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Transient Beam Loading Effect in Constant Gradient Accelerator Structure

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In this paper the general solution of the transient beam loading problem for a constant gradient travelling wave accelerator structure (SLAC-type structure) is obtained starting with the energy conservation law. In particular, for the thermionic microwave gun serving as a injector where the beam pulse current increases with time, the beam loading problem is treated firstly. Then, the solution is used to analyze the accelerator of Beijing Free Electron Laser and the result shows the beam energy decrease with the increase of the pulse current.

Key words: linac, beam loading effect, constant gradient accelerator structure.

In linear accelerators, the distribution of rf field strength changes with time along the accelerating sections due to the interaction between electrons and rf field, so that the latter electrons passing through the accelerating section will obtain smaller energy than former electrons. That is usually called beam loading effect [1]. Most linacs operate in pulse mode; once the rf power is turned on, the rf field increases timely in every section until it reaches its steady state only after one filling time t_F. For the same reason, after the injection of electron beam, beam induced field also needs one filling time t_F to reach its steady state. The electron beam is acted on by both rf field and beam induced field. The transient beam loading effect is defined as beam energy gain variation when above fields are in their setting up. In particular, for the accelerator using electron beam from thermionic cathode rf gun where

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the current increases with time, the beam induced field, as well as the beam energy varies all time, cannot keep stable. In the following, we will start with the energy conservation law to find the solution of the transient beam loading problem for a constant gradient section (SLAC type). Then, the solution for the time-increased injection beam will be given. Finally, the solution will be used to analyze the accelerator of Beijing Free Electron Laser (BFEL).

1. GENERAL SOLUTION OF TRANSIENT BEAM LOADING EFFECT [2]

In the presence of an electron beam in a accelerating waveguide, the rf power lose per unit length is given with

$$\frac{\mathrm{d}P}{\mathrm{d}z} = \left(\frac{\mathrm{d}P}{\mathrm{d}z}\right)_{\mathrm{wall}} + \left(\frac{\mathrm{d}P}{\mathrm{d}z}\right)_{\mathrm{beam}} = -2\alpha(z)P(z,t) - I(t)E(z,t), \tag{1}$$

where the first term $(\mathrm{d}P/\mathrm{d}z)_{\mathrm{wall}}$ is the power dissipated in the accelerating section walls; $\alpha(z)$ is the attenuation function of the section, for the constant gradient section, $\alpha(z)$ is a slowly varying function along the section,

$$\alpha(z) = \frac{\alpha_0}{(1-2\alpha_0 z)}. \tag{2}$$

The attenuation constant for the entire section of length L is defined as

$$\tau = \int_0^l \alpha(z)\mathrm{d}z. \tag{3}$$

The second term $(\mathrm{d}P/\mathrm{d}z)_{\mathrm{beam}}$ in Eq. (1) is the power absorbed by the beam, $I(t)$ is beam current. $E(z,t)$ is the amplitude of rf electric field along the axis, for the constant gradient section,

$$P(z,t) = \frac{E^2(z,t)}{2\alpha(z)R_{\mathrm{m}}}, \tag{4}$$

where R_{m} is shunt impedance of the section. By differentiating $P(z,t)$ with respect to z, one obtains

$$\frac{\mathrm{d}P(z,t)}{\mathrm{d}z} = \frac{\partial P(z,t)}{\partial z} + \frac{\partial P(z,t)}{\partial t}\frac{\mathrm{d}t}{\mathrm{d}z}, \tag{5}$$

$$\frac{\mathrm{d}t}{\mathrm{d}z} = \frac{1}{v_{\mathrm{g}}(z)} = \frac{2Q}{\omega}\alpha(z), \tag{6}$$

where, v_{g} is group velocity, ω is circular frequency, Q is unloaded quality factor. Thus Eq. (1) becomes

$$\frac{\partial P(z,t)}{\partial z} + \frac{1}{v_{\mathrm{g}}(z)}\frac{\partial P(z,t)}{\partial t} = -2\alpha(z)P(z,t) - I(t)E(z,t). \tag{7}$$

By substituting Eq. (4) into Eq. (7), one obtains

$$\frac{\partial E(z,t)}{\partial z} + \frac{1}{v_{\mathrm{g}}(z)}\frac{\partial E(z,t)}{\partial t} = -\alpha(z)R_{\mathrm{m}}I(t). \tag{8}$$

We can solve Eq. (8) by performing the Laplace transformation with respect to time [2],

$$\frac{\partial E(z,s)}{\partial z} + \frac{s}{v_g(z)} E(z,s) = -\alpha(z) R_m I(s). \tag{9}$$

The general solution of Eq. (9) is [3]

$$E(z,s) = e^{-\int_0^z p\,dz}\left(-\int_0^z q e^{\int_0^z p\,dz} dz + c\right),$$
$$p = \frac{s}{v_g(z)},$$
$$q = -\alpha(z) R_m I(s). \tag{10}$$

By integrating Eq. (10) and using the initial condition $E(0, s)$ at $z = 0$, we obtain

$$E(z,s) = E(0,s)e^{-t_z s} - \frac{\omega R_m I(s)}{2Qs}(1 - e^{-t_z s}), \tag{11}$$

where t_z is the interval during which the rf power propagates from 0 to z:

$$t_z = \int_0^z \frac{dz}{v_g(z)} = -\frac{Q}{\omega} \ln[1 - (1 - e^{-2\tau})z/L]. \tag{12}$$

The energy gain of the synchronous electron ($v \approx c$) passing through the section is given by integrating Eq. (11):

$$U(s) = \int_0^L E(z,s)dz. \tag{13}$$

From Eq. (12) we have

$$z = \frac{L}{1 - e^{-2\tau}}(1 - e^{-t_z\omega/Q}),$$

$$dz = \frac{\omega L}{(1 - e^{-2\tau})Q} e^{-t_z\omega/Q} dt_z. \tag{14}$$

Substituting Eq. (14) into Eq. (13) and integrating for the right side, we obtain

$$U(s) = E(0,s)\frac{\omega L}{(1 - e^{-2\tau})(s + \omega/Q)Q}[1 - e^{-t_F(s+\omega/Q)}]$$
$$- R_m I(s)\frac{\omega L}{2sQ(1 - e^{-2\tau})}[(1 - e^{-t_F\omega/Q})$$
$$- \frac{\omega}{Q(s + \omega/Q)}(1 - e^{-t_F(s+\omega/Q)})], \tag{15}$$

where t_F is the filling time of the section:

$$t_F = t_z(L) = 2\tau \frac{Q}{\omega}. \tag{16}$$

For simplicity, we assume that the rf pulse is an ideal step-function without dispersive effects, and that all electrons travel with the speed of light, thus $E(0,t)$ and $I(t)$ are step functions as follows:

$$\begin{aligned} E(0,t) &= E_0 H(t); \\ I(t) &= I_0 H(t - t_0), \end{aligned} \tag{17}$$

where t_0 is the time when the beam is injected, I_0 is the average current, E_0 is the amplitude of the electric field at $z = 0$, for the constant gradient section,

$$E_0 = (1 - e^{-2\tau})^{1/2} \left(\frac{P_0 R_m}{L} \right)^{1/2}; \tag{18}$$

$H(t)$ is the unit step function. Performing the Laplace transformation on Eq. (17), we obtain

$$\begin{aligned} E(0,s) &= \frac{E_0}{s}; \\ I(s) &= \frac{I_0}{s} e^{-st_0}. \end{aligned} \tag{19}$$

Substituting Eq. (19) into Eq. (18), one obtains

$$\begin{aligned} U(s) = {} & E_0 \frac{\omega L}{(1 - e^{-2\tau})(s + \omega/Q)sQ} (1 - e^{-2\tau} e^{-st_F}) \\ & - R_m I_0 \frac{\omega L e^{-st_0}}{2s^2 Q(1 - e^{-2\tau})} [(1 - e^{-2\tau}) \\ & - \frac{\omega}{Q(s + \omega/Q)} (1 - e^{-2\tau} e^{-st_F})]. \end{aligned} \tag{20}$$

By performing the inverse Laplace transformation, we obtain the beam energy gain as a function of time,

$$\begin{aligned} U(t) = {} & \frac{E_0 L}{(1 - e^{-2\tau})} [(1 - e^{-t\omega/Q}) H(t) - e^{-2\tau}(1 - e^{-(t-t_F)\omega/Q})] H(t - t_F)] \\ & - \frac{R_m I_0 L}{2(1 - e^{-2\tau})} [(1 - e^{-(t-t_0)\omega/Q}) - \frac{\omega}{Q} e^{-2\tau}(t - t_0)] H(t - t_0) \\ & - \frac{R_m I_0 L}{2(1 - e^{-2\tau})} e^{-2\tau} \Big[(-1 + e^{-(t-t_0-t_F)\omega/Q}) \\ & + \frac{\omega}{Q} e^{-2\tau}(t - t_0 - t_F) \Big] H(t - t_0 - t_F), \end{aligned} \tag{21}$$

where the first term is the contribution of the rf field, the second and the third terms come from the beam induced field. The energy gain does not keep stable until $(t - t_0) > t_F$ when rf field and beam

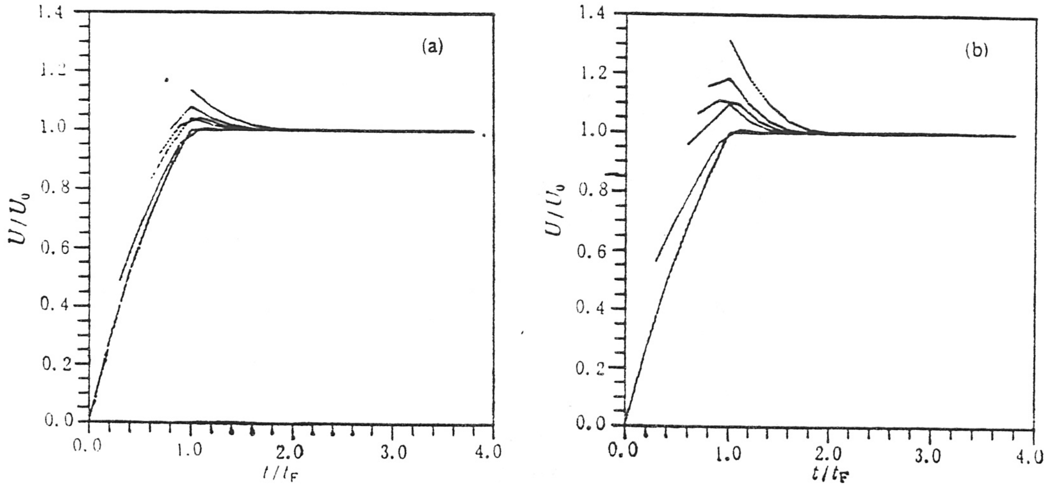

Fig. 1
Transient beam-loading curves for
the different beam currents.

induced field are both in steady state:

$$U_0 = E_0 L - \frac{R_m I_0 L}{2}\left(1 - \frac{2\tau e^{-2\tau}}{1 - e^{-2\tau}}\right). \tag{22}$$

It can be seen from Eq. (22) that the energy gain at the steady state decreases linearly with injected beam current.

2. BEAM LOADING CALCULATION FOR BFEL ACCELERATOR

Having obtained the general solution of transient beam loading effect, let us now apply it to an example: BFEL accelerator. The BFEL accelerator is equipped with a SLAC type constant gradient accelerating section [4], its main parameters are [5]: $f = 2856$ MHz, $Q = 13000$, $\tau = 0.57$, $R_m = 60$ MΩ/m, $L = 3.05$ m and $t_F = 0.83$ μs.

The injector of BFEL us an microwave electron gun [6] with its output beam energy is 1.1 MeV. The injected electrons are of the velocity near to c. Although there is some phase slippage in the beginning of the accelerator, the electrons still meet the requirement of Eq. (13). The design beam current of BFEL is 100/200 mA. Corresponding to different time of injection, the energy gain of electrons, (U/U_0), at the different position of a beam pulse is shown in Fig. 1. Typically, $P_0 = 10$ MW, the curves correspond to the various injection times of $t/t_F = 0$, 0.3, 0.6, 0.7, 0.8 and 1.0, respectively. Figure 1(a) is the case of 100 mA. We see that there is an optimum injection time (t_{opt}) the energy spread caused by beam loading effect can be compensated to be the smallest. For 100 mA, $t_{opt} = 0.7\ t_F$. Figure 1(b) is the case of 200 mA where $t_{opt} = 0.6\ t_F$. For multi-section accelerators, beam loading induced energy spread could be reduced further by adjusting the beam injection time of energy section.

Figure 2 shows the beam loading curves of the BFEL accelerator, where U/U_0 is so called steady-state energy gain. Those curves are used for adjusting output energy of the accelerator.

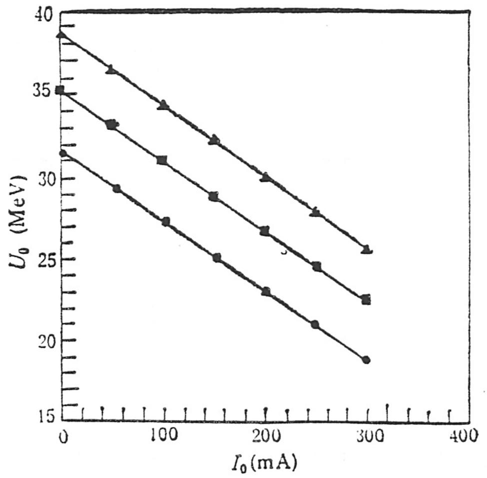

Fig. 2
Beam-loading curve of the BFEL accelerator.
• P_0 = 8 MW; ■ P_0 = 10 MW; ▲ P_0 = 12 MW.

3. GENERAL SOLUTION OF BEAM LOADING FOR TIME-INCREASED CURRENT

For most of electron accelerators, the injectors are usually diode or triode guns and bunchers. After the buncher, the pulse current can be represented with Eq. (17). In BFEL, a thermionic cathode microwave electron gun is chosen as high brightness injector. Its output current increases within the pulse as a result of the back-bombardment effect [7] and can be written as follows,

$$\begin{aligned} I(t) &= I_0[1 + k(t - t_0)]H(t - t_0) \\ &= I_0 H(t - t_0) + I_0 k(t - t_0)H(t - t_0), \end{aligned} \tag{23}$$

where k is current increase rate in unit of $\%/\mu$s. Let us consider only the added term. The Laplace transformation of the second term is

$$\Delta I(s) = \frac{I_0 k}{s^2} e^{-st_0}, \tag{24}$$

Substituting Eq. (24) into Eq. (15), we have

$$\Delta U(s) = -R_m I_0 k \frac{\omega L e^{-st_0}}{2s^3 Q(1 - e^{-2\tau})}\left[(1 - e^{-2\tau}) - \frac{\omega}{Q(s + \omega/Q)}(1 - e^{-2\tau}e^{-st_F})\right], \tag{25}$$

The inverse Laplace transformation of Eq. (25) is

$$\begin{aligned} \Delta U(t) = &- \frac{R_m I_0 k L}{2(1 - e^{-2\tau})}\left[-\frac{Q}{\omega}(1 - e^{-(t-t_0)\omega/Q}) \right. \\ &\left. + (t - t_0) - \frac{\omega}{2Q} e^{-2\tau}(t - t_0)^2\right] H(t - t_0) \end{aligned}$$

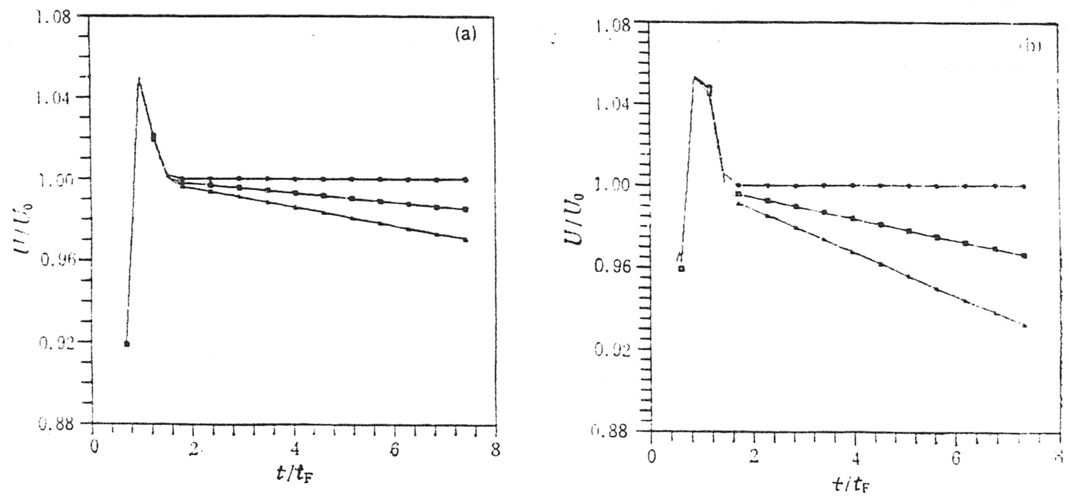

Fig. 3

Transient beam-loading curves for increased currents.
• $k = 0$; ■ $k = 2\%/\mu s$; ▲ $k = 4\%/\mu s$.

$$- \frac{R_m I_0 k L}{2(1 - e^{-2\tau})} e^{-2\tau} \left[\frac{Q}{\omega} (1 - e^{-(t - t_0 - t_F)\omega / Q}) - (t - t_0 - t_F) + \frac{\omega}{2Q} e^{-2\tau} (t - t_0 - t_F)^2 \right] H(t - t_0 - t_F). \quad (26)$$

Equation (26) describes the energy variation due to the increase of the current within a pulse. This solution will be applied to BFEL accelerator. Since the pulse length (4-6μs) is much longer than filling time, we pay more attention to the case after the accelerator is entirely filled up by rf power. For a typical current increment, the ramp of energy gain is drawn in Fig. 3 with the initial current of 100 mA and 200 mA respectively. It is found that the longer the pulse is, the more the energy goes down. The time-increased beam is generally not suitable for accelerators due to beam loading. This is one of the main reasons why the thermionic cathode microwave gun is confined to operating at narrow macropulse.

We must point out that the time-increased current induced energy variation discussed here is considered only in the accelerating section. The actual situation is more complicated since there is also beam loading effect in the cavity of the microwave gun. The electrons energies are varied with time in the macropulse, which means the injection beam energy is not the same within the macropulse, and the variation of energy gain is different from Eq. (26). Fortunately, the difference is much smaller than the result of Eq. (26), as the microwave gun consists of only one cavity, but whole accelerating section of 86 cavities.

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