

# Microscopic Investigation of High-Spin States for the Even-Even Nuclei (III) Application to Isotopes $^{126}\text{Ba}$ , $^{128}\text{Ba}$ and $^{130}\text{Ba}$

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Starting from the shell model configuration and effective nucleon-nucleon interaction, a microscopic approach is applied to study the high-spin states of the isotopes  $^{126}\text{Ba}$ ,  $^{128}\text{Ba}$  and  $^{130}\text{Ba}$ . The low-lying states and the band built upon aligned  $h_{11/2}$  proton pair configurations are considered. Energy levels and backbendings are well reproduced. For the yrast band, the dependence of the strength of interband interaction on coupling operator is discussed.

**Key words:** isotopes  $^{126}\text{Ba}$ ,  $^{128}\text{Ba}$  and  $^{130}\text{Ba}$ , microscopic investigation, yrast band.

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## 1. INTRODUCTION

In our recent paper [1], a theoretical approach was proposed to study the properties of high-spin states of even-even nuclei which are treated as a system of valence neutrons and protons. The calculated results of neutron-pair aligned bands for rare earths  $^{154}\text{Dy}$ ,  $^{156}\text{Er}$  and  $^{158}\text{Yb}$  were reported in our previous publication [2]. This present work is one of our series studies. In the present work we study the excitation spectra of  $^{126-130}\text{Ba}$  even-mass isotopic chain in the  $A \sim 130$  region. The emphasis

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is placed on the observed backbendings formed by a broken pair of protons.

In the mass region of  $A \sim 130$ , there are quite rich experimental data, and as well as detailed theoretical analyses.

The backbending phenomenon has been observed in the nucleus  $^{126}\text{Ba}$ . For the isotopes  $^{128,130}\text{Ba}$ , there are, respectively, two bands which exhibit backbendings due to the band crossing ground bands. These nuclei are good testing lasting examples of theoretical models. By using the particle-rotor model, Flaum *et al.* made studies and calculated the S band of  $^{126}\text{Ba}$  [3]. They pointed out that the S band shows the features of two-proton alignment. For the  $^{128}\text{Ba}$  or  $^{130}\text{Ba}$  isotope, it is believed that the two high-spin bands are based, respectively, on the alignment states of two protons and two neutrons [4,5]. According to the above analyses of the experimental data and the fact that the intruder orbits are of crucial importance in the yrast spectroscopy [6], the present work will study the high-spin band formed by alignment of proton pair in the intruder  $h_{11/2}$  orbit. Moreover, the calculated results of low-lying excitation levels will also be given within our unified theoretical framework.

## 2. FORMALISM

Since the details of the approach have been presented in [1], we only illustrate an outline here for the present interest of studying  $^{126-130}\text{Ba}$  isotopes.

We consider a valence nucleon system. For  $^{126-130}\text{Ba}$  isotopes, this means that the effects of degrees of freedom in the closed major shell on the excited states under discussion can be neglected, and the closed core is thus assumed to be inert. In such a case, the configuration space may be expressed as

$$(2d_{5/2}1g_{7/2}1h_{11/2}2d_{3/2}3s_{1/2})^{n_p}, \quad (2.1a)$$

$$(2d_{5/2}1g_{7/2}1h_{11/2}2d_{3/2}3s_{1/2})^{n_p}, \quad (2.1b)$$

where  $n_n(n_p)$  is the number of valence neutrons (protons). In general, the effective valence nucleon Hamiltonian can be written as

$$H_t = H_t^{(n)} + H_t^{(p)} + H_t^{(np)}, \quad (2.2)$$

with

$$H_t^{(\sigma)} = \sum_a E_a^{(\sigma)} a_a^{(\sigma)\dagger} a_a^{(\sigma)} + \sum_{\alpha\beta\gamma\delta} P_{\alpha\beta\gamma\delta}^{(\sigma)} a_\alpha^{(\sigma)\dagger} a_\beta^{(\sigma)\dagger} a_\gamma^{(\sigma)} a_\delta^{(\sigma)}, (\sigma = n, p) \quad (2.3)$$

$$H_t^{(np)} = \sum_{\alpha\beta\gamma\delta} P_{\alpha\beta\gamma\delta}^{(np)} a_\alpha^{(n)\dagger} a_\beta^{(p)\dagger} a_\gamma^{(p)} a_\delta^{(n)}, \quad (2.4)$$

where the interaction matrix elements  $P_{\alpha\beta\gamma\delta}^{(\sigma)}$  and  $P_{\alpha\beta\gamma\delta}^{(np)}$  depend on the types of interactions employed. In practical calculations, the interactions between identical nucleons are taken to be pairing force, quadrupole pairing force and quadrupole-quadrupole force, and the neutron-proton interaction is of quadrupole-quadrupole type. The corresponding strength parameters are labeled as  $g_\sigma$ ,  $G'_\sigma$ ,  $K'_\sigma$  ( $\sigma = n, p$ ) and  $K'_{np}$ .

The state space given in Eq.(2.1) is extremely large. It is necessary to truncate appropriately the space to a tractable subspace in order to study the properties of excited states up to the first backbending energy region. Since the viewpoint of band crossing between ground and S band has been accepted as the formation of first backbending, one deals with the nucleus as a system in which a broken nucleon pair couples to the core formed by all other nucleons to describe the S band. There are several approaches for the core description. In the present approach we work in the framework of the

Table 1  
Single particle energies of the valence nucleons (MeV).

$nlj$	$2d_{3/2}$	$1g_{7/2}$	$1h_{11/2}$	$2d_{5/2}$	$3s_{1/2}$
neutron	4.00	4.88	5.70	6.24	7.24
proton	4.54	4.00	5.80	7.47	6.54

Table 2  
The parameters of nucleon-nucleon effective interaction (MeV).

nucleon	$g_n$	$G'_n$	$K'_n$	$g_p$	$G'_p$	$K'_p$	$K'_{np}$
$^{126}\text{Ba}$	0.041	0.051	0.020	0.041	0.051	0.020	0.028
$^{128}\text{Ba}$	0.040	0.051	0.020	0.040	0.051	0.020	0.026
$^{130}\text{Ba}$	0.040	0.050	0.020	0.040	0.050	0.020	0.028

phenomenological IBM +  $2q.p.$  model. That is, we assume that the s-d truncation may be regarded as an appropriate approximation to the core space. This implies that the truncation problem in the shell model space can be solved by investigating the microscopic structures of the s and d bosons. Therefore, our present approach, which describes high-spin states in the basis of Eqs.(2.1) and (2.2), is, in fact, an extension of microscopic IBM approach to the case of including explicitly a pair of fermions.

For the definition of s and d bosons, the present work follows the procedure of the microscopic IBM approach, which is based on the Dyson boson mapping [8]. According to the expansion theory, the original valence nucleon description of nucleus can be first transformed into corresponding ideal boson-fermion description. In the coupled-boson representation, consideration of coherency due to interactions in the Hamiltonian (2.2) and properties of s, d bosons, the coupled-bosons with a strong collectivity are defined as s and d bosons. Then, we make truncation of the space of state.

For our principal purpose here, the truncated state space can be decomposed into two smaller subspaces.

$$\Sigma = \Sigma_1 \oplus \Sigma_2 \quad (2.5)$$

where  $\Sigma_1 = \{(sd)^N\}$  and  $\Sigma_2 = \{(sd)^{N-1} + 2q.p.\}$ . The former contains no fermion and thus, is used to describe low-lying state. The latter has pair of protons and describes the S band. If the bosons consisting of protons and neutrons, respectively neutron are not distinguished, the basis states can be express as:

(1) IBM-1 state  $\alpha$  with spin  $J$ :  $|\alpha JM\rangle$ ;

(2) Direct product of IBM-1 state  $\beta$  with spin  $K$  and proton pair state with spin  $L$ :  $|\beta K, j_p^2 L; JM\rangle$ .  
Using  $U(5)$  representation for the core, the above basis states have the following explicit forms

$$|\alpha JM\rangle = |N n_d \nu n_\Delta J M\rangle; \quad (2.6)$$

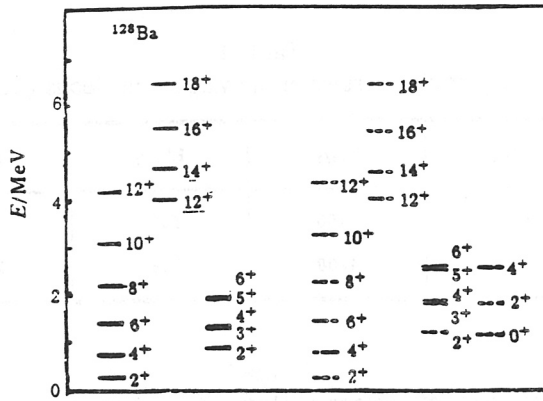


Fig. 1

The experimental data are taken from [5].

— Exp.; --- Cal..

$$|\beta K, j_p^2 L; JM\rangle = \sum_{m_1 m_2} \langle K m_1 L m_2 | JM \rangle |N - 1 n_d \nu n_\Delta K m_1\rangle |j_p^2 L m_2\rangle; \quad (2.7)$$

with

$$|j_p^2 L m_2\rangle = \frac{1}{\sqrt{2}} (\xi_i^{(p)\dagger} \xi_i^{(p)\dagger})_{L m_2} |0\rangle, \quad (2.8)$$

where  $j_p$  is restricted to the intruder  $h_{11/2}$  orbit.

In the state space given by Eq.(2.5), the effective Hamiltonian is written as

$$h = h_{sd} + h_f + V_{Bf} + h_{mix} \quad (2.9)$$

where  $h_{sd}$  is just the most general IBM-1 Hamiltonian,  $h_f$ ,  $V_{Bf}$  and  $h_{mix}$  are, respectively, the fermion

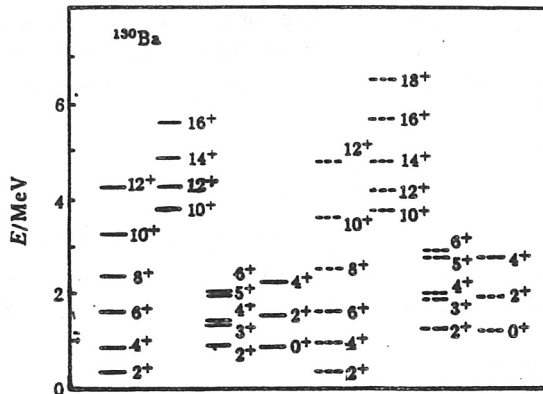


Fig. 2

The experimental data are taken from [4].

— Exp.; --- Cal..

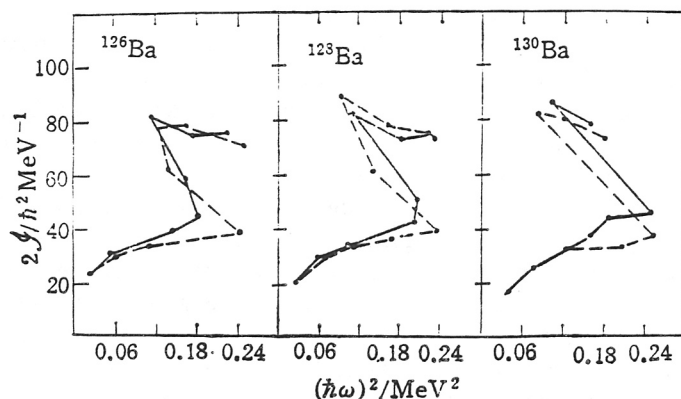


Fig. 3

The experimental data are taken from [3-5]: — Exp.; --- Cal..

part, boson-fermion interaction and coupling terms due to configuration mixing. Their expressions can be found in [1].

The present study proceeds as the following: based on a given effective Hamiltonian for the valence nucleon system, the microscopic structures of s and d bosons are then defined in such a way that one can carry out the truncation of state space and derive corresponding effective operator of the Hamiltonian (2.9) in full space. After that, the eigenvalue equation of operator (2.9) will be solved in the model space spanned by basis vectors (2.5). The obtained eigenvalue spectrum are compared directly with experimental data. The calculated results, including the eigenfunctions, are also analyzed and discussed.

### 3. CALCULATED RESULTS AND DISCUSSIONS

In the calculations of  $^{126}\text{Ba}$ ,  $^{128}\text{Ba}$  and  $^{130}\text{Ba}$ , the single-particle energy for the unclosed nucleon shells and the strengths of the nucleon-nucleon effective interaction are employed according to the corresponding values used in [9]. Considering that the proton and neutron bosons are not distinguished in the present calculation, the interaction strengths are slightly adjusted, see Tables 1 and 2.

The calculated results of  $^{128,130}\text{Ba}$  spectra are shown, respectively, in Figs. 1 and 2 with comparison to experimental data. The result for  $^{126}\text{Ba}$  isotope is similar. Since we have no experimental data for the quasi- $\beta$  band in  $^{128}\text{Ba}$ , only theoretical predictions are given in the figure. Moreover, the  $J - \omega^2$  curves for yrast bands are plotted in Fig. 3, which clearly indicate an agreement between our predictions and the observations.

Figures 1-3 qualitatively show that a good description is obtained by our approach for the excited states up to the energy regions of the first backbending. The calculated results are in fairly satisfactory agreement with experiment, especially the first backbending of high-spin states are reproduced. The variation trends of moments of inertia with increasing  $\omega^2$  are in accordance with observations. For the spectra below 2 MeV, the structure of each quasi-band has the same feature as obtained from experiment. For example, the staggering occurs in the quasi- $\gamma$  bands, although the approach somewhat overestimates the intrinsic excitation energies of these bands. We compared our calculations with the phenomenological IBM +  $2q.p.$  studies of yrast bands [7,10]. We noticed that our microscopic study give similar results comparing to the experiments, as in the phenomenological model. From the viewpoint of quantitative investigations, some deviations appear in the states 2 MeV and below the band crossing. The calculated excitation energies for these states are slightly higher than corresponding data and the moments of inertia are systematically small.

There are at least two possible reasons that may be able to explain the above deviations in yrast bands. They are the interaction between different bands and the truncation of states space. The band interaction used in the present study is weaker than one we needed. It leads to sharp backbendings as plotted in Fig. 3. Also due to the weak band interaction, the states before band crossing in the yrast bands belong consist of almost pure  $(sd)^N$  configuration. There is a number of work which suggested that such configurations are suitable only for describing the excited states below 2 MeV. For states above 2 MeV, one cannot neglect the contribution from other excitation modes. This implies the deviations can be caused by the s-d truncation too.

In the calculation of interband coupling, the coupling operator takes the form  $[b_l^\dagger(\bar{a}\bar{a})_l]_0 + \text{h.c.}$  ( $b_l^\dagger = s^\dagger, d^\dagger$ ) referring to an phenomenological work [10], i.e., as an approximation the higher order terms in  $h_{\text{mix}}$  of Eq.(2.9) are neglected. However, each member of the s-band states has several components discriminated by different angular momentum given by two-proton coupling. In the component analysis we found that those components with proton angular momenta  $L = 0$  and 2 are considerably small. On the other hand, the  $L > 2$  components do not contribute to the matrix elements of the band coupling operator due to the limitation  $l \leq 2$  in the coupling operator and selection rules. This indicates that the present approximation is restricted in describing such an even-even nucleus in which the band coupling is weak. In the case of strong coupling, it is needed to evaluate the contribution of higher order terms in the coupling operator and/or to study the effects of higher angular momentum bosons.

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