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EFR-DWBA Analysis of the α-Particle-Induced Reactions on Nuclei at 31.2 MeV

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The data of α-particle-induced reactions on $^{10,11}B$ at 31.2 MeV have been analyzed by using the program MARS-SATURN EFR-DWBA. The results show that direct stripping reactions are the main reaction mechanism in $^{10,11}B(\alpha, x)^{*}C$. However, there are direct stripping plus heavy particle stripping in the reactions $^{11}B(\alpha, d_0)^{13}C$ and $^{11}B(\alpha, p_0)^{14}C$. In addition, the mechanism of compound nucleus seem to be taken into account for the peak of intermediate angular region in the reaction $^{11}B(\alpha, p_0)^{14}C$. Finally, the agreement between theoretical calculation and experimental data is more satisfactory in those reactions: $^{10}B(\alpha, p_0)^{13}C$, $^{10}B(\alpha, d_1)^{12}C^{*}$ and $^{11}B(\alpha, d_0)^{13}C$.

Key words: α-particle, nuclear reactions, EFR-DWBA analysis.

1. INTRODUCTION

Reference [6] pointed out that the study of the complex particle-induced reactions must take into account the effect of EFR-DWBA. Besides the stripping mechanism, the EFR-DWBA calculations of the others reaction mechanisms should be taken into account. As one of the complex particles-particle, when $E_{\alpha} \leq 30$ MeV, the mechanism of α-article induced reactions on nuclei B has been studied systematically [1-3]. The papers pointed out that the anomaly of the reaction cross section in the

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Table 1
The optical potential parameters were used in the calculations.

channel	ejection	V_0 (MeV)	r_0 (fm)	a_0 (fm)	W (MeV)	r_w (fm)	a_w (fm)	r_C (fm)	Ref.
$^{11,10}B + \alpha$	α	212	1.37	0.52	4.83	1.70	0.52	1.30	[2]
					9.00				
$t + {}^{12}C_{0,1}$	$t_{0,1}$	133.3	1.13	0.686	15.60	1.75	0.760	1.3	[3]
$t + {}^{11}C_0$	t_0	160.5	1.40	0.626	17.60	1.40	0.626	1.25	[2]
$d + {}^{13}C$	d_0	114.0	1.60	0.600	10.00	1.60	0.600	1.30	[2]
$d + {}^{12}C_{0,1}$	$d_{0,1}$	106.5	1.60	0.600	10.00	1.60	0.600	1.30	[3]
$p + {}^{14}C$	p_0	46.0	1.31	0.620	10.50	1.31	0.620	1.30	[7]
$p + {}^{13}C$	p_0	62.4	1.52	0.680	7.20	1.52	0.680	1.30	[7]

backward angles are very clear, the anomalous characters decrease with the increase of α-particle energies, thus when the study of the mechanisms of α-particle induced reactions on nuclei B, not only stripping reaction but also the exchanged process should be taken into account. Same framework of the calculation program has been used, the interactions of the projectile and target have been changed from $A(a,b)B$ to $a(A,b)B$ for calculating the exchanged process. In the present work, the method of EFR-DWBA was used to analyze the mechanisms of α-particles induced reactions on nuclei B at 31.2 MeV for eight angular distribution [4] of the reactions $^{11}B(\alpha,d_0)^{13}C$ and $^{11}B(\alpha,p_0)^{14}C$.

2. THEORETICAL FORMULAS

About the derived process of fundamental formulas are given in [5]. The calculated program of the MARS-SATURN was used in the preset work. The program consists of two main parts as follows:

(1) SATURN was used to calculate the interaction of wave functions of the bound states, and the form factors are then obtained by integrating the wave functions:

$$F^{jl_sA_aB_b}_{l_a,l_b}(r_a,r_b) = \sum_{l_1m_1l_2m_2} d^{jl_sA_aB_b}_{l_1m_1l_2m_2} \sum_{k}\Big[\sum_{A_{12}} r_a^{A_{12}} r_b^{l_1+l_2-A_{12}} \times \sum_{\Lambda_a\Lambda_b}\{CFAC(\Lambda_a\Lambda_b;\Lambda_{12};l_1l_2l)\cdot GEOK(\Lambda_a\Lambda_b;l_al_b;kl_1l_2l)\}\Big] \times G_k^{l_1m_1l_2m_2}(r_a,r_b). \qquad (1)$$

The geometrical factor CFAC is given in the following:

$$CFAC(\Lambda_a\Lambda_b;l_1l_2l) = \sum_{\lambda_1\lambda_2} \delta_{\lambda_1+\lambda_2;A_{12}} D_{l_1\lambda_1\lambda_1'} D_{l_2\lambda_2\lambda_2'}\, s_1^{\lambda_1}s_2^{\lambda_2}t_1^{\lambda_1'}t_2^{\lambda_2'} \times \hat{\lambda}_1\cdot\hat{\lambda}_2\cdot\hat{\lambda}_1'\cdot\hat{\lambda}_2'\cdot\hat{l}_1\cdot\hat{l}_2\cdot\hat{\Lambda}_a\cdot\hat{\Lambda}_b(\lambda_10\lambda_20|\lambda_a0)(\lambda_1'0\lambda_2'0|\Lambda_b0) \times \begin{Bmatrix}\lambda_1 & \lambda_2 & \Lambda_a\\ \lambda_1' & \lambda_2' & \Lambda_b\\ l_1 & l_2 & l\end{Bmatrix}, \qquad (2)$$

with

$$D_{l\lambda\lambda'} \equiv \delta_{\lambda+\lambda';l}[(2l+1)!/(2\lambda+1)!(2\lambda'+1)!]^{\frac{1}{2}};$$
$$\hat{\lambda} \equiv (2\lambda+1)^{\frac{1}{2}}.$$

For the factor:

$$GEOK(\Lambda_a\Lambda_b;l_al_b;k;l_1l_2l) = \frac{1}{2}\cdot i^{(l_1+l_2)-(l_a+l_b)}(-)^{k+l}\cdot(2k+1)$$

$$\cdot(\Lambda_a0k0|l_a0)(\Lambda_b0k0|l_b0)\cdot W(l_a\Lambda_al_b\Lambda_b;kl), \tag{3}$$

$$d^{jl_sl_A_aB_b}_{l_1m_1l_2m_2} = C^{(1)}_{I_>l_<jl_1m_1}\cdot C^{(2)}_{s_>s_<sl_2m_2}\cdot(-)^{s+l_2-l_x}\cdot W(l_1l_2;s;ll_x), \tag{4}$$

The integral kernel is defined as follows:

$$G^{l_1m_1l_2m_2}_k(r_ar_b) = \int_{-1}^{1} W_{l_1m_1}(r_1)\cdot W_{l_2m_2}(r_2)P_k(\mu)d\mu, \tag{5}$$

where μ is the cosine of the angle between the vector r and r'. The preceding formulas are mainly used to described the reactions between $A(a,b)B$ and $a(A,b)B$, where a, b or A, B are the projectile and ejectile respectively. In direct stripping reactions, we have the relation $a = x + b$, $B = x + A$; in the heavy particles stripping reactions, we have the relation $A = x + b$, $B = x + a$. The details of Jacobian transformations are given in [5,6].

(2) MARS is used to calculate the cross sections. MARS is based on the calculated results of above form factors; the cross sections are defined by the following expression:

$$\frac{d\sigma(\theta)}{d\Omega} = C_1^2S_1C_2^2S_2\cdot\frac{\mu_a\mu_b}{(2\pi\hbar^2)^2}\cdot\frac{K_b}{K_a}\cdot\frac{(2I_>+1)(2S_>+1)}{(2I_A+1)(2S_a+1)}\cdot\left(\frac{4\pi}{K_aK_b}\right)^2$$
$$\times\sum_{jl_sm_l}\left|\sum_{l_al_b}(-)^{l_b+m_l}\cdot\hat{l}_a\cdot\hat{l}_b(l_a0l_bm_l|lm_l)I^l_{l_al_b}\cdot G_{l_b-m_l}\cdot P_{l_bm_l}(\theta)\right|^2. \tag{6}$$

For the form factors of EFR-DWBA $I^{j\,l_s}_{l_al_b}$ should be written as follows:

$$I^{jl_s}_{l_al_b} = J\iint\chi^{(-)*}_{l_b}(K_b,r_b)F^{jl_sl_A_aB_b}_{l_al_b}(r_ar_b)\chi^{(+)}_{l_a}(K_a,r_a)r_ar_bdr_adr_b,$$

In Eq. (6) G_{lm} is expressed by

$$G_{lm} = (-)^{(m-|m|)/2}\cdot[(l-|m|)!]^{1/2}. \tag{7}$$

In one-nucleon or multi-nucleons (under the cluster approximation), the transferred reactions have:

$$C_i^2S_i = C^{(i)2},$$

If $C^{(1)} = C^{(2)} = 1$ is taken, Eq. (7) is equal to Eq. (1). The Jacobian transformation of stripping reaction is $J = (aB/xT)^3$. In the present program, the form factors for the NR calculations are also considered. But, we do not give the details, as they are given in [5,7].

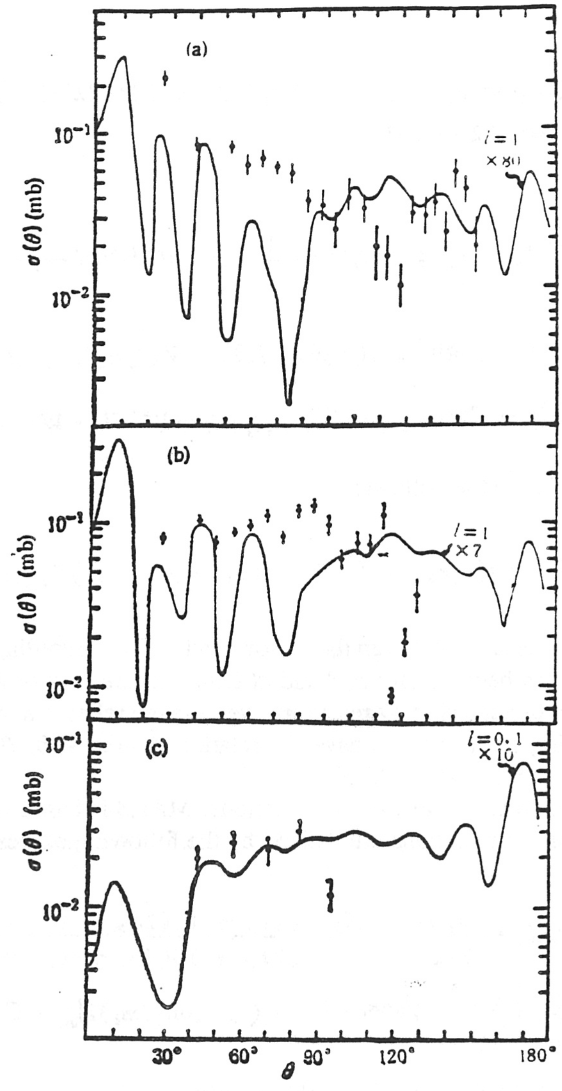

Fig. 1
The angular distributions of the reaction. ${}^{11}_{3/2-}B(\alpha, t_0){}^{12}_{0+}C_{g.s.}$ (a); ${}^{11}_{3/2-}B(\alpha, t_1){}^{12}_{2+}C_{1st}$ (b); ${}^{10}_{3+}B(\alpha, t_0){}^{11}_{3/2-}C$ (c).

3. RESULTS OF CALCULATION AND DISCUSSIONS

As is well known, the mechanism of α-particles induced reactions on 1p shell nuclei have four different types [6]. In direct stripping reactions, there are two mechanisms: cluster stripping and heavy replacement. There are two mechanisms in the exchanged process: heavy particle stripping and general replacement. Because the matrix element of the present program depends only on the potential given by the Jacobian coordinate relations and is independent of the relative wave functions, heavy replacement and general replacement mechanisms cannot be calculated in the present program, but heavy particle stripping reactions can be calculated due to the fact that the interaction potential between *b* and *x* in the nucleus A has the same coordinate relationship in nucleus a. Within the framework of

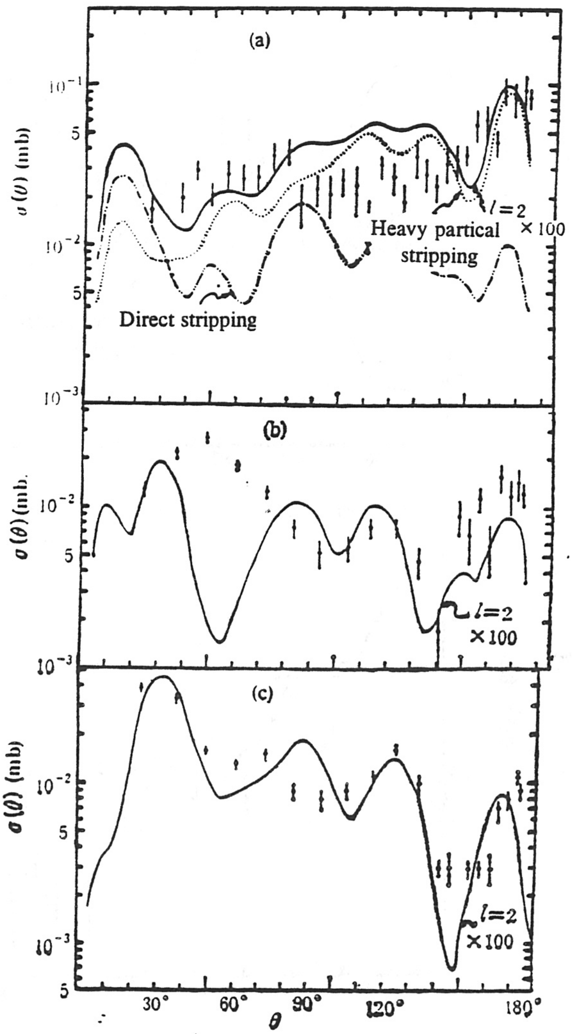

Fig. 2

The angular distributions of the reactions ${}^{11}_{3/2-}B(\alpha, d_0){}^{13}_{1/2-}C$ (a); ${}^{10}_{3+}B(\alpha, d_0){}^{12}_{0+}C_{g.s.}$ (b); ${}^{10}_{3+}B(\alpha, d_1){}^{12}_{2+}C_{1st}$ (c).

three-dimensions, the direct and the exchanged processes are independent of each other; coherence between them also cannot take place. So on the basis of experimental results, the calculations of direct stripping and exchanged processes are taken into account, and we then proceed to sum incoherently. The angles in the heavy particles stripping reactions are the reverse of the angles of direct stripping reaction, i.e., $\Theta^* = \pi - \Theta$, Θ^* are distribution angles of heavy particles stripping reactions.

We worked in conjunction with the Mathematics Department, Shanghai University (Jading Campus). At first, the EFR-DWBA program of MARS-SATURN type was successfully transplanted on the Hypermicrocomputer HP type and tested by standard data. Then the one-nucleon transferred reaction was calculated for ${}^{11,10}B(\alpha,t_{0,1}){}^{12,11}C$, the optical parameters were taken from [2], they are shown in the Appendix Table 1. The calculated results are shown in Fig. 1. When calculating, the imaginary part of optical potential has been slightly adjusted so that it can coincide with the

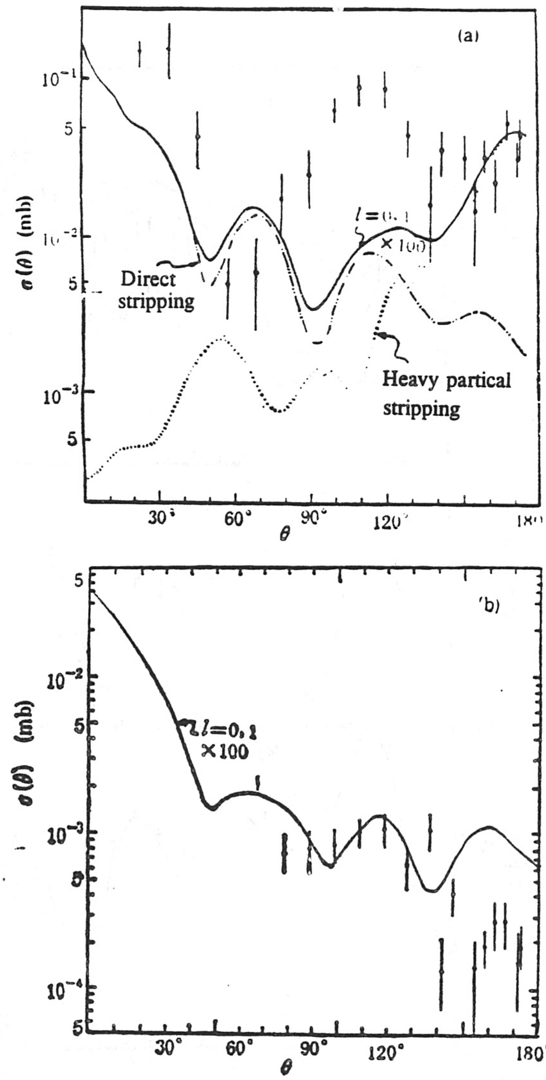

Fig. 3

The angular distributions of the reactions ${}^{11}_{3/2-}$B(α, p_0)${}^{14}_{0+}$C (a); ${}^{10}_{3+}$B(α, p_0)${}^{13}_{0+}$C (b).

experimental data. The calculated results show that the coincidence between the calculated and the experimental values are very satisfactory. The results also ascertain that the direct stripping reaction mechanism is the main mechanism. Of course, there is a little non-coincidence in the intermediate part of angles, possibly because the stripping mechanism is taken into account only in the direct process and the mechanism of heavy replacement is not considered, or else other mechanisms, for example, little amount of compound nucleus mechanism should be taken into account.

In the calculations for the reactions ^{11}B(α,d_0)^{13}C and ^{10}B(α,d_0)^{12}C, the angular distributions of the reaction ^{11}B(α,d_0)^{13}C show anomaly of backward angles cross section, hence, in the calculations besides the stripping of direct process, the exchanged process should also be considered. Therefore, we calculate the direct stripping and heavy particle stripping reactions respectively. The results are shown in Figs. 2 and 3. The coincidence between the incoherent sum results of direct stripping and heavy particles stripping and the experimental data are very good.

Table 2

The interaction potential parameters of the bound states and between two clusters.

	ε_0(MeV)	r_0(fm)	a_0(fm)	V_0(MeV)	W(MeV)	r_w(fm)	a_0(fm)	r_C(fm)
$\alpha + {}^{9}Be$	−10.646	1.500	0.600	19.980	8.700	1.500	0.600	1.300
$\alpha + {}^{10}Be$	−12.011	1.500	0.600	21.434	7.700	1.500	0.600	1.300
$d + {}^{9}Be$	−15.815	1.470	0.670	29.972	8.000	1.470	0.670	1.300
$p + {}^{10}Be$	−11.228	1.200	0.650	41.589	7.700	1.200	0.620	1.300
$t + {}^{11}Be$	−18.784	1.400	0.700					
$t + {}^{10}B$	−23.875	1.400	0.700					
$d + {}^{11}B$	−18.619	1.300	0.500					
$d + {}^{10}B$	−25.186	1.300	0.500	41.075	7.700	1.300	0.500	1.300
$p + {}^{11}B$	−15.957	1.200	0.630	47.200	7.700	1.200	0.630	1.300
	−11.720	1.090	0.590	47.317	7.790	0.980	0.570	1.290
$p + {}^{10}B$	−8.690	1.200	0.630	34.243	7.700	1.200	0.630	1.300
$t + p$	−19.814	1.150	0.200	72.121	7.700	1.150	0.200	1.200
$d + d$	−23.848	1.260	0.800	64.747	7.700	1.260	0.800	1.300

In the reaction ${}^{10}B(\alpha,d_0){}^{12}C$ besides the direct mechanism accounted for other mechanisms seems necessary to be considered for the peak shape in the neighborhood of 50°, for example, heavy replacement or general replacement mechanisms, but these mechanisms cannot be calculated due to the limit of the present program. There is only direct stripping mechanism in the reaction ${}^{10}B(\alpha,d_1){}^{12}C^*$. For the reaction ${}^{11}B(\alpha,p_0){}^{14}C$, the incoherent sum of direct and heavy particles stripping have been calculated. The coincidence between the calculated and the experimental values in the forward and backward angles are satisfactory, but the distribution peak shape in the 80°-130° angles accounted for the compound nucleus mechanism is important, it means to show the complex of three-nucleons transferred mechanism. In the reaction ${}^{10}B(\alpha,p_0){}^{13}C$, the direct stripping mechanism is the sole mechanism. In addition, the angular distributions of reaction cross sections of the reaction ${}^{11}B(\alpha,d_0){}^{13}C$ as compared with the reaction ${}^{11}B(\alpha,p_0){}^{14}C$, the anomaly in the backward angles assured that the reaction ${}^{11}B(\alpha,d_0){}^{13}C$ is stronger than the reaction ${}^{11}B(\alpha,p_0){}^{14}C$. It seems that though separated energy of the latter one is lower than the former one, the cluster structure of the target is that the probability of the ${}^{11}B \rightarrow d + {}^{9}Be$ is larger than the ${}^{11}B \rightarrow p + {}^{10}Be$. This is just the reason that in the exchanged process the cross section of the reaction ${}^{11}B(\alpha,d_0){}^{13}C$ is more apparent than that of the reaction ${}^{11}B(\alpha,p_0){}^{14}C$. In addition, in the reaction ${}^{11}B(\alpha,p_0){}^{14}C$, the clear peak shape in intermediate angular parts accounted for the fact that considerable part of p_0 came from the formation mechanism of compound nucleus.

Regarding the calculations of two-nucleon or three-nucleons transferred reactions, the optical potential parameters were taken from [2,3] and some of the parameters were slightly adjusted to coincide with the experimental data. All parameters taken are shown in Tables 1 and 2. About absolute cross sections besides the spectroscopic factors of one-nucleon transferred reaction can take from the table of nuclear data, but for three-nucleons or two-nucleons transferred reactions the spectroscopic factors cannot be obtained from anywhere. Moreover, the calculated framework of the spectroscopic factors of multi-nucleons transferred reaction was not included in the present program. Thus the cross sections were normalized to the third peak of the angular distributions. The total normalization factors include the spectroscopic factors, so that the values are too large. They may be due to the following reasons:

(1) When the calculations of wave functions have not taken reasonable optical parameters of elastic scattering in appropriate energies, so there are effects on the calculation of wave function.

(2) As stated in this paper, during the calculation of direct reaction and exchanged reactions, the heavy replacement and general replacement mechanisms have not been considered due to the limit of the present program. These may have important effects on the absolute cross sections of α-particles induced reactions.

(3) The difference in the situations of peaks and valleys explain that others mechanisms should be taken into account, for example, the formation mechanism of compound nucleus should not be ignored.

All the above situations have some effects on the absolute cross sections, thus here the normalization factors including the spectroscopic factors and the effects of indetermination of optic potential parameters and complex mechanisms have not been taken into account.

CONCLUSIONS

The method of EFR-DWBA were used to analyze the α-particles induced reactions on nuclei B at 31.2 MeV, the semiquantitating results have been obtained: The calculated results coincided satisfactorily with the experimental data. Analysis showed for that in reactions $^{11,10}B(\alpha,t_{0,1})^{12,11}C$, $^{10}B(\alpha,d_{0,1})^{12}C_{0,1}$ and $^{10}B(\alpha,p_0)^{13}C$ the direct stripping mechanism is the main or even the sole mechanism. For the mechanism in the reaction $^{11}B(\alpha,d_0)^{13}C$, it is the result of incoherent sum of direct stripping and heavy particle stripping. In the reaction $^{11}B(\alpha,p_0)^{14}C$, the incoherent sum results of direct stripping and heavy particles stripping can better coincide with experimental data in the forward and backward angles. When only the mechanism of compound nucleus is reasonably considered, then the structure of the cross sections distribution peak in the angles of 80°-130° may be understood. Finally, for the reactions $^{10}B(\alpha,d_1)^{12}C^{*}$, $^{11}B(\alpha,d_0)^{13}C$ and $^{10}B(\alpha,p_0)^{13}C$, the coincidence between theoretical calculations and experimental data are very satisfactory.

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