High Energy Physics and Nuclear Physics

Volume 19, Number 2

Generalization and Application of the Noether's Identities

Li Ziping

(Department of Applied Physics, Beijing Polytechnic University, Beijing, China)

In this paper the generalized Noether's identities for nonlocal transformations are derived from the properties of the action of a system under local and nonlocal transformations. In the application of the theory to the Yang-Mills field with high-order derivatives, a new conservative PBRS charge, which differs from the BRS conservative charge, and a new conservative charge connecting with nonlocal transformations are found.

Key words: generalized Noether's identities, nonlocal transformation, Yang-Mills field with higher-order derivatives, conservative charge.

1. INTRODUCTION

The Noether identities [1] related to the invariance of the action of a system under an infinite continuous group play an important role [2] in various physical fields (e.g., gauge field theories). The traditional Noether identities and their generalization [3] were based on the local transformation. However, in the Yang-Mills field theory and in the study of the conformal symmetry of the gauge field theory [5,6] some nonlocal transformations were discussed. Therefore, it becomes necessary to study the properties of a system in general nonlocal transformations. The field theories with high-order derivatives have drawn more attention [7]. In this paper we will discuss the properties of a system with high-order derivatives under a general nonlocal transformation, and generalize the Noether's identity to that system. Applying this theory to the Yang-Mills field theory with high-order derivatives, we

Received on May 18, 1994.

obtain a new conservative PBRS charge, which differs from the BRS conservative charge, and a new conservative charge connecting with nonlocal transformations that cannot be derived from the first Noether's theorem. In fact, we present a new method for finding the conservative laws of a system.

2. GENERALIZATION OF NOETHER'S IDENTITIES

Discuss a dynamical system, described by fields ψ^α ($\alpha = 1, 2, \ldots, n$), with the action:

$$I = \int_D d^4x \mathscr{L}(x, \psi^\alpha(x), \psi^\alpha_{,\mu}(x), \cdots \psi^\alpha_{,\mu(m)}(x), \cdots) \tag{1}$$

where the Lagrangian density $\mathscr{L}$ contains the high-order derivative:

$$\psi^\alpha_{,\mu} = \partial_\mu \psi^\alpha_; \quad \psi^\alpha_{,\mu(m)} = \partial_{\mu(m)}\psi^\alpha = \underbrace{\partial_\mu \partial_\nu \cdots \partial_\lambda}_{m} \psi^\alpha, \tag{2}$$

where $\partial_\mu = \partial/\partial x^\mu$, $x^\mu = (t, x)$. The flat metric of space-time is $\eta_{\mu\nu} = (+---)$. Denote by N the highest derivative of ψ^α in $\mathscr{L}$. Assume that all the functions and their high-order derivatives appearing in this paper are smooth enough.

Consider the properties of the action under general local and nonlocal transformations with the following general forms of infinitesimal transformations:

$$\begin{cases} x^{\mu'} = x^\mu + \Delta x^\mu = x^\mu + R^\mu_\sigma \varepsilon^\sigma(x) \\ \psi^{\alpha'}(x') = \psi^\alpha(x) + \Delta\psi^\alpha(x) = \psi^\alpha(x) + A^\alpha_\sigma \varepsilon^\sigma(x) + \int_D d^4y F(x,y) B^\alpha_\sigma(y) \varepsilon^\sigma(y) \end{cases} \tag{3}$$

where
$$R^\mu_\sigma = r^{\mu(l)}_\sigma \partial_{\mu(l)}, \quad A^\alpha_\sigma = a^{\mu(m)}_\sigma \partial_{\mu(m)}, \quad B^\alpha_\sigma = b^{\mu(n)}_\sigma \partial_{\mu(n)},$$
$$r^{\mu(l)}_\sigma = \overbrace{r^{\mu\nu\cdots\lambda}_\sigma}^{l}, \quad a^{\mu(m)}_\sigma = \overbrace{a^{\mu\nu\cdots\lambda}_\sigma}^{m}, \quad b^{\mu(n)}_\sigma = \overbrace{b^{\mu\nu\cdots\lambda}_\sigma}^{n}, \tag{4}$$

where $r^{\mu(l)}_\sigma$, $a^{\mu(m)}_\sigma$, and $b^{\mu(n)}_\sigma$ are the functions of x, ψ^α, and $\psi^\alpha_{,\mu(m)}$. ε^σ ($\sigma = 1, 2, \ldots, s$) are the infinitesimal functions of the space-time coordinates. On the boundary of the domain D their all-order derivatives are vanishing. Assume that the variation of action (1) under transformation (3) is:

$$\Delta I = \int_D [\partial_\mu(\Lambda^\mu_\sigma \varepsilon^\sigma) + W] d^4x, \tag{5}$$

where

$$W = U_\sigma \varepsilon^\sigma(x) + \int_D d^4y V_\sigma(x,y) \varepsilon^\sigma(y), \tag{6}$$

$$\Lambda^\mu_\sigma = \lambda^{\mu\nu(i)}_\sigma(x) \partial_{\nu(i)}, \quad U_\sigma = u^{\mu(j)}_\sigma(x) \partial_{\mu(j)}, \quad V_\sigma = v^{\mu(k)}_\sigma(x,y) \partial_{\mu(k)}, \tag{7}$$

where $\lambda^{\mu\nu(i)}_\sigma$, $u^{\mu(j)}_\sigma$, and $v^{\mu(k)}_\sigma$ are the functions of x, ψ^α, and $\psi^\alpha_{,\mu(m)}$. For a weak invariant system, $W \doteq 0$, where $\doteq$ denotes that the equality holds along the movement orbit of the system [8]. Under the transformation (3), we have [9] from (1), (3), and (5):

$$\begin{aligned} &\int_D d^4x \left\{ \frac{\delta I}{\delta \psi^\alpha} \left[(A^\alpha_\sigma - \psi^\alpha_{,\mu} R^\mu_\sigma) \varepsilon^\sigma + \int_D d^4y F(x,y) B^\alpha_\sigma(y) \varepsilon^\sigma(y) \right] + \partial_\mu (j^\mu_\sigma \varepsilon^\sigma(x)) \right. \\ &\left. + \partial_\mu \left[\sum_{m=0}^{N-1} \Pi^{\mu\nu(m)}_\alpha \partial_{\nu(m)} \int_D d^4y F(x,y) B^\alpha_\sigma(y) \varepsilon^\sigma(y) \right] \right\} \\ &= \int_D d^4x \left[U_\sigma(x) \varepsilon^\sigma(x) + \int_D d^4y V_\sigma(x,y) \varepsilon^\sigma(y) \right], \end{aligned} \tag{8}$$

where

$$\frac{\delta I}{\delta \psi^\alpha} = (-1)^m \partial_{\mu(m)} \mathscr{L}_\alpha^{\mu(m)}, \tag{9}$$

$$\mathscr{L}_\alpha^{\mu(m)} = \frac{1}{m!} \sum \frac{\partial \mathscr{L}}{\partial \psi^\alpha_{,\mu(m)}}, \tag{10}$$

$$\Pi_\alpha^{\mu\nu(m)} = \sum_{l=0}^{N-(m+1)} (-1)^l \partial_{\lambda(l)} \mathscr{L}_\alpha^{\mu\nu(m)\lambda(l)}, \tag{11}$$

$$j_\sigma^\mu = \mathscr{L} R_\sigma^\mu + \sum_{m=0}^{N-1} \Pi_\alpha^{\mu\nu(m)} \partial_{\nu(m)} (A_\sigma^\alpha - \psi^\alpha_{,\rho} R_\sigma^\rho) - \Lambda_\sigma^\mu. \tag{12}$$

By making use of Gauss's theorem, the fourth term $\partial_\mu(j_\sigma^\mu \varepsilon^\sigma(x))$ in (8) can be changed to the integral along the boundary of the domain D. The integral is vanishing owing to the boundary condition of $\varepsilon^\sigma(x)$. Through the integration by parts for the rest terms in (8), we know that the integrals of those terms on the boundary are vanishing also owing to the boundary condition of $\varepsilon^\sigma(x)$. Now, from the functional derivative of the results with respect to $\varepsilon^\rho(z)$ we obtain:

$$\tilde{A}_\rho^\alpha(z)\left(\frac{\delta I}{\delta \psi^\alpha(z)}\right) - \tilde{R}_\rho^\mu(z)\left(\psi^\alpha_{,\mu}(z)\frac{\delta I}{\delta \psi^\alpha(z)}\right) + \int_D \mathrm{d}^4x \tilde{B}_\rho^\alpha(z)\left(F(x,z)\frac{\delta I}{\delta \psi^\alpha(x)}\right)$$

$$+\int_D \mathrm{d}^4x \tilde{B}_\rho^\alpha(z)\left\{\partial_\mu\left[\sum_{m=0}^{N-1} \Pi_\alpha^{\mu\nu(m)} \partial_{\nu(m)} F(x,z)\right]\right\} = \tilde{U}_\rho(z) + \int_D \mathrm{d}^4x \tilde{V}_\rho(x,z), (\rho = 1,2,\cdots,s) \tag{13}$$

where $\tilde{A}_\rho^\alpha$, $\tilde{B}_\rho^\alpha$, $\tilde{R}_\rho^\alpha$, $\tilde{U}_\rho$, and $\tilde{V}_\rho$ are the adjoint operators of A_ρ^α, B_ρ^α, U_ρ, and V_ρ. Thus, we obtain the generalized second Noether theorem: If the variation of action (1) of a system under transformation (3) satisfies (5), for the system there exist s identities (13) with derivatives and integrals, including functional derivative $\delta I/\delta\psi^\alpha$. The identities (13) are called the generalized Noether identities, or generalized Bianch identities. The existence of the generalized Noether identities, which give some relations among the functional derivatives $\delta I/\delta\psi^\alpha$, does not depend on whether or not the field quantities ψ^α satisfy the field equation. For the invariant systems under the transformations (3) ($W = 0$), the generalized Noether identities become

$$\tilde{A}_\rho^\alpha(z)\left(\frac{\delta I}{\delta \psi^\alpha(z)}\right) - \tilde{R}_\rho^\mu(z)\left(\psi^\alpha_{,\mu}(z)\frac{\delta I}{\delta \psi^\alpha(z)}\right) + \int_D \mathrm{d}^4x \tilde{B}_\rho^\alpha(z)\left(F(x,z)\frac{\delta I}{\delta \psi^\alpha(x)}\right)$$

$$+\int_D \mathrm{d}^4x \tilde{B}_\rho^\alpha(z)\left\{\partial_\mu\left[\sum_{m=0}^{N-1} \Pi_\alpha^{\mu\nu(m)} \partial_{\nu(m)} F(x,z)\right]\right\} = 0 \quad (\rho = 1,2,\cdots,s) \tag{14}$$

3. CONSERVATIVE LAWS

Consider the systems noninvariant under the local transformation, such as the massive Yang-Mills field, whose Lagrangian is noninvariant under the gauge transformation; the system of the massive Fermi fields, the gauge field whose Lagrangian is gauge-invariant but noninvariant under the chiral transformation for the Fermi fields; the nonabelian gauge field, whose Lagrangian is invariant for BRS transformation but noninvariant only for the transformation of the gauge field; and so on. Discuss a special form of (3) that is related to the gauge transformation, where $\Delta x^\mu = 0$ and

$$\psi^{\alpha\prime}(x) = \psi^\alpha(x) + A_\sigma^\alpha \varepsilon^\sigma(x) = \psi^\alpha(x) + (a_\sigma^\alpha + a_\sigma^{\alpha\mu}\partial_\mu)\varepsilon^\sigma(x), \tag{15}$$

where both a_σ^α and $a_\sigma^{\alpha\mu}$ are the functions of x, ψ^α, and $\psi^\alpha_{,\mu(m)}$. Denote by $\delta\mathscr{L} = U_\sigma\varepsilon^\sigma(x)$ the variation of the Lagrangian $\mathscr{L}$ of the system under the transformation (15), where $U_\sigma = u_\sigma^{\mu(j)}\partial_{\mu(j)}$ and $u_\sigma^{\mu(j)}$ are the functions of x, ψ^α, and $\psi^\alpha_{,\mu(m)}$. There is a fundamental identity for the variation of action (1) under transformation (15):

$$\frac{\delta I}{\delta\psi^\alpha} a_\sigma^\alpha \varepsilon^\sigma(x) + \frac{\delta I}{\delta\psi^\alpha} a_\sigma^{\alpha\mu}\partial_\mu\varepsilon^\sigma(x) + \partial_\mu\left[\sum_{m=0}^{N-1} \Pi_\alpha^{\mu\nu(m)}\partial_{\nu(m)}A_\sigma^\alpha\varepsilon^\sigma(x)\right] = U_\sigma\varepsilon^\sigma(x). \quad (16)$$

In this case the generalized Noether identities (13) become

$$a_\sigma^\alpha\frac{\delta I}{\delta\psi^\alpha} - \partial_\mu\left(a_\sigma^{\alpha\mu}\frac{\delta I}{\delta\psi^\alpha}\right) = \tilde{U}_\sigma(1), \quad (17)$$

where $\tilde{U}_\sigma$ is the adjoint operator U_σ, and $\tilde{U}_\sigma(1)$ is that of 1 [9]. Multiplying (17) by $\varepsilon^\sigma(x)$ and subtracting (16), we ascertain when $u_\sigma^{\mu\nu}$ and $u_\sigma^{\mu\nu\lambda}$ are totally symmetric for the superscripts μ, ν, and λ:

$$\begin{aligned}
&\partial_\mu J^\mu = 0, \\
&J^\mu = \sum_{m=0}^{N-1} \Pi_\alpha^{\mu\nu(m)}\partial_{\nu(m)}A_\sigma^\alpha\varepsilon^\sigma(x) + a_\sigma^{\alpha\mu}\frac{\delta I}{\delta\psi^\alpha}\varepsilon^\sigma(x) - u_\sigma^\mu\varepsilon^\sigma(x) \\
&\quad + (\partial_\nu u_\sigma^{\mu\nu})\varepsilon^\sigma(x) - u_\sigma^{\mu\nu}\partial_\nu\varepsilon^\sigma(x) + u_\sigma^{\mu\nu\lambda}\partial_\nu\partial_\lambda\varepsilon^\sigma(x) \\
&\quad + (\partial_\nu\partial_\lambda u_\sigma^{\mu\nu\lambda})\varepsilon^\sigma(x) - (\partial_\nu u_\sigma^{\mu\lambda})(\partial_\lambda\varepsilon^\sigma(x)) + \cdots\cdots.
\end{aligned} \quad (18)$$

If $\varepsilon^\sigma(x) = \varepsilon_0^\rho\zeta_\rho^\sigma(x)$ in the transformation (15), where ε_0^ρ is a constant parameter, and $\zeta_\rho^\sigma(x)$ is a given function of x, ψ^α, and $\psi^\alpha_{,\mu(m)}$, we have $\delta I/\delta\psi^\alpha = 0$ on the movement orbit of the system, and obtain the (weak) conservative laws:

$$\begin{aligned}
&\partial_\mu\Bigg[\sum_{m=0}^{N-1} \Pi_\alpha^{\mu\nu(m)}\partial_{\nu(m)}A_\sigma^\alpha\zeta_\rho^\sigma - u_\sigma^\mu\zeta_\rho^\sigma + (\partial_\nu u_\sigma^{\mu\nu})\zeta_\rho^\sigma - u_\sigma^{\mu\nu}\partial_\nu\zeta_\rho^\sigma + u_\sigma^{\mu\nu\lambda}\partial_\nu\partial_\lambda\zeta_\rho^\sigma + (\partial_\nu\partial_\lambda u_\sigma^{\mu\nu\lambda})\zeta_\rho^\sigma \\
&\quad - (\partial_\nu u_\sigma^{\mu\nu\lambda})\,\partial_\lambda\zeta_\rho^\sigma + \cdots\cdots\Bigg] = 0. \quad (\rho = 1, 2, \cdots, s)
\end{aligned} \quad (19)$$

The reason we call them the (weak) conservative laws is that we have used the equation of motion of the system in obtaining (19). This method for obtaining the conservative laws is totally different from the first Noether theorem. The latter is based on the invariance of the action of a system under a finite continuous group. However, we study a system that is noninvariant under the local transformations. In the condition, for example, $u_\sigma^{\mu(j)}$ is symmetric for some superscripts as discussed previously, and the conservative movement quantities of the system can be derived from the generalized Noether identities. Thus, we present a new method for finding the conservative laws.

4. PBRS CONSERVATIVE CHARGES IN THE YANG-MILLS THEORY WITH HIGH-ORDER DERIVATIVES

The classical Lagrangian for the Yang-Mills field with second-order derivatives is:

$$\mathscr{L}_{YM} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \kappa D_{b\lambda}^a F^{b\mu\nu} D^{a\lambda} F^{e\mu\nu}, \quad (20)$$

where

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f_{bc}^a A_\mu^b A_\nu^c, \quad (21)$$

$$D_{b\mu}^a = \delta_b^a\partial_\mu + f_{bc}^a A_\mu^c, \quad (22)$$

where f^a_{bc} denote the construction constants of the nonabelian gauge group. $\mathcal{L}_{YM}$ is invariant under the following gauge transformation:

$$\delta A^a_\mu = D^a_{b\mu}\varepsilon^b(x)$$

where $\varepsilon^b(x)$ is an arbitrary function of space-time. The invariance of the Lagrangian $\mathcal{L}_{YM}$ under the local gauge transformation shows that there exist the proper constraints of the system in the phase space. All constraints are first-class. We have to choose gauge condition for quantizing the fields. In the Coulomb gauge, in terms of the Dirac theory for a constraint system and the quantized method of the path integral, we obtain the effective Lagrangian of the system from the generating functional of the Green function [10]:

$$\mathcal{L}_{eff} = \mathcal{L}_{YM} + \mathcal{L}_{gh} - \frac{1}{2\alpha_0}(\partial^i A^a_i)^2,$$
$$\mathcal{L}_{gh} = -\partial^i \bar{C}^a D^a_{bi} C^b, \tag{23}$$

where $\bar{C}^a$ and C^b are the ghost fields, and α_0 is a parameter. The effective Lagrangian $\mathcal{L}_{eff}$ is invariant under the following BRS transformation:

$$\begin{cases} \delta A^a_\mu = D^a_{b\mu} C^b \tau, \\ \delta C^a = \frac{1}{2} f^a_{bc} C^b C^c \tau, \\ \delta \bar{C}^a = -\frac{1}{\alpha_0}\partial^\mu A^a_\mu \tau \end{cases} \tag{24}$$

where τ is the Grassmann parameter. From the first Noether theorem, we have the conservative current in the Coulomb gauge condition along the movement orbit of the system:

$$\begin{aligned} J^\nu &= \frac{\partial \mathcal{L}_{eff}}{\partial A^a_{\mu,\nu}} D^a_{b\mu} C^b + \frac{\partial \mathcal{L}_{eff}}{\partial A^a_{\mu,\nu\rho}} \partial_\rho (D^a_{b\mu} C^b) - \partial_\rho \left(\frac{\partial \mathcal{L}_{eff}}{\partial A^a_{\mu,\nu\rho}}\right) D^a_{b\mu} C^b \\ &+ \frac{\partial \mathcal{L}_{eff}}{\partial C^a_{,\nu}} \delta C^a + \frac{\partial \mathcal{L}_{eff}}{\partial \bar{C}^a_{,\nu}} \delta \bar{C}^a \\ &= J^\nu_1 - \frac{1}{2}\partial^\nu \bar{C}^a f^a_{bc} C^b C^c + D^{a\nu}_b C^b \partial^0 A^a_0, \end{aligned} \tag{25}$$

where

$$\begin{aligned} J^\nu_1 = \frac{1}{\kappa^2}\{[f^m_{ca} A^{c\rho}\partial_\rho F^{m\mu\nu} + f^m_{acl} f^m_{fd} A^c_\lambda A^{\lambda f} F^{d\mu\nu} - \partial_\rho(\partial^\rho F^{a\mu\nu} + f^a_{cd} A^{\rho c} F^{d\mu\nu})] D^a_{b\mu} C^b \\ + (\partial^\rho F^{a\mu\nu} + f^a_{cd} A^{\rho c} F^{d\mu\nu})\partial_\rho (D^a_{b\mu} C^b)\} - F^{a\nu\mu} D^a_{b\mu} C^b, \end{aligned} \tag{26}$$

Therefore, the BRS conservative charge is

$$Q = \int_V d^3x J^0 = \int_V d^3x \left[J^0_1 - \frac{1}{2} f^a_{bc}\partial^0 \bar{C}^a C^b C^c + D^{a0}_b C^b \partial^0 A^a_0\right]. \tag{27}$$

Note that in the BRS transformation, the ghost fields are invariant. Only the Yang-Mills field transforms as follows:

$$\begin{cases}\delta A^a_\mu = D^a_{b\mu} C^b \tau, \\ \delta C^a = \delta \bar{C}^a = 0.\end{cases} \tag{28}$$

so that the effective Lagrangian (23) is noninvariant:

$$\begin{aligned}\delta \mathscr{L}_{\text{eff}} &= F(\theta) + u^\mu_a \partial_\mu \theta^a + u_a \partial^\mu \partial_\mu \theta^a \\ &= F(\theta) - \frac{1}{\alpha_0}(\partial^i A^a_i)\partial^\mu \partial_\mu \theta^a - \frac{1}{\alpha_0} f^a_{bc}(\partial^i A^b_i) A^c_j \partial^j \theta^a - f^a_{bc} \partial^\mu \bar{C}^c C^b \partial_\mu \theta^a,\end{aligned} \tag{29}$$

where $\theta^a = C^a\tau$, and $F(\theta)$ does not contain the derivative of θ^a. From (19) and (20), we obtain the conservative current in the gauge constraint (the Coulomb gauge) along the movement orbit of the system:

$$J^\nu_P = J^\nu_1 + f^a_{bc} \partial^\nu \bar{C}^c C^b C^a, \tag{30}$$

J^ν_P is called PBRS conservative current. The PBRS conservative charge is

$$Q_P = \int_V d^3x J^0_P = \int_V d^3x [J^0_1 + f^a_{bc} \partial^0 \bar{C}^c C^b C^a] \tag{31}$$

Obviously, the PBRS conservative charge is different from the BRS charge Q.

In the quantum theory, the BRS charge is used to choose the physical states. We found another new PBRS charge that gives an additional condition of the physical states. The role played by the PBRS charge in the quantization should be studied further.

5. NONLOCAL TRANSFORMATIONS AND CONSERVATIVE CHARGES

$\mathscr{L}_{\text{YM}}$ and $\mathscr{L}_{\text{gh}}$ in the effective Lagrangian (23) of the Yang-Mills field with second-order derivatives are invariant under the following transformation:

$$A^{a\prime}_\mu(x) = A^a_\mu(x) + D^a_{\sigma\mu}\varepsilon^\sigma(x), \tag{32a}$$

$$C^{a\prime}(x) = C^a(x) + ig(T_\sigma)^a_b C^b(x)\varepsilon^\sigma(x), \tag{32b}$$

$$\partial^\mu \bar{C}^{a\prime}(x) = \partial^\mu \bar{C}^a - ig\partial^\mu \bar{C}^b(x)(T_\sigma)^a_b \varepsilon^\sigma(x) \tag{32c}$$

where T_σ ($\sigma = 1, 2, \ldots, N$) are the representation matrices of the generators of the gauge group, and $\varepsilon^\sigma(x)$ are the infinitesimal arbitrary functions. (32c) can be rewritten as follows:

$$\bar{C}^{a\prime}(x) = \bar{C}^a(x) - ig\bar{C}^b(x)(T_\sigma)^a_b \varepsilon^\sigma(x) + \frac{ig}{\Box}\partial_\mu[\bar{C}^b(x)(T_\sigma)^a_b \partial^\mu \varepsilon^\sigma(x)], \tag{32c'}$$

where $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$. (32c′) may be rewritten as follows [4]:

$$\bar{C}^{a\prime}(x) = \bar{C}^a(x) - ig\bar{C}^b(x)(T_\sigma)^a_b \varepsilon^\sigma(x) + ig\int d^4y \Delta_0(x,y)\partial_\mu[\bar{C}^b(y)(T_\sigma)^a_b \partial^\mu \varepsilon^\sigma(y)], \tag{32c''}$$

where

$$\Box\Delta_0(x,y) = i\delta^{(4)}(x-y). \tag{33}$$

(32c″) is a nonlocal transformation. Under the transformations (32a), (32b), and (32c″) we obtain from the generalized identities (13) and the effective Lagrangian (23):

$$\widetilde{D}^a_{\rho\mu}\left(\frac{\delta I_{\mathrm{eff}}}{\delta A^a_\mu(z)}\right) + ig(T_\rho)^a_b\frac{\delta I_{\mathrm{eff}}}{\delta C^a(z)}C^b(z) - ig\bar{C}^b(z)(T_\rho)^a_b\frac{\delta I_{\mathrm{eff}}}{\delta\bar{C}^a(z)}$$
$$+\int_D d^4x\widetilde{B}^a_\rho(z)\left[\partial_\mu\left(\frac{\partial\mathscr{L}_{\mathrm{eff}}}{\partial\bar{C}^a_{,\mu}}\right)\Delta_0(x,z)\right] = \frac{1}{\alpha_0}\widetilde{D}^a_{\rho i}\partial^i(\partial^k A^a_k), \tag{34}$$

where

$$\widetilde{D}^a_{\rho\mu} = -\delta^a_\rho\partial_\mu + f^a_{\rho c}A^c_\mu, \tag{35}$$

$$B^a_\rho(y) = ig\partial_\mu[\bar{C}^b(y)(T_\rho)^a_b\partial^\mu]. \tag{36}$$

In the gauge constraint (the Coulomb gauge) and along the movement orbit of the system we have from (34):

$$\partial^{\mu_z}\int_D d^4x\bar{C}^b(z)(T_\rho)^a_b\partial_{\mu_z}\left[\partial_{\nu_z}\left(\frac{\partial\mathscr{L}_{\mathrm{eff}}}{\partial\bar{C}^a_{,\nu_z}}\right)\Delta_0(x,z)\right] = 0, \tag{37}$$

and the conservative charge:

$$Q' = \int_V d^3z\int_D d^4x\bar{C}^b(z)(T_\rho)^a_b\partial_{0z}\left[\partial_{\nu_z}\left(\frac{\partial\mathscr{L}_{\mathrm{eff}}}{\partial\bar{C}^a_{,\nu_z}}\right)\Delta_0(x,z)\right] = \mathrm{const}. \tag{38}$$

Substituting (23) into (38) we obtain the conservative charge:

$$Q' = \int_V\int_D d^3z d^4x\bar{C}^b(z)(T_\rho)^a_b(\partial_\nu D^{a\nu}_e C^e)\partial_{0z}\Delta_0(x,z) = \mathrm{const}. \tag{39}$$

REFERENCES

[1] Noether, E., *Nachr. Ges. Wiss. Gottingen, Math.-Phys. Kl.*, **2**(1918): p. 235.

[2] Li Ziping, *Classical and Quantum Constraint Systems and Their Symmetric Properties*. Beijing: Beijing Polytechnic University Press, 1993.

[3] Li Ziping, *High Energy Phys. and Nucl. Phys.* (Chinese Edition), **12**(1988): p. 784.

[4] Kuang Yuping and Yi Yuping, *High Energy Phys. and Nucl. Phys.* (Chinese Edition), **4**(1980): p. 286.

[5] Fradkin, E. S., and Palchik, M. Ya., *Phys. Lett.*, **147B**(1984): p. 86.

[6] Palchik, M. Ya., *Yad. Fiz.*, **42**(1985): p. 522.

[7] Li Ziping, *Scientia Sinica*, **A9**(1992): p. 977.

[8] Lusanna, L., *Riv. Nuovo Cimento*, **14**(1991): p. 1.

[9] Li Ziping, *Int. J. Theor. Phys.*, **26**(1987): p. 853.

[10] Gitman, D. M., and Tyutin, I. V., *Quantization of Fields with Constraints*, Berlin: Springer-Verlag, 1990.

[11] Li Ziping, *J. Phys., A: Math. Gen.*, **24**(1991): p. 4261.