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High Energy Unequal Nuclei Collision and Relaxation Time in a Relativistic Kinetic Equation

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In this paper, the space-time evolution of high energy heavy ion collision is described by the Relativistic Kinetic Equation. The rapidity distributions of the final state particles for 200A GeV ^{16}O and ^{32}S beams are analyzed in the central rapidity region. The relaxation times of various systems are determined.

Key words: high energy unequal nuclei collision, relativistic kinetic equation, relaxation time.

1. INTRODUCTION

In high energy heavy ion collisions, a huge amount of kinetic energy is transformed into thermal energy, and high temperature and high energy density matter is produced. At present, studying the space-time evolution of such hot dense matter is an important subject in high energy physics. The space-time evolution of equilibrium hot dense matter has been studied by many authors. In particular much attention has been paid to the investigation of the ideal relativistic hydrodynamic model. However, it is uncertain whether the produced hot dense matter would stay in a thermodynamical equilibrium state or not. In recent years, people tend to start the study from a system staying in non-equilibrium state [1]. We have described the relaxation process which happens before the system

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reaches the local thermodynamic equilibrium state by Fokker-Planck equation, and have also given the relaxation time for various systems.

Noting that a more general method to deal with the relaxation process is by kinetic equation, in this paper, the space-time evolution of high energy heavy ion collisions is described by the relativistic kinetic equation, and the rapidity distribution of final state particles near the central rapidity region for ^{16}O and ^{32}S beams at 200 A GeV is analyzed. The relaxation times of various systems are given.

2. RELATIVISTIC KINETIC EQUATION IN HIGH ENERGY UNEQUAL NUCLEI COLLISION

For the high energy unequal nuclei collision process we take the collision picture of [3]. An A − A′ collision is completed through two steps: (1) A′ nucleons of the nucleus A collide with the nucleons of nucleus A′ (i.e., an "A′ − A′" collision process). (2) After the "A′ − A′" collision is completed, the remaining nucleons of nucleus A collide with the secondary particles produced in "A′ − A′" collision.

The high temperature and high energy density matter produced in the central rapidity region is hadronic matter (pions). We try to describe the space-time evolution of this hadronic matter system by the relativistic kinetic equation. The relativistic kinetic equation is [4]:

$$p^\mu \partial_\mu f(x,p) + mF^\mu(x,p)\frac{\partial}{\partial p^\mu} f(x,p) = C(x,p), \tag{1}$$

where $f(x,p)$ is the distribution function of the hadrons, F^μ is the four-dimension force acting on the hadrons, m is the pion mass.

Taking the relaxation-time approximation, the collision term $C(x,p)$ can be expressed by [4]:

$$C(x,p) = -(f(x,p) - f_{eq}(x,p))/\tau', \tag{2}$$

and the relativistic kinetic equation becomes

$$p^\mu \partial_\mu f(x,p) + mF^\mu(x,p)\frac{\partial}{\partial p^\mu} f(x,p) = -(f(x,p) - f_{eq}(x,p))/\tau', \tag{3}$$

where τ' is the relaxation time of the hadronic gas, $f_{eq}(x,p)$ the hadron distribution function of the hadronic gas in local thermodynamical equilibrium. The form of $f_{eq}(x,p)$ is

$$f_{eq}(x,p) = 1/[\exp(\beta(x)p^\mu u_\mu(x)) - 1], \tag{4}$$

where $\beta(x) = T(x)^{-1}$, $T(x)$ is the temperature of the hadronic gas in local thermodynamics equilibrium, $u(x)$ the four-dimensional velocity of the pion.

It can be seen from Eq. (3) that the solution of the equation depends on the choice of temperature $T(x)$ in Eq. (4). $T(x)$ is originally the temperature of the system in local equilibrium. However, since in the relaxation-time approximation only the system near local equilibrium is discussed, one simply call $T(x)$ the temperature of the system in this case [5].

Taking account of the fact that the expansion of the high temperature and high energy density matter is mainly along the longitudinal direction i.e. along the beam direction, the equation can be considered in 1 + 1 dimension. In the relaxation-time approximation, one dimensional time-dependent kinetic equation can be obtained from (1) as

$$(m+\varepsilon)\frac{\partial f}{\partial t}+p\frac{\partial f}{\partial x}-m\frac{\partial V}{\partial x}\frac{\partial f}{\partial p}=-(f-f_{eq})/\tau'. \tag{5}$$

Here it is assumed that the interaction of other hadrons on a single hadron can be described by a mean field. In Eq. (5), V represents the potential energy in the mean field of the hadrons in the hadronic gas. As an example, this potential energy can be taken as the phenomenological Yukawa potential

$$V=-V_0\frac{\exp(x/x_N)}{x/x_N}, \tag{6}$$

where V_0 represents the depth of the potential well, x_N is its width.

The kinematic variables used currently in high energy collision phenomenology are the light cone variables (y,τ), the transformation from (t,x) and (ε,p) to (y,τ) is correspondingly

$$\left.\begin{aligned} t&=\tau\cosh y\\ x&=\tau\sinh y\end{aligned}\right\}, \tag{7}$$

$$\left.\begin{aligned}\frac{\partial}{\partial t}&=\cosh y\frac{\partial}{\partial\tau}-\frac{1}{\tau}\sinh y\frac{\partial}{\partial y}\\ \frac{\partial}{\partial x}&=-\sinh y\frac{\partial}{\partial\tau}+\frac{1}{\tau}\cosh y\frac{\partial}{\partial y}\end{aligned}\right\} \tag{8}$$

and

$$\left.\begin{aligned}\varepsilon&=m_T\cosh y\\ p&=m_T\sinh y\end{aligned}\right\}. \tag{9}$$

Due to the one-dimensional motion in consideration, the transverse momentum of the pion vanishes, so we have

$$\left.\begin{aligned} m_T&=(p_T^2+m_0^2)^{1/2}=m_0\\ \frac{\partial}{\partial\varepsilon}&=-\frac{1}{m_0}\sinh y\frac{\partial}{\partial y}\\ \frac{\partial}{\partial p}&=\frac{1}{m_0}\cosh y\frac{\partial}{\partial y}\end{aligned}\right\}, \tag{10}$$

hence

$$f_{eq}(y,\tau)=1\Big/\left[\exp\left(\frac{m_0}{T}\right)-1\right], \tag{11}$$

$$\begin{aligned}\frac{\partial V}{\partial x}&=\frac{V_0}{\tau\sinh y}\exp(\tau\sinh y/x_N)\left[\frac{x_N}{\tau}\sinh y(\mathrm{cth}^2 y-1)-1\right]\\ &=A(y,\tau).\end{aligned} \tag{12}$$

Substituting (6)-(12) into (5), we obtain the relativistic kinetic equation in light cone variables (y,τ)

$$\frac{\partial f(y,\tau)}{\partial\tau}=W_1\frac{\partial f(y,\tau)}{\partial y}+W_2(f(y,\tau)-f_{eq}(y,\tau)). \tag{13}$$

This equation is the time evolution equation of the distribution function $f(y,\tau)$ for pions. The coefficient

W_1 and W_2 is correspondingly

$$W_1 = \left(\frac{\cosh y \cdot \sinh y}{\tau} + \frac{A(y,\tau) \cdot \cosh^2 y}{m_0}\right) \Big/ (1 + \cosh^2 y) \tag{14}$$

and

$$W_2 = -\frac{1}{m_0 \tau'} \Big/ (1 + \cosh^2 y). \tag{15}$$

The solution of Eq. (13) can be obtained by the finite difference method of the partial differential equation.

Consider the first stage of the A − A′ collision, the time evolution equation of the distribution function $f_1(y,\tau)$ of pions is

$$\frac{\partial f_1(y,\tau)}{\partial \tau} = W_1 \frac{\partial f_1(y,\tau)}{\partial y} + W_2(f_1(y,\tau) - f_{eq}(y,\tau)). \tag{16}$$

If the nucleus-nucleus collision is attributed to nucleon-nucleon collisions, the initial condition for solving Eq. (16) may be taken as

$$f_{10}(y) = N\rho_{10}(y), \tag{17}$$

where $\rho_{10}(y)$ is the distribution function of the pions in a nucleon-nucleon collision at the corresponding incident energy [3]

$$\rho_{10}(y) = \int\left[n_C \frac{\mathrm{d}W_C}{\mathrm{d}y}(y) + n_p \frac{\mathrm{d}W_p}{\mathrm{d}y}(y - y_p) + n_T \frac{\mathrm{d}W_T}{\mathrm{d}y}(y - y_T)\right] \cdot \frac{p(n_C)p(n_p)p(n_T)}{p(n)} \delta(n - n_C - n_p - n_T)\mathrm{d}n_C \mathrm{d}n_p \mathrm{d}n_T, \tag{18}$$

N is the effective number of nucleons participating in the collision [3].

According to the relativity theory, using the light cone variables, the motion of particles is restricted inside the light cone, the equation of the light cone line is $x^{\pm} = 1$, the boundary condition for solving Eq. (16) can be taken as

$$\left.\begin{array}{l} f_1(x,p)|_{x^{\pm}=1} = 0 \\ \left.\dfrac{\partial f_1(x,p)}{\partial x}\right|_{x^{\pm}=1} = 0 \end{array}\right\}. \tag{19}$$

Consider the second stage of collision, the time evolution equation of the distribution function $f_2(y,\tau)$ of pions is

$$\frac{\partial f_2(y,\tau)}{\partial \tau} = W_1 \frac{\partial f_2(y,\tau)}{\partial y} + W_2(f_2(y,\tau) - f_{eq}(y,\tau)). \tag{20}$$

Since in this stage the hadron-nucleon collisions are in consideration, the initial condition for solving Eq. (20) is taken as

$$f_{20}(y) = n'\rho_{20}(y), \tag{21}$$

where n' is the number of effective nucleon participating in the collisions in this stage, $\rho_{20}(y)$ is the distribution function of pions given by the hadron-nucleon collision at the corresponding energy[3]

$$\rho_{20}(y) = \rho_0'\left(1 - \frac{\overline{m}_\pi \cosh(y + y_k/2)}{\overline{m}_N \cosh(y_k/2)}\right)^3. \tag{22}$$

The boundary condition for solving Eq. (20) can be taken as

$$\left.\begin{aligned} f_2(x,p)|_{x'^{\pm}=1} &= 0, \\ \left.\frac{\partial f_2(x,p)}{\partial x}\right|_{x'^{\pm}=1} &= 0 \end{aligned}\right\} \tag{23}$$

where [3]:

$$\left.\begin{aligned} x'^{+} &= t' + x' = (t - t_1) + (x - x_1) \\ x'^{-} &= t' - x' = (t - t_1) - (x - x_1) \end{aligned}\right\}. \tag{24}$$

and

$$\left.\begin{aligned} x_1 &= -(t_1 - d_t)\mathrm{th}y_B^* \\ d_t &= \frac{2R_A}{\sinh y_B^*}, \quad t_1 = \tau_{10}\cosh y \end{aligned}\right\}, \tag{25}$$

R_A is the radius of nucleus A, y_B^* is the rapidity of the nucleus in the equal velocity frame, τ_{10} is the time for the points in A′ − A′ collision to reach local equilibrium through thermalization.

If we consider simultaneously the contributions of the two collision process included in A − A′, then, after all the collisions are finished, the rapidity distribution function $F(y,\tau)$ of the final state particles is the convolution of $f_1(y,\tau)$ and $f_2(y,\tau)$, i.e.,

$$F(y,\tau) = \int_{y_1}^{y_2} C f_1(\mathscr{Y},\tau) f_2(y - \mathscr{Y},\tau)\, d\mathscr{Y}, \tag{26}$$

where C is the normalization factor.

3. CONCLUSIONS AND DISCUSSION

Using numerical method, starting from Eqs. (16), (20) and the conditions for fixing their solution, we could, in principle, using Eq. (26), find the solution of the high temperature and high density system in local equilibrium at any time after all the collisions are ended.

In Figs. 1 and 2 are shown the results of our calculation for ^{16}O + Ag/Br and ^{32}S + Ag/Br at beam energy 200A GeV. It can be seen from Eq. (3) that the solution of the equation depends on two parameters, i.e., the temperature T and relaxation time τ'. We need the solution of the system at the time when it fragments into observed particles, and pay attention to the fact that in phenomenological works, we usually take the fragmentation temperature of the heat density hadronic matter less than 120 MeV. In our work the choice of fragmentation temperature is restricted by this value. By comparing repeatedly the solution of Eqs. (16), (20) and (26) with experimental data, we finally get T and τ' self consistently.

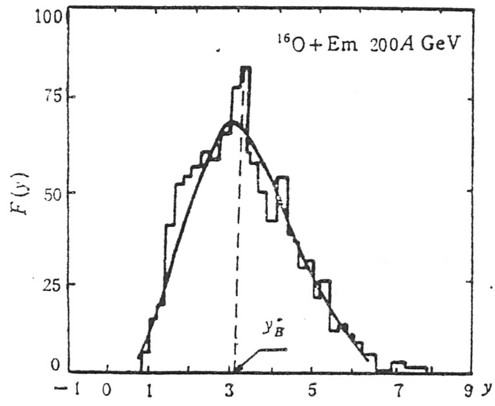

Fig. 1

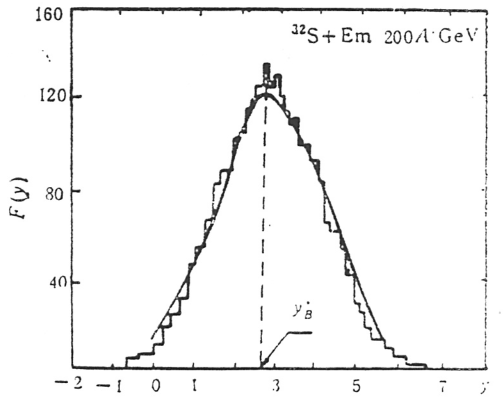

Fig. 2

Figure 1 gives the rapidity distribution of the final state particles in ^{16}O + Ag/Br, where the solid line is the result of our calculation at the time $\tau = 1.16$ fm for the system approaching its equilibrium with temperature $T = 96$ MeV and relaxation time $\tau' = 1.36$ fm. Figure 2 gives the result for ^{32}S + Ag/Br at $\tau = 1.18$ fm for temperature $T = 96$ MeV and $\tau' = 1.38$ fm. Evidently, the results of calculation for ^{16}O + Ag/Br and ^{32}S + Ag/Br indicate that the important parameter, relaxation time τ', depends on the temperature of the system, the incident beam energy and the colliding nucleon number. At a given temperature of the system and incident beam energy, the larger the mass number of the colliding nuclei, the longer the relaxation process. Naturally, this needs to be tested by more experimental data.

Let us note that we have not analyzed how the final state particles freeze out from the system in the above discussion. This is a question which has not been clarified in various model calculations at present.

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