High Energy Physics and Nuclear Physics

Volume 19, Number 1

# Study of the Boson Resonance Produced in the Three-Step Two-Body Decay Process of J/ψ

Shen Qixing, Yu Hong, and Zhang Lin

(Institute of High Energy Physics, The Chinese Academy of Sciences, Beijing, China)

**The three-step two-body decay process of J/ψ, $J/\psi \to V + X$, $X \to P_1 + Y$, $Y \to P_2 + P_3$ is discussed by using the generalized moment analysis method. The spin, parity of the boson resonance X and the ratios of helicity amplitudes of the process can be determined in terms of the measurement of the corresponding moments except for very special cases.**

**Key words: generalized moment analysis, spin, parity, helicity amplitude.**

## 1. INTRODUCTION

The existence of the glueballs (pure gluons bound states), hybrids (bound states which include quarks and gluons, such as, $q\bar{q}g$ or qqqg), and four quark states is an important prediction of the quantum chromodynamics (QCD) theory [1]. Potential model, bag model, and lattice gauge theory expect that the mass of these new hadronic states may lie in the range of 1 to 2.5 GeV [2].

The J/ψ decay process is considered to be an ideal place to search for these new hadronic states. The reasons are as follows: (1) The expected mass of the new hadronic states are in the range of 1 to 2.5 GeV. Hence they may be produced in the J/ψ decay process. (2) The perturbative QCD predicts that the branching ratio of the J/ψ radiative decay processes is about 7% [2]. Most of these final-state hadrons produced in company with photons are the hadronic states with spin-parity $J^{PC} = 2^{++}$, $0^{-+}$

Received on February 15, 1994. Supported by the National Natural Science Foundation of China and the Grant LWTZ-1298 of the Chinese Academy of Sciences.

and $0^{++}$ [3] which are just the low energy states of glueballs. (3) The QCD also predicts that the branching ratio of the $J/\psi$ hadronic decay processes via the three-gluons decay is about 66% [2], and the lowest order Feymann diagram of these processes is the $J/\psi$ decay into a hybrid and a usual meson. (4) For the $J/\psi$ particle produced in $e^+e^-$ collisions, its energy and momentum are fixed, $(E,p) = (m_J,0)$ in the center-of-mass system of $e^+e^-$; The polarization of $J/\psi$ is also fixed, i.e., the polarization state with helicity 0 has no contribution and the states with helicity $\pm 1$ have the same probability. In addition, $J/\psi$ is a $c\bar{c}$ bound state, namely it is a so-called $SU(3)$ flavor singlet. All of these properties are favorable for the determination of the characteristics of the final-state hadrons produced in the $J/\psi$ decay processes.

In the last ten years, Mark II, Crystal Ball, Mark III, DM2, and BES have done a series of work to search for and identify new hadronic states in the $J/\psi$ decay processes, respectively. At least three candidates of the new hadronic states have been proposed up to now. They are $\iota/\eta(1440)$ discovered by Mark II [4]; $\Theta/f_2(1720)/f_0(1710)$ discovered by Crystal Ball [5]; and $\xi(2230)$ discovered by Mark III [6]. But many characteristics of these new hadronic states have not been fixed yet. For example, near the mass region of the $\iota/\eta(1440)$ there are at least three resonances, not just one resonance $\iota/\eta(1440)$ [7]; It still need to be tested by further experiments that the $\Theta(1720)$ has spin 0 or 2, or it is a mixed structure of $2^{++}$ and $0^{++}$; The existence of the $\xi(2230)$ has been confirmed by BES from three channels, $J/\psi \to \gamma K\bar{K}$, $\gamma\pi\pi$, $\gamma p\bar{p}$, but it is difficult to determine whether the spin of the $\xi(2230)$ is 2 or 4 by using the helicity amplitude analysis method from the 9 million $J/\psi$ events.

As pointed out in [8], the systematic studies of the three-step two-body processes of $J/\psi$

$$e^+ + e^- \to J/\psi \to \gamma + X, X \to P_1 + Y, Y \to P_2 + P_3, \tag{1}$$

and

$$e^+ + e^- \to J/\psi \to V + X, X \to P_1 + Y, Y \to P_2 + P_3, \tag{2}$$

(where V is vector meson, $P_i$ represents the pseudoscalar meson) provide a way to determine the properties of the boson resonance X, which is useful for the study of the new hadronic states. In [8], above processes have been discussed by using the angular distribution analysis method. The result shows that the spin, parity, and the ratios of the helicity amplitude of the boson resonance X can be determined by fitting the experimental angular distribution, provided that Y is a known particle.

Some experimental analyses show that the generalized moment analysis method is more effective than the angular distribution analysis method for the determination of the characteristics of the resonance [9], especially if the statistics is low and contributions from different spin states overlap [10]. Therefore, processes (1) and (2) are discussed by using the generalized moment analysis method in this paper. The results show that the spin and parity of the boson resonance X can be distinguished conveniently in terms of the measurement of the moments except for a very special case. In addition, since all the helicity amplitude ratios can be expressed by some moments independently, their measurements are simpler and are more useful for reducing the measurement error.

## 2. DERIVATION OF FORMULAS AND DISCUSSIONS

We suppose that the particle Y in process (2) is usual meson. Because Y decades into two pseudoscalar mesons, its spin-parity $J_Y^{P_Y C_Y}$ can only be $0^{++}$, $1^{--}$, $2^{++}$. Due to limited space we only discuss two common cases, $J_Y^{P_Y} = 0^+$ and $1^-$, in this paper.

We take the c.m. system of $e^+e^-$ and choose the direction of the incident positron as the $z$-axis and the vector meson V lying in the $x$-$z$ plane. In this frame we have the matrix elements as follows:

$$\langle V_{\lambda_V} X_{\lambda_X} | T_1 | \psi_{\lambda_J} \rangle \sim A_{\lambda_V,\lambda_X} D^{1*}_{\lambda_J,\lambda_V-\lambda_X}(0,\theta_V,0),$$

$$\langle P_1 Y_{\lambda_Y} | T_2 | X_{\lambda_X} \rangle \sim B^{J_X}_{\lambda_Y,0} D^{J_X *}_{\lambda_X,\lambda_Y}(\phi_1, \theta_1, -\phi_1),$$

$$\langle P_2 P_3 | T_3 | Y_{\lambda_Y} \rangle \sim D^{J_Y *}_{\lambda_Y,0}(\phi_2, \theta_2, -\phi_2), \tag{3}$$

and

$$I_{\lambda_J,\lambda_J'}(\theta_V) \sim \frac{1}{4} \sum_{r,r'} \langle \psi_{\lambda_J} | T | e^-_r e^+_{r'} \rangle \langle \psi_{\lambda_J'} | T | e^-_r e^+_{r'} \rangle^* \sim 2 |p_-|^2 \delta_{\lambda_J,\lambda_J'} \delta_{\lambda_J,\pm 1}. \tag{4}$$

where $\Theta_V$ is the angle between the moving direction of the vector meson V and the incident positron. The meaning of other symbols has been given in [8]. Therefore, in the c.m. system of $e^+e^-$ the angular distribution of process (2) can been written as

$$W(\theta_V, \theta_1, \phi_1, \theta_2, \phi_2) \sim \sum_{\lambda_J = \pm 1, \lambda_V, \lambda_X, \lambda_X', \lambda_Y, \lambda_Y'} A_{\lambda_V,\lambda_X} A^*_{\lambda_V,\lambda_X'} B^{J_X}_{\lambda_Y,0}$$
$$\cdot B^{J_X^*}_{\lambda_Y' 0} D^{1*}_{\lambda_J,\lambda_V-\lambda_X}(0, \theta_V, 0) D^{1}_{\lambda_J,\lambda_V-\lambda_X'}(0, \theta_V, 0) D^{J_X^*}_{\lambda_X \lambda_Y}(\phi_1, \theta_1, -\phi_1)$$
$$\cdot D^{J_X}_{\lambda_X',\lambda_Y'}(\phi_1, \theta_1, -\phi_1) D^{J_X^*}_{\lambda_Y,0}(\phi_2, \theta_2, -\phi_2) D^{J_Y}_{\lambda_Y',0}(\phi_2, \theta_2, -\phi_2). \tag{5}$$

We now define the moments

$$M(jlmin) = \int d\theta_V \sin\theta_V d\phi_1 d\theta_1 \sin\theta_1 d\phi_2 d\theta_2 \sin\theta_2 D^j_{0,-m}(0, \theta_V, 0)$$
$$\cdot D^l_{m,n}(\phi_1, \theta_1, -\phi_1) D^i_{n,0}(\phi_2, \theta_2, -\phi_2) W(\theta_V, \theta_1, \phi_1, \theta_2, \phi_2). \tag{6}$$

After integrating all the angles we have

$$M(jlmin) \sim \sum_{\lambda_J = \pm 1, \lambda_V, \lambda_X, \lambda_X', \lambda_Y, \lambda_Y'} A_{\lambda_V,\lambda_X} A^*_{\lambda_V,\lambda_X'} B^{J_X}_{\lambda_Y,0} B^{J_X^*}_{\lambda_Y',0}$$
$$\cdot \langle 1\lambda_J j 0 | 1 \lambda_J \rangle \langle 1 \lambda_V - \lambda_X' j - m | 1 \lambda_V - \lambda_X \rangle \langle J_X \lambda_X' l m | J_X \lambda_X \rangle$$
$$\cdot \langle J_X \lambda_Y' l n | J_X \lambda_Y \rangle \langle J_Y \lambda_Y' i n | J_Y \lambda_Y \rangle \langle J_Y 0 i 0 | J_Y 0 \rangle. \tag{7}$$

where $\langle j_1 m_1 j_2 m_2 | jm \rangle$ is the Clebsch-Gordan coefficient.

In the following we will discuss the two cases of $J_Y^{P_Y} = 0^+$ and $1^-$, respectively.

### 2.1. $J_Y^{P_Y} = 0^+$

In this case the parity conservation gives the following relations:

$$A_{\lambda_V,\lambda_X} = P_X (-1)^{J_X} A_{-\lambda_V,-\lambda_X}$$
$$B^{J_X}_{0,0} = -P_X (-1)^{J_X} B^{J_X}_{0,0}. \tag{8}$$

where $\lambda_V = 0, \pm 1$, $\lambda_X = 0, \pm 1, \cdots, \pm J_X$. Hence process (2) is forbidden for the resonance X with $J_X^{P_X} = 0^+$, $1^-$ or $2^+$. The possible lowest states are those with $J_X^{P_X} = 0^-$, $1^-$ and $2^-$.

### 2.1.1. $J_X^{P_X} = 0^-$

In this case there are two non-zero moments:

$$M(00000) \sim 4|A_{1,0}|^2|B_{0,0}^0|^2,$$
$$M(20000) \sim \frac{2}{5}|A_{1,0}|^2|B_{0,0}^0|^2. \quad (9)$$

### 2.1.2. $J_X^{P_X} = 1^+$

There are three independent helicity amplitudes: $A_{1,0}$, $A_{1,1}$ and $A_{0,1}$ for the process $J/\psi \to V + X$ and there is only one helicity amplitude $B_{0,0}^1$ for the process $X \to P_1 + Y$ according to Eq.(8). Therefore, we have the following moments:

$$M(00000) \sim 4(|A_{1,1}|^2 + |A_{1,0}|^2 + |A_{0,1}|^2)|B_{0,0}^1|^2,$$
$$M(02000) \sim -\frac{4}{5}(|A_{1,1}|^2 - 2|A_{1,0}|^2 + |A_{0,1}|^2)|B_{0,0}^1|^2,$$
$$M(20000) \sim \frac{2}{5}(-2|A_{1,1}|^2 + |A_{1,0}|^2 + |A_{0,1}|^2)|B_{0,0}^1|^2,$$
$$M(22000) \sim \frac{2}{25}(2|A_{1,1}|^2 + 2|A_{1,0}|^2 - |A_{0,1}|^2)|B_{0,0}^1|^2,$$
$$M(22\pm100) \sim \frac{6}{25}\mathrm{Re}(A_{1,1}A_{1,0}^*)|B_{0,0}^1|^2,$$
$$M(22\pm200) \sim \frac{6}{25}|A_{0,1}|^2|B_{0,0}^1|^2. \quad (10)$$

### 2.1.3. $J_X^{P_X} = 2^-$

When $J_X = 2$ there are four independent helicity amplitudes: $A_{1,0}$, $A_{1,1}$, $A_{1,2}$ and $A_{0,1}$ for the process $J/\psi \to V + X$. From Eq.(7) we obtain the following moments:

$$M(00000) \sim 4(|A_{1,2}|^2 + |A_{1,1}|^2 + |A_{1,0}|^2 + |A_{0,1}|^2)|B_{0,0}^2|^2,$$
$$M(02000) \sim -\frac{4}{7}(2|A_{1,2}|^2 - |A_{1,1}|^2 - 2|A_{1,0}|^2 - |A_{0,1}|^2)|B_{0,0}^2|^2,$$
$$M(20000) \sim \frac{2}{5}(|A_{1,2}|^2 - 2|A_{1,1}|^2 + |A_{1,0}|^2 + |A_{0,1}|^2)|B_{0,0}^2|^2,$$
$$M(22000) \sim -\frac{2}{35}(2|A_{1,2}|^2 + 2|A_{1,1}|^2 - 2|A_{1,0}|^2 - |A_{0,1}|^2)|B_{0,0}^2|^2,$$
$$M(22\pm100) \sim \frac{2\sqrt{3}}{35}\mathrm{Re}(A_{1,1}A_{1,0}^* - \sqrt{6}A_{1,2}A_{1,1}^*)|B_{0,0}^2|^2,$$
$$M(22\pm200) \sim \frac{2}{35}[3|A_{0,1}|^2 - 2\sqrt{6}\mathrm{Re}(A_{1,2}A_{1,0}^*)]|B_{0,0}^2|^2,$$
$$M(24000) \sim \frac{2}{105}(|A_{1,2}|^2 + 8|A_{1,1}|^2 + 6|A_{1,0}|^2 - 4|A_{0,1}|^2)|B_{0,0}^2|^2,$$

$$M(24\pm100) \sim \frac{2}{21}\sqrt{\frac{3}{5}}\,\mathrm{Re}(\sqrt{6}\,A_{1,1}A^{*}_{1,0} + A_{1,2}A^{*}_{1,1})\,|B^{2}_{0,0}|^{2},$$

$$M(24\pm200) \sim \frac{4}{21}\sqrt{\frac{3}{5}}\left[\,|A_{0,1}|^{2} + \sqrt{\frac{3}{2}}\,\mathrm{Re}(A_{1,2}A^{*}_{1,0})\right]|B^{2}_{0,0}|^{2}. \tag{11}$$

Therefore, when $J_Y^{P_Y} = 0^+$, we can easily determine the spin and parity $J_X^{P_X}$ of the boson resonance X. The reason is that for $J_X^{P_X} = 0^-$, all of the moments with $l = 2$ and 4 are equal to zero, and there is a simple relation:

$$M(00000) = 10M(20000). \tag{12}$$

However, these properties cannot be satisfied simultaneously for $J_X^{P_X} = 1^+$ and $2^-$. Therefore, through the measurement of moments, the boson resonance X with $J_X^{P_X} = 0^-$ can be distinguished from the resonances with $J_X^{P_X} = 1^+$ and $J_X^{P_X} = 2^-$. In order to distinguish the state of $J_X^{P_X} = 1^+$ from that of $J_X^{P_X} = 2^-$ we measure the moments with $l = 4$ first. This resonance must be the one with $J_X^{P_X} = 2^-$ if there is one nonzero moment with $l = 4$; If all of the moments with $l = 4$ are equal to zero and $A_{1,1} \neq 0$, one can see that the spin-parity of X is $1^+$ or $2^-$ from one of the following two relations:

$$\frac{M(00000) - 7M(02000) - 10M(20000) + 70M(22000)}{M(00000) - 10M(20000)} = \begin{cases} 0 & \text{for } 2^- \\ \frac{12}{5} & \text{for } 1^+ \end{cases} \tag{13}$$

$$\frac{M(00000) + 5M(02000) - 10M(20000) - 50M(22000)}{7M(02000) - 70M(22000)} = \begin{cases} \frac{12}{7} & \text{for } 2^- \\ 0 & \text{for } 1^+ \end{cases} \tag{14}$$

If $A_{1,1} = 0$ but $2|A_{1,0}|^2 \neq |A_{0,1}|^2$, we can still distinguish the resonance of $J_X^{P_X} = 1^+$ from the resonance of $J_X^{P_X} = 2^-$ by using the relation:

$$\frac{2M(00000) + 5M(02000) - 50M(22200)}{M(22000)} = \begin{cases} 0 & \text{for } 2^- \\ 100 & \text{for } 1^+ \end{cases} \tag{15}$$

We cannot distinguish the state of $J_X^{P_X} = 1^+$ from the state of $J_X^{P_X} = 2^-$ only is a very special case that all of the moments of $l = 4$ are equal to zero, $A_{1,1} = 0$, and $2|A_{1,0}|^2 = |A_{0,1}|^2$.

We now define the following independent helicity amplitude ratios $x$, $y$, $z_1$, $z_2$, $\xi_J$ and $\eta_J$ ($J = 1, 2$):

$$xe^{i\phi_x} = \frac{A_{1,1}}{A_{1,0}},\quad ye^{i\phi_y} = \frac{A_{1,2}}{A_{1,0}},\quad z_1e^{i\phi_{z_1}} = \frac{A_{0,0}}{A_{1,0}},$$

$$z_2e^{i\phi_{z_2}} = \frac{A_{0,1}}{A_{1,0}},\quad \xi_Je^{i\phi_J} = \frac{B^J_{0,0}}{B^J_{1,0}},\quad \eta_Je^{i\phi'_J} = \frac{B^J_{2,0}}{B^J_{1,0}}. \tag{16}$$

After determining the spin-parity of the resonance X, one can determine the helicity amplitude ratios from the following relations.

For $J_X^{P_X} = 1^+$, one finds

$$x^2 = \frac{M(00000) - 10M(20000)}{M(00000) + 5M(02000)},$$

$$z_2^2 = \frac{50M(22200)}{M(00000) + 5M(02000)},$$

$$x\cos\phi_x = \frac{50M(22100)}{M(00000) + 5M(02000)}. \tag{17}$$

For $J_X^{P_X} = 2^-$, we have

$$x^2 = \frac{245}{3T}[M(02000) - 10M(22000)],$$

$$y^2 = \frac{14}{T}[M(00000) - 7M(02000) + 15M(24000) + 20M(22000)],$$

$$z_2^2 = \frac{70}{T}[5M(22200) + 2\sqrt{15}M(24200)],$$

$$x\cos\phi_x = \frac{70}{3T}[5\sqrt{3}M(22100) + 9\sqrt{10}M(24100)],$$

$$xy\cos(\phi_y - \phi_x) = \frac{70}{T}[\sqrt{15}M(24100) - 5\sqrt{2}M(22100)]. \tag{18}$$

where

$$T = 7M(00000) - 49M(02000) + 630M(24000) + 840M(22000). \tag{19}$$

### 2.2. $J_Y^{P_Y} = 1^-$

In this case, the parity conservation gives the following relations:

$$A_{\lambda_V,\lambda_X} = P_X(-1)^{J_X}A_{-\lambda_V,-\lambda_X}$$

$$B^{J_X}_{\lambda_Y,0} = -P_X(-1)^{J_X}B^{J_X}_{-\lambda_Y,0}. \tag{20}$$

Therefore, only process (2) with $J_X^{P_X} = 0^+$ is forbidden.

#### 2.2.1. $J_X^{P_X} = 0^-$

In this case we have four moments:

$$M(00000) \sim 4|A_{1,0}|^2|B^0_{0,0}|^2,$$

$$M(00020) \sim \frac{8}{5}|A_{1,0}|^2|B^0_{0,0}|^2,$$

$$M(20000) \sim \frac{2}{5}|A_{1,0}|^2|B^0_{0,0}|^2,$$

$$M(20020) \sim \frac{4}{25}|A_{1,0}|^2|B^0_{0,0}|^2. \tag{21}$$

#### 2.2.2. $J_X^{P_X} = 1^-$

There are four independent helicity amplitudes: $A_{1,0}$, $A_{1,1}$, $A_{0,0}$ and $A_{0,1}$ for the process $J/\psi \to V + X$ and there is still one independent helicity amplitude $B^1_{1,0}$ for the process $X \to P_1 + Y$ due to

Eq.(20). From Eq.(7) we can obtain sixteen moments. Below we only list eight of them which will be used in the later calculation:

$$M(00000) \sim 4(2|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,0}|^2 + 2|A_{0,1}|^2)|B^1_{1,0}|^2,$$

$$M(02000) \sim \frac{4}{5}(|A_{1,1}|^2 - 2|A_{1,0}|^2 - |A_{0,0}|^2 + |A_{0,1}|^2)|B^1_{1,0}|^2,$$

$$M(02020) \sim -\frac{4}{25}(|A_{1,1}|^2 - 2|A_{1,0}|^2 - |A_{0,0}|^2 + |A_{0,1}|^2)|B^1_{1,0}|^2,$$

$$M(02022) \sim \frac{12}{25}(|A_{1,1}|^2 - 2|A_{1,0}|^2 - |A_{0,0}|^2 + |A_{0,1}|^2)|B^1_{1,0}|^2,$$

$$M(20000) \sim \frac{4}{5}(-2|A_{1,1}|^2 + |A_{1,0}|^2 - |A_{0,0}|^2 + |A_{0,1}|^2)|B^1_{1,0}|^2,$$

$$M(22000) \sim -\frac{2}{25}(2|A_{1,1}|^2 + 2|A_{1,0}|^2 - 2|A_{0,0}|^2 - |A_{0,1}|^2)|B^1_{1,0}|^2,$$

$$M(22100) \sim \frac{6}{25}\mathrm{Re}[A_{0,1}A^*_{0,0} - A_{1,1}A^*_{1,0}]|B^1_{1,0}|^2,$$

$$M(22200) \sim \frac{6}{25}|A_{0,1}|^2|B^1_{1,0}|^2. \qquad (22)$$

### 2.2.3. $J_X^{P_X} = 1^+$

We have $A_{0,0} = 0$ and there are two independent helicity amplitudes $B^1_{1,0}$ and $B^1_{0,0}$ for the process $X \to P_1 + Y$ from Eq.(20). In this case the total number of moments is twenty-one. The seven moments listed below are useful for the later calculation:

$$M(02020) \sim -\frac{4}{25}(|A_{1,1}|^2 - 2|A_{1,0}|^2 + |A_{0,1}|^2)(|B^1_{1,0}|^2 + 2|B^1_{0,0}|^2),$$

$$M(02022) \sim -\frac{12}{25}(|A_{1,1}|^2 - 2|A_{1,0}|^2 + |A_{0,1}|^2)|B^1_{1,0}|^2,$$

$$M(22022) \sim \frac{6}{125}(2|A_{1,1}|^2 + 2|A_{1,0}|^2 - |A_{0,1}|^2)|B^1_{1,0}|^2,$$

$$M(22100) \sim -\frac{6}{25}\mathrm{Re}[A_{1,1}A^*_{1,0}](|B^1_{1,0}|^2 - |B^1_{0,0}|^2),$$

$$M(22121) \sim \frac{18}{125}\mathrm{Re}[A_{1,1}A^*_{1,0}]\mathrm{Re}[B^1_{1,0}B^{1*}_{0,0}],$$

$$M(22122) \sim \frac{18}{125}\mathrm{Re}[A_{1,1}A^*_{1,0}]|B^1_{1,0}|^2,$$

$$M(22222) \sim \frac{18}{125}|A_{0,1}|^2|B^1_{1,0}|^2. \qquad (23)$$

### 2.2.4. $J_X^{P_X} = 2^+$

There are twenty-eight moments when $J_X^{P_X} = 2^+$. The following two moments are used later:

$$M(00000) \sim 4(2|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,0}|^2 + 2|A_{0,1}|^2 + 2|A_{1,2}|^2)|B^2_{1,0}|^2,$$

$$M(02000) \sim \frac{4}{7}(|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,0}|^2 + |A_{0,1}|^2 - 2|A_{1,2}|^2)|B^2_{1,0}|^2,$$

$$M(02022) \sim -\frac{12}{35}(|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,0}|^2 + |A_{0,1}|^2 - 2|A_{1,2}|^2)|B^2_{1,0}|^2,$$

$$M(20000) \sim \frac{4}{5}(-2|A_{1,1}|^2 + |A_{1,0}|^2 - |A_{0,0}|^2 + |A_{0,1}|^2 + |A_{1,2}|^2)|B^2_{1,0}|^2,$$

$$M(22000) \sim -\frac{2}{35}(2|A_{1,1}|^2 - 2|A_{1,0}|^2 + 2|A_{0,0}|^2 - |A_{0,1}|^2 + 2|A_{1,2}|^2)|B^2_{1,0}|^2,$$

$$M(22200) \sim -\frac{2}{35}[2\sqrt{6}\,\mathrm{Re}(A_{1,2}A^*_{1,0}) + 3|A_{0,1}|^2]|B^2_{1,0}|^2,$$

$$M(22100) \sim \frac{2\sqrt{3}}{35}\mathrm{Re}[A_{1,1}A^*_{1,0} - A_{0,1}A^*_{0,0} - \sqrt{6}\,A_{1,1}A^*_{1,2}]|B^2_{1,0}|^2,$$

$$M(04000) \sim \frac{16}{63}(4|A_{1,1}|^2 - 6|A_{1,0}|^2 - 3|A_{0,0}|^2 + 4|A_{0,1}|^2 - |A_{1,2}|^2)|B^2_{1,0}|^2,$$

$$M(24100) \sim -\frac{8}{21}\sqrt{\frac{2}{5}}\mathrm{Re}\left[A_{1,1}A^*_{1,0} - A_{0,1}A^*_{0,0} + \frac{1}{\sqrt{6}}A_{1,2}A^*_{1,1}\right]|B^2_{1,0}|^2,$$

$$M(24200) \sim -\frac{8}{21}\sqrt{\frac{2}{5}}\left[\mathrm{Re}(A_{1,2}A^*_{1,0}) - \frac{\sqrt{6}}{3}|A_{0,1}|^2\right]|B^2_{1,0}|^2. \tag{24}$$

### 2.2.5. $J_X^{P_X} = 2^-$

In the cases of $J_Y^{P_X} = 1^-$ and $2^-$, because $A_{0,0} = 0$, $B^2_{0,0} \neq 0$, there are four independent helicity amplitudes for the process $J/\psi \to V + X$ and two independent helicity amplitudes for the process $X \to P_1 + Y$. There are thirty-nine moments in these cases. The fourteen moments of them which will be used later are:

$$M(00000) \sim 4(|A_{1,1}|^2 + |A_{1,0}|^2 + |A_{0,1}|^2 + |A_{1,2}|^2)(2|B^2_{1,0}|^2 + |B^2_{0,0}|^2),$$

$$M(00020) \sim -\frac{8}{5}(|A_{1,1}|^2 + |A_{1,0}|^2 + |A_{0,1}|^2 + |A_{1,2}|^2)(|B^2_{1,0}|^2 - |B^2_{0,0}|^2),$$

$$M(02000) \sim \frac{4}{7}(|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,1}|^2 - 2|A_{1,2}|^2)(|B^2_{1,0}|^2 + |B^2_{0,0}|^2),$$

$$M(02020) \sim -\frac{4}{35}(|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,1}|^2 - 2|A_{1,2}|^2) \times (|B^2_{1,0}|^2 - 2|B^2_{0,0}|^2),$$

$$M(02021) \sim \frac{4\sqrt{3}}{35}(|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,1}|^2 - 2|A_{1,2}|^2)\mathrm{Re}(B^2_{1,0}B^{2*}_{0,0}),$$

$$M(02022) \sim \frac{12}{35}(|A_{1,1}|^2 + 2|A_{1,0}|^2 + |A_{0,1}|^2 - 2|A_{1,2}|^2)|B^2_{1,0}|^2,$$

$$M(04022) \sim -\frac{8\sqrt{15}}{315}(4|A_{1,1}|^2 - 6|A_{1,0}|^2 + 4|A_{0,1}|^2 - |A_{1,2}|^2)|B^2_{1,0}|^2,$$

$$M(22022) \sim -\frac{6}{175}(2|A_{1,1}|^2 - 2|A_{1,0}|^2 - |A_{0,1}|^2 + 2|A_{1,2}|^2)|B^2_{1,0}|^2,$$

$$M(22122) \sim \frac{6\sqrt{3}}{175}\mathrm{Re}[A_{1,1}A^*_{1,0} - \sqrt{6}A_{1,1}A^*_{1,2}]|B^2_{1,0}|^2,$$

$$M(22200) \sim -\frac{2}{35}[2\sqrt{6}\,\mathrm{Re}(A_{1,2}A^*_{1,0}) - 3|A_{0,1}|^2](|B^2_{1,0}|^2 + |B^2_{0,0}|^2),$$

$$M(22220) \sim \frac{2}{175}[2\sqrt{6}\,\mathrm{Re}(A_{1,2}A^*_{1,0}) - 3|A_{0,1}|^2](|B^2_{1,0}|^2 - 2|B^2_{0,0}|^2),$$

$$M(22222) \sim -\frac{6}{175}[2\sqrt{6}\,\mathrm{Re}(A_{1,2}A^*_{1,0}) - 3|A_{0,1}|^2]|B^2_{1,0}|^2,$$

$$M(24122) \sim \frac{4}{35}\sqrt{\frac{2}{3}}\mathrm{Re}\left[A_{1,1}A^*_{1,0} + \frac{1}{\sqrt{6}}A_{1,1}A^*_{1,2}\right]|B^2_{1,0}|^2,$$

$$M(24222) \sim \frac{4}{35}\sqrt{\frac{2}{3}}\left[\mathrm{Re}(A_{1,0}A^*_{1,2}) + \frac{\sqrt{6}}{3}|A_{0,1}|^2\right]|B^2_{1,0}|^2. \tag{25}$$

From Eqs.(21)-(25) we can see that there is an obvious characteristic for the resonance X of $J_X^{P_X} = 0^-$, that is, all of the moments with $l > 0$, $m > 0$, and $n > 0$ equal zero and only the four nonzero moments satisfy a simple relation:

$$2M(00000) = 5M(00020) = 20M(20000) = 50M(20020). \tag{26}$$

However the above relations cannot be satisfied simultaneously for the resonances X of $J_X^{P_X} = 1^\pm$ or $J_X^{P_X} = 2^\pm$. Therefore, the $0^-$ state can be distinguished from the states of $1^\pm$ and $2^\pm$ through the measurement of the moments. One sees that all of moments with $l > 2$ equal to zero for the $J_X = 1$ state, and there are eighteen moments with $l > 2$ for the $2^-$ state and twelve moments with $l > 2$ for the $2^+$ state. If there exists one non-zero measured value of the moments with $l > 2$ the spin of this boson resonance X must be 2. After determining the spin, one can fix the parity of the boson resonance by using the following two relations.

$$\frac{M(02020)}{M(02022)} = \begin{cases} -\frac{1}{3} & \text{for } J_X^{P_X} = 1^- \\ \frac{1}{3}(1 + 2\xi_1^2) \geqslant \frac{1}{3} & \text{for } J_X^{P_X} = 1^+ \end{cases} \tag{27}$$

$$\frac{M(02000)}{M(02022)} = \begin{cases} -\frac{5}{3} & \text{for } J_X^{P_X} = 2^+ \\ \frac{5}{3}(1 + \xi_2^2) \geqslant \frac{5}{3} & \text{for } J_X^{P_X} = 2^- \end{cases} \tag{28}$$

After the spin and parity of the resonance are determined, the helicity amplitude ratios can be obtained by the measurement of the moments. For $J_X^{P_X} = 1^-$, we have

$$x^2 = \frac{1}{T_1}[M(00000) - 10M(20000) + 5M(02000) - 50M(22000)],$$

$$z_1^2 = \frac{1}{T_1}[M(00000) - 10M(20000) - 10M(02000) + 100M(22000)],$$

$$z_2^2 = \frac{150}{T_1} M(22200),$$

$$z_1 z_2 \cos(\phi_{z_1} - \phi_{z_2}) - x\cos\phi_x = \frac{150}{T_1} M(22100). \tag{29}$$

where

$$T_1 = M(00000) + 5M(20000) - 10M(02000) - 50M(22000). \tag{30}$$

When $J_X^{P_X} = 1^+$, we obtain

$$x^2 = \frac{25}{T_2}[10M(22022) - M(02022)],$$

$$z_2^2 = \frac{250}{T_2} M(22222),$$

$$\xi_1^2 = \frac{3}{5}\frac{M(22100)}{M(22122)} + 1,$$

$$x\cos\phi_x = \frac{250}{T_2} M(22122),$$

$$\xi_1 \cos\phi_1 = \frac{M(22121)}{M(22122)}. \tag{31}$$

where

$$T_2 = 25[5M(22022) + M(02022) + 5M(22222)]. \tag{32}$$

For $J_X^{P_X} = 2^+$, we have expressions as follows:

$$x^2 = \frac{5}{4T_3}[M(00000) - 10M(20000) - 7M(02000) + 70M(22000)],$$

$$y^2 = \frac{3}{16T_3}[4M(00000) - 40M(02000) - 9M(04000)],$$

$$z_1^2 = \frac{5}{4T_3}[-M(00000) + 10M(20000) + 14M(02000) - 140M(22000)],$$

$$z_2^2 = \frac{105}{28T_3}[3\sqrt{15}M(24200) - 10M(22200)],$$

$$y\cos\phi_y = -\frac{35}{112T_3}[40\sqrt{6}M(22200) + 27\sqrt{10}M(24200)],$$

$$xy\cos(\phi_y - \phi_x) = -\frac{105}{56T_3}[3\sqrt{15}M(24100) + 20\sqrt{2}M(22100)],$$

$$x\cos\phi_x - z_1 z_2 \cos(\phi_{z_1} - \phi_{z_2}) = \frac{5}{8T_3}[20\sqrt{3}M(22100) - 27\sqrt{10}M(24100)]. \quad (33)$$

where
$$T_3 = M(00000) - 5M(02000) - \frac{25}{4}M(20000) + \frac{175}{2}M(22000) - \frac{81}{16}M(04000). \quad (34)$$

Finally, when $J_X^{P_X} = 2^-$, we can obtain the following relations:

$$x^2 = \frac{1225}{12T_4}[M(02022) - 10M(22022)],$$

$$y^2 = \frac{7}{8T_4}[4M(00000) - 10M(00020) - 100M(02022) + 9\sqrt{15}M(04022)],$$

$$z_2^2 = \frac{175}{2T_4}[9M(24222) + 5M(22222)],$$

$$\xi_2^2 = \frac{M(02000) + 5M(02020)}{5M(02022)},$$

$$x\cos\phi_x = \frac{350\sqrt{3}}{24T_4}[10M(22122) + 27\sqrt{2}M(24122)],$$

$$y\cos\phi_y = \frac{175\sqrt{6}}{24T_4}[-8M(22200) + 20M(22220) + 27M(24222)],$$

$$\xi_2\cos\phi_2 = \frac{\sqrt{3}M(02021)}{M(02022)}. \quad (35)$$

where

$$T_4 = \frac{7}{4}M(00000) - \frac{35}{8}M(00020) + \frac{175}{4}M(02022) + \frac{189\sqrt{15}}{8}M(04022). \quad (36)$$

## 3. SUMMARY

We discuss the three-step two-body decay process (2) of J/$\psi$ by using the generalized moment analysis. The corresponding moment expressions for the cases of $J_Y^{P_Y} = 0^+, 1^+$, and $J_X^{P_X} = 0^-, 1^\pm, 2^\pm$, have been given. The spin and parity of the boson resonance X can be determined by the measurement of these moments except for a very special case. And the corresponding experimental value of the helicity amplitude ratios can also be obtained. Because each helicity amplitude ratio can be expressed in terms of the moments independently, we can expect that the generalized moment analysis method may be more convenient than the angular distribution analysis method for the measurement and the reduction of the measurement error.

The above discussion for process (2) is also applicable to process (1) if $A_{0,0} = A_{0,1} = 0$.

## REFERENCES

[1] H. Fritszch and P. Minkowski, *Nuovo Cimento*, **30A**(1975), p. 393; P. G. O. Freund and Y. Nambu, *Phys. Rev. Lett.*, **34**(1975), p. 1645; D. Horn and J. Mandula, *Phys. Rev.*, **D17**(1978), p. 365; M. Chanowitz and S. Sharpe, *Nucl. Phys.*, **B222**(1983), p. 211; R. L. Jaff and K. Johnson, *Phys. Lett.*, **60B**(1976), p. 201; R. L. Jaffe, *Phys. Rev.*, **D15**(1977), p. 267, p. 281.

[2] L. Köpke and N. Wermes, *Phys. Rep.*, **174**(1989), p. 67.

[3] A. Billoire *et al.*, *Phys. Lett.*, **80B**(1978), p. 381; R. Lancaze and H. Navelet, *Nucl Phys.*, **B186**(1981), p. 247.

[4] D. L. Scharre *et al.*, *Phys. Lett.*, **97B**(1980), p. 329.

[5] C. Edwards *et al.*, *Phys. Rev. Lett.*, **48**(1982), p. 458.

[6] K. F. Einweiler, *SLAC-PUB-3202* (1983).

[7] J. E. Augustin *et al.*, *Phys. Rev.*, **D46**(1992), p. 1951; Z. Bai *et al.*, *Phys. Rev. Lett.*, **65**(1990), p. 2507.

[8] Sheng Qixing and Yu Hong, *High Energy Phys. and Nucl. Phys.* (Chinese Edition), **16**(1992), p. 219.

[9] Yu Hong, *High Energy Phys. and Nucl. Phys.* (Chinese Edition), **13**(1989), p. 87.

[10] Liang Ping Chen, *SLAC-REPORT-386* (1991); Zhu Yucan, *Inner Report of Institute of High Energy Phys.* (1993).