{\mathrm{Free}}(x), \\
\psi^{(N)}(x) &= i\int_{-\infty}^{+\infty} d^4y G^R_\psi \gamma^\mu A_\mu^{(0)}(y) \psi^{(N-1)}(y) \\
&\quad + \int_{-\infty}^{+\infty} d^4y_1 d^4y_2 G^R_\psi(x-y_1) G^R(y_1-y_2) \gamma^\mu \sum_{m=0}^{N-2} \sum_{n=0}^{N-2-m} \bar{\psi}^{(n)} \\
&\quad \times (y_2) \gamma_\mu \psi^{(N-2-m-n)}(y_2) \psi^{(m)}(y_1), \\
A_\mu^{(N)}(x) &= -i\int_{-\infty}^{+\infty} d^4y G^R(x-y) \sum_{m=0}^{N-1} \bar{\psi}^{(m)}(y) \gamma_\mu \psi^{(N-1-m)}(y). \\
&\quad (N = 1,2,3,\cdots)
\end{aligned} \tag{8}$$

where the second term in $\psi^{(N)}(x)$ is zero when $N = 1$. Eq. (8) shows that $\psi^{(N)}(x)$ and $A_\mu^{(N)}(x)$ are successively determined when the $A_\mu^{(0)}(x)$ and $\psi^{(0)}(x)$ are given.

Eqs. (4), (7) and (8) are also valid for the quantized fields. The problem of quantization of high order fields $\psi^{(N)}(x)$ and $A_\mu^{(N)}(x)$ is solved provided that the quantization conditions for the free fields $\psi^{(0)}(x)$ and $A_\mu^{(0)}(x)$ are specified. The quantization conditions for the free fields are

$$\{\psi^{(0)}(\boldsymbol{x},t), \psi^{(0)\dagger}(\boldsymbol{y},t)\} = \delta^3(\boldsymbol{x}-\boldsymbol{y}), [A_\mu^{(0)}(\boldsymbol{x},t), \Pi_\nu^{(0)}(\boldsymbol{y},t)] = i\delta_{\mu\nu}\delta^3(\boldsymbol{x}-\boldsymbol{y}). \tag{9}$$

and other (anti) commutators which equal to zero. Besides, we need the quantized Lorentz condition $\langle 0|\partial_\lambda A^{0\lambda}(x)|0\rangle = 0$. Since there is no any relation between $\psi^{(0)}(x)$ and $A_\mu^{(0)}$, they are always commutative to each other. When $\psi^{(N)}(x)$ and $A_\mu^{(N)}(x)$, $(N = 1, 2, \ldots)$ are quantized, we finally obtain the quantization of $\psi(x)$ and $A_\mu(x)$. The advantage of this quantization approach is that it does not concern whether $\psi(x)$ and $A_\mu(x)$ are independent fields or not.

3. EQUIVALENCY OF TWO QUANTIZATION SCHEMES

The second quantization of fields describes the quantum behavior of fields, which is independent of how to deal it mathematically, i.e., independent of the quantization scheme. So we have to prove that the two quantization schemes are equivalent.

In the Heisenberg picture, field operator equations are

$$\frac{\partial}{\partial t}\psi(\boldsymbol{x},t) = -i[\psi(\boldsymbol{x},t),\hat{H}], \quad \frac{\partial}{\partial t}\Pi_\mu(\boldsymbol{x},t) = -i[\Pi_\mu(\boldsymbol{x},t),\hat{H}]. \tag{10}$$

where $\hat{H}$ is the total Hamiltonian of QED and $\Pi_\mu(x) = \dot{A}_\mu(x)$. By making use of the quantization condition (1), Eq. (10) leads to the equation of motion (3). The formal solution of Eq. (3) is the same as formula (4), which can be expanded in power series according to Eq. (7); then, we obtain Eq. (8). Substituting (7) into quantization condition (1) we obtain the commutative or anticommutative relation between the different order expansions of $\psi(x)$ and $A_\mu(x)$. The zeroth order relations are

$$\{\psi^{(0)}(x,t),\psi^{(0)\dagger}(y,t)\} = \delta^3(x-y),[A_\mu^{(0)}(x,t),\Pi_\nu^{(0)}(y,t)] = i\delta_{\mu\nu}\delta^3(x-y). \quad (11)$$

other zeroth order fields are commutative or anticommutative. The high order terms are

$$\sum_{K=0}^{N}\{\psi^{(K)}(x,t),\psi^{(N-K)\dagger}(y,t)\} = 0,$$
$$\sum_{K=0}^{N}[A_\mu^{(K)}(x,t),\Pi_\nu^{(N-K)}(y,t)] = 0. \quad (12)$$
$$\cdots\cdots$$

The zeroth order quantization conditions (11) are the same as the quantization condition (9) for the free fields. In terms of Eqs. (9) and (8), we obtain the commutative or anticommutative relation between $\psi^{(N)}(x)$ and $A_\mu^{(N)}(x)$ ($N = 0, 1, 2, \ldots$). Then, we substitute them into Eq. (12) for check. If all identities are satisfied, we are convinced that two quantization schemes are equivalent.

The problem is that Eq. (12) contains infinite number of equalities, one cannot complete the proof by calculations order by order. Induction is also not suitable here. We have to use the method of reduction to absurdity.

Assume that $\psi^{(N)}(x)$ and $A_\mu^{(N)}(x)$ ($N = 0, 1, 2, \ldots$) do not satisfy Eq. (12). Obviously, quantization condition (1) is not compatible with the solution (4) of the field equation. If we can prove that they are compatible, then we can surely tell that this assumption is wrong, and the two quantization schemes are equivalent.

Introduce a unitary transformation $U(t)$ which depends on time only. Under transformation $U(t)$, a field quantity $\hat{O}$ becomes $\hat{O}'$:

$$\hat{O}' = U(t)\hat{O}U^\dagger(t), \quad (13)$$

$\hat{O}$ represent $\psi(x)$, $\psi'^{+}(x)$, $A_\mu(x)$, $\Pi_\nu(x)$, $\hat{H}$, ... Under transformation (13), the quantization condition (1) is unchanged formally, the nonvanishing terms are

$$\{\psi'(x,t),\psi'^\dagger(y,t)\} = \delta^3(x-y),[A'_\mu(x,t),\Pi'_\nu(y,t)] = i\delta_{\mu\nu}\delta^3(x-y). \quad (14)$$

If $U(t)$ takes the following form

$$U(t) = Te^{-i\int_{-\infty}^{t}\hat{H}'_I(t_1)dt_1}, \quad (15)$$

where $\hat{H}'_I = U\hat{H}_IU^\dagger$, $\hat{H}_I = -ie\int d^3yA_\mu(y,t)\bar{\psi}(y,t)\gamma^\mu\psi(y,t)$ is the interacting part of the Hamiltonian, the transformation (13) changes the Heisenberg picture into interaction picture. In the interaction picture the quantized field Eq. (10) becomes

$$\frac{\partial}{\partial t}\psi'(x,t) = -i[\psi'(x,t),H'_\psi(t)],\ \frac{\partial}{\partial t}\Pi'_\mu(x,t) = -i[\Pi'_\mu(x,t),H'_A(t)], \tag{16}$$

where $\Pi'_\mu(x,t) = \partial/\partial t A'_\mu(x) = -i[A'_\mu(x), H'_A(t)]$. Following the quantization condition (14), Eq. (16) are the free field equations satisfied by $\psi'(x)$ and $A'_\mu(x)$:

$$\begin{aligned} &\partial_\lambda \partial^\lambda A'_\mu(x) = 0, \\ &(\gamma^\mu \partial_\mu + m)\psi'(x) = 0. \end{aligned} \tag{17}$$

Obviously, the quantization condition (14) is compatible with the field Eq. (17). Using inverse transformation $U^{-1}(t)$, the quantization condition transforms into Eq. (1), and the motion equation becomes Eq. (3), they are still compatible. The equivalency of two quantization schemes is proved.

4. QUANTIZATION OF DEPENDENT INTERACTING FIELDS

Eq.(4) shows that $A_\mu(x)$ and $\psi(x)$ are not independent of each other when $A^{ex}_\mu(x) = 0$. To distinguish $\psi(x)$ from that in case $A^{ex}_\mu \neq 0$, we denote $\psi(x)$ here by $\bar{\psi}(x)$, then Eq. (4) becomes

$$\begin{aligned} &A_\mu(x) = -ie\int_{-\infty}^{+\infty} d^4y G^R(x-y)\bar{\Psi}(y)\gamma_\mu \Psi(y), \\ &\Psi(x) = \Psi^{(0)}(x) + e^2\int_{-\infty}^{+\infty} d^4y_1 d^4y_2 G^R_\psi(x-y_1)G^R(y_1-y_2)\gamma^\lambda \bar{\Psi}(y_2)\gamma_\lambda \Psi(y_2)\Psi(y_1). \end{aligned} \tag{18}$$

For the quantization of the dependent field quantities, the quantization condition of free (9) is still valid, but now $A^{(N)}_\mu(x) \equiv 0$. The nonvanishing term is

$$\{\Psi^{(0)}(x,t),\Psi^{(0)\dagger}(y,t)\} = \delta^3(x-y). \tag{19}$$

In terms of Eq. (19) we can quantize $\Psi^{(N)}(x)$ and $A^{(N)}_\mu$ $(N = 1, 2, \ldots)$ order by order,

$$\begin{aligned} \Psi^{(N)}(x) &= \int_{-\infty}^{+\infty} d^4y_1 d^4y_2 G^R_\psi(x-y_1)G^R(y_1-y_2)\gamma^\mu \sum_{m=0}^{N-2}\sum_{n=0}^{N-2-m} \bar{\Psi}^{(n)} \\ &\quad \times (y_2)\gamma_\mu \Psi^{(N-2-m-n)}(y_2)\Psi^{(m)}(y_1), \\ A^{(N)}_\mu(x) &= -i\int_{-\infty}^{+\infty} d^4y G^R(x-y)\sum_{m=0}^{N-1}\bar{\Psi}^{(m)}(y)\gamma_\mu \Psi^{(N-1-m)}(y) \\ &\qquad (N = 1,2,\cdots). \end{aligned} \tag{20}$$

where $\Psi^{(1)}(x) = 0$. Since we have Eq. (7), the quantization problem of $A_\mu(x)$ and $\psi(x)$ are hence solved. The left problem is whether the quantization condition (1) can be generalized to the case of dependent interacting fields. The answer is negative. We shall show that the quantization condition (1) is not compatible with the solution (18) specified by field Eq. (3). In other words, $\{\Psi^{(N)}, N = 1, 2, \ldots\}$ and $\{A^{(0)}_\mu(x) = 0, A^{(N)}_\mu(x), N = 1, 2, \ldots\}$ cannot satisfy all the equations in formulas (11) and (12). Obviously, the quantization condition (1) requires $A^{(0)}_\mu(x) \neq 0$, and $[A^{(0)}_\mu(x,t), \dot{A}^{(0)}_\nu(y,t)] = i\delta_{\mu\nu}\delta^3(x-y)$, but for dependent interacting fields, $A^{(0)}_\mu = 0$, so relations (11) are not satisfied for gauge fields. How about the matter fields? Is the quantization condition (1) suitable for field $\Psi(x)$? The answer is no, that is,

$$\{\Psi(x,t),\Psi^{\dagger}(y,t)\} \neq \delta^3(x-y). \tag{21}$$

This inequality can be proved with method of reduction to absurdity. If relation (21) is not true, then we have

$$\{\Psi(x,t),\Psi^{\dagger}(y,t)\} = \delta^3(x-y). \tag{22}$$

Substituting the expansion $\Psi(x) = \sum_{n=0}^{\infty} e^n\Psi^{(n)}(x)$ into Eq. (22), we obtain

$$\begin{aligned}&\{\Psi^{(0)}(x,t),\Psi^{(0)\dagger}(y,t)\} = \delta^3(x-y),\\ &\sum_{K=0}^{N}\{\Psi^{(K)}(x,t),\Psi^{(N-K)\dagger}(y,t)\} = 0 \qquad (N=1,2,\cdots).\end{aligned} \tag{23}$$

Let $\psi(x)$ be the field when $A_\mu^{(0)} \neq 0$. $\psi(x)$ satisfies the quantization condition (1),

$$\{\psi(x,t),\psi^{\dagger}(y,t)\} = \delta^3(x-y).$$

Substituting the expansion (7) into the above relation we obtain the relations which is the same form as relation (23)

$$\begin{aligned}&\{\psi^{(0)}(x,t),\psi^{(0)\dagger}(y,t)\} = \delta^3(x-y),\\ &\sum_{K=0}^{N}\{\psi^{(K)}(x,t),\psi^{(N-K)\dagger}(y,t)\} = 0 \quad (N=1,2,\cdots).\end{aligned} \tag{24}$$

We shall show in the following that Eqs. (23) and (24) are not compatible. Comparing Eqs. (4) and (8) with Eqs. (18) and (20), we have

$$\begin{aligned}\psi^{0}(x) &= \Psi^{(0)}(x),\\ \psi^{(1)}(x) &= \Psi^{(1)}(x) + i\int_{-\infty}^{+\infty} d^4y G_\psi^R(x-y)A_\mu^{(0)}(y)\gamma^\mu\Psi^{(0)}(y),\\ \psi^{(2)}(x) &= \Psi^{(2)}(x) + i\int_{-\infty}^{+\infty} d^4y G_\psi^R(x-y)A_\mu^{(0)}(y)\gamma^\mu\Psi^{(1)}(y)\\ &\quad - \int_{-\infty}^{+\infty} d^4y_1 d^4y_2 G_\psi^R(x-y_1)A_\mu^{(0)}(y_1)\gamma^\mu G_\psi^R(y_1\\ &\quad - y_2)A_\nu^{0}(y_2)\gamma^\nu\Psi^{(0)}(y_2),\\ &\cdots\cdots\end{aligned} \tag{25}$$

which shows that $\psi^{(N)}(x)$ can be expressed in terms of $\Psi^{(N)}(x)$ $(N = 1, 2, \ldots)$ and $\psi^{(N)}(x)$. With the help of Eq. (23), we calculate

$$E^{(N)}(x,y) = \sum_{K=0}^{N}\{\psi^{(K)}(x,t),\psi^{(N-K)\dagger}(y,t)\}. \tag{26}$$

If $E^{(N)}(x, y) = 0$ $(N = 1, 2, \ldots)$, it means that Eq. (23) is compatible with Eq. (24).

Inversely, if we find any $E^{(N)}(x, y)$ which is not zero, then Eqs. (23) and (24) are not compatible. Since Eq. (24) is proven to be correct, so relation (23) must be wrong, i.e., the original assumption that Eq. (21) is not true is not correct. We conclude that one can use quantization condition (1) for field $\Psi(x)$. After tedious calculation we have

$$\begin{aligned} E^{(1)}(x,y) &= 0, \\ E^{(2)}(x,y) &= i\int_{-\infty}^{+\infty} d^4z_1 d^4z_2 \theta(t-t_1)\theta(t-t_2)S(x-z_1)D(z_1-z_2)\gamma^\lambda \\ &\quad \cdot [iS(z_1-z_2) - \psi^{(0)}(z_1)\psi^{(0)\dagger}(z_2)\gamma^4]\gamma_\lambda S(z_2-y)\gamma^4 \neq 0. \end{aligned} \tag{27}$$

So we know that Eq. (21) is correct.

5. EXAMPLE OF QED$_2$

Massless Quantum Electrodynamic in 1+1 dimension (QED$_2$) [7] is the example that provides an exact operator solution. The Lagrangian with minimal electromagnetic coupling is

$$\mathscr{L} = i:\bar{\psi}(x,A)\gamma^\mu(\partial_\mu + ieA_\mu)\psi(x,A): - \frac{1}{4}:F_{\mu\nu}(x)F^{\mu\nu}(x): \tag{28}$$

where γ matrices are chosen as: $\gamma^0 = \sigma_1$, $\gamma^1 = i\sigma_2$, $\gamma^5 = \gamma^0\gamma^1 = -\sigma_3$. Since $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$, the metric tensor is

$$g_{\mu\nu} = (2\delta_{\mu,0}\delta_{\nu,0} - 1)\delta_{\mu\nu}. \tag{29}$$

The equations of motion satisfied by fields are [5]

$$\begin{aligned} i\gamma^\mu:(\partial_\mu + ieA_\mu(x))\psi(x,A): &= 0, \\ \partial_\lambda\partial^\lambda A^\mu(x) &= e:\bar{\psi}(x,A)\gamma^\mu\psi(x,A):. \end{aligned} \tag{30}$$

The Lorentz gauge condition is $\langle|\partial_\lambda A^\lambda(x)|\rangle = 0$, the formal solution of Eq. (30) is

$$A_\mu(x) = A_\mu^{ex}(x) + e\int_{-\infty}^{+\infty} d^2y G^R(x-y):\bar{\psi}(y,A)\gamma_\mu\psi(y,A):. \tag{31}$$

where $G^R(x - y)$ is the retarded Green function which satisfies the equation

$$\partial_\lambda\partial^\lambda G^R(x-y) = \delta^2(x-y)$$

and takes the form

$$G^R(x-y) = \frac{1}{2}\theta[(x-y)^+]\theta[(x-y)^-]. \tag{32}$$

where Θ is the step function, and $x^+ \equiv x^0 + x^1$, $x^- \equiv x^0 - x^1$ are the light cone coordinates. Substituting expression (31) into the Dirac equation in Eq. (30), we obtain the equation satisfied by

$\psi(x,A)$:

$$i\gamma^{\mu}:\left\{\partial_{\mu}+ieA_{\mu}^{ex}(x)+ie^{2}\int_{-\infty}^{+\infty}d^{2}yG^{R}(x-y)\bar{\psi}(y,A)\gamma_{\mu}\psi(y,A)\right\}\psi(x,A):=0 \tag{33}$$

Eq. (33) is a closed (nonlinear) differential-integral equation for $\psi(x,A)$. Generally it is impossible to solve it exactly. But for the massless 1+1 dimensional QED, it can be solved exactly. For dependent interacting fields, $A_{\mu}^{ex}(x) = 0$, we have to use the quantization approach of free fields to quantize field $\psi(x,A)$. The quantization condition for 1+1 dimensional free Fermion field is [1]

$$\{\psi(x^{1},t),\psi^{\dagger}(y^{1},t)\}=\delta(x^{1}-y^{1}). \tag{34}$$

where $\psi(x)$ satisfies the 1+1 dimensional massless Dirac equation

$$i\gamma^{\mu}\partial_{\mu}\psi(x)=0. \tag{35}$$

The exact solution of field Eq. (33) can be expressed as [2]

$$\psi(x,A)=e^{i\sqrt{\pi}\gamma^{5}\tilde{\Sigma}^{+}(x)}\psi(x)e^{i\sqrt{\pi}\gamma^{5}\tilde{\Sigma}^{-}(x)}=:e^{i\sqrt{\pi}\gamma^{5}\tilde{\Sigma}(x)}\psi(x):. \tag{36}$$

where $\tilde{\Sigma}(x) = \tilde{\Sigma}^{+}(x) + \tilde{\Sigma}^{-}(x)$. $\tilde{\Sigma}^{+}(x)$ and $\tilde{\Sigma}^{-}(x)$ represent the creation and annihilation operators in the configuration space of a certain quantum field. $\tilde{\Sigma}(x)$ and $A_{\mu}(x)$ are related by equation

$$\epsilon_{\mu\nu}\partial^{\nu}\tilde{\Sigma}(x)=\frac{e}{\sqrt{\pi}}A_{\mu}(x), \tag{37}$$

where $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}$, and $\epsilon_{01} = 1$. The structure of $\tilde{\Sigma}$ is [3]

$$\begin{aligned}
\tilde{\Sigma}(x)&=\frac{e^{2}}{4\pi}\{x^{+}\Xi(x^{-})+x^{-}\Xi(x^{+})\},\\
\Xi(x^{+})&=\frac{i}{2\sqrt{\pi}}\int_{K\to0}^{\infty}(p^{1})^{-3/2}dp^{1}[\hat{c}^{*}(-p^{1})e^{ip^{1}x^{+}}-\hat{c}(-p^{1})e^{-ip^{1}x^{+}}],\\
\Xi(x^{-})&=\frac{-i}{2\sqrt{\pi}}\int_{K\to0}^{\infty}(p^{1})^{-3/2}dp^{1}[\hat{c}^{*}(p^{1})e^{ip^{1}x^{-}}-\hat{c}(p^{1})e^{-ip^{1}x^{-}}].
\end{aligned} \tag{38}$$

where $\hat{c}^{*}(p^{1})$ and $\hat{c}(p)$ represent the creation and annihilation operators of a certain boson in momentum space, the non-vanishing commutator is

$$[\hat{c}(p^{1}),\ \hat{c}^{*}(q^{1})]=\delta(p^{1}-q^{1}). \tag{39}$$

The relation of $\hat{c}(p)$ with the creation and annihilation operators $\hat{a}^{*}(p^{1})$, $\hat{a}(p^{1})$, $\hat{b}^{*}(p^{1})$, $\hat{b}(p^{1})$ of free massless fermion and antifermion is

$$\begin{aligned}
\hat{c}(p^{1})=\frac{i}{\sqrt{|p^{1}|}}\int_{-\infty}^{+\infty}dk^{1}\{&\theta(k^{1}p^{1})[\hat{b}^{*}(k^{1})\hat{b}(k^{1}+p^{1})-\hat{a}^{*}(k^{1})\hat{a}(k^{1}+p^{1})]\\
&+\theta[k^{1}(p^{1}-k^{1})]\hat{a}(p^{1}-k^{1})\hat{b}(k^{1})\}.
\end{aligned} \tag{40}$$

The solution of Dirac Eq. (35) satisfied by free-fermion is where $u(p)^1 \begin{bmatrix} \theta(-p^1) \\ \theta(p^1) \end{bmatrix}$, $px = p^0x^0 - p^1x^1$, $p^0 = |p^1|$. The quantization condition which is equivalent to Eq. (34) is

$$\{\hat{a}(p^1), \hat{a}^*(q^1)\} = \delta(p^1 - q^1),$$
$$\{\hat{b}(p^1), \hat{b}^*(q^1)\} = \delta(p^1 - q^1). \tag{41}$$

Other anticommutators are all zero. In terms of Eqs. (38-41), we can find the commutative relation among $\tilde{\Sigma}(x)$, $\tilde{\Sigma}(y)$, $\psi(x)$, and $\psi^{\dagger}(y)$ [3]:

$$[\tilde{\Sigma}(x), \tilde{\Sigma}(y)] = \frac{1}{4i}\left(\frac{e^2}{4\pi}\right)^2 [(x-y)^2]^2 D(x-y),$$

$$[\tilde{\Sigma}(x), \psi(y)] = -\frac{e^2}{4\sqrt{\pi}}(x-y)^2\{\tilde{D}(x-y) + \gamma^5 D(x-y)\}\psi(y),$$

$$[\tilde{\Sigma}(x), \psi^{\dagger}(y)] = \frac{e^2}{4\sqrt{\pi}}(x-y)^2\psi^{\dagger}(y)\{\tilde{D}(x-y) + \gamma^5 D(x-y)\}. \tag{42}$$

where $D(\xi) = 1/2\, \epsilon(\xi^0)\theta(\xi^2)$, $\tilde{D}(\xi) = -1/2\epsilon(\xi^1)\theta(-\xi^2)$, and $\epsilon(\xi^a) = \theta(\xi^a) - \theta(-\xi^a)$. If $x^0 = y^0 = t$, $x^1 \neq y^1$, the interchange relations of wave function components are

$$\psi_1(x^1,t,A)\psi_1^*(y^1,t,A) = -e^{-i\frac{e^2}{4}\epsilon(x^1-y^1)(x^1-y^1)^2}\psi_1^*(y^1,t,A)\psi_1(x^1,t,A),$$
$$\psi_2(x^1,t,A)\psi_2^*(y^1,t,A) = -e^{i\frac{e^2}{4}\epsilon(x^1-y^1)(x^1-y^1)^2}\psi_2^*(y^1,t,A)\psi_2(x^1,t,A),$$
$$\{\psi_j(x^1,t,A), \psi_k^*(y^1,t,A)\} = 0 \quad j \neq k. \tag{43}$$

Obviously, there is no such simple relation like $\{\psi(x^1, t, A), \psi^{\dagger}(y^1, t, A)\} = \delta(x^1 - y^1)|_{x_1 \neq y_1} = 0$.

6. CONCLUSIONS

The usual quantization condition (1) obtained by the canonical quantization of fields in QED works only if the $\psi(x)$ and $A_\mu(x)$ are independent of each other. This has escaped notice for a long time. Is the usual canonical quantization approach still applicable when fields $\psi(x)$ and $A_\mu(x)$ are not independent, i.e., $A_\mu^{ex}(x)$ in Eq. (4), and $A_\mu(x)$ is determined completely by $\psi(x)$? This is open question. From the structure of motion Eq. (3) satisfied by fields, we have the expansion (7), so there is an alternative quantization approach which does not concern the independence of free fields and leads to the quantization of interacting fields $\psi(x)$ and $A_\mu(x)$ [1-3]. But the reasonableness of this approach needs to be proved. Our work answers these questions clearly and provides a theoretical basis for the free field quantization scheme.

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